Gamow-Teller and Spin-Dipole Resonances and Experimental Methods for Spin Excitations

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Giant resonances and sum rule

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What is a Resonance?

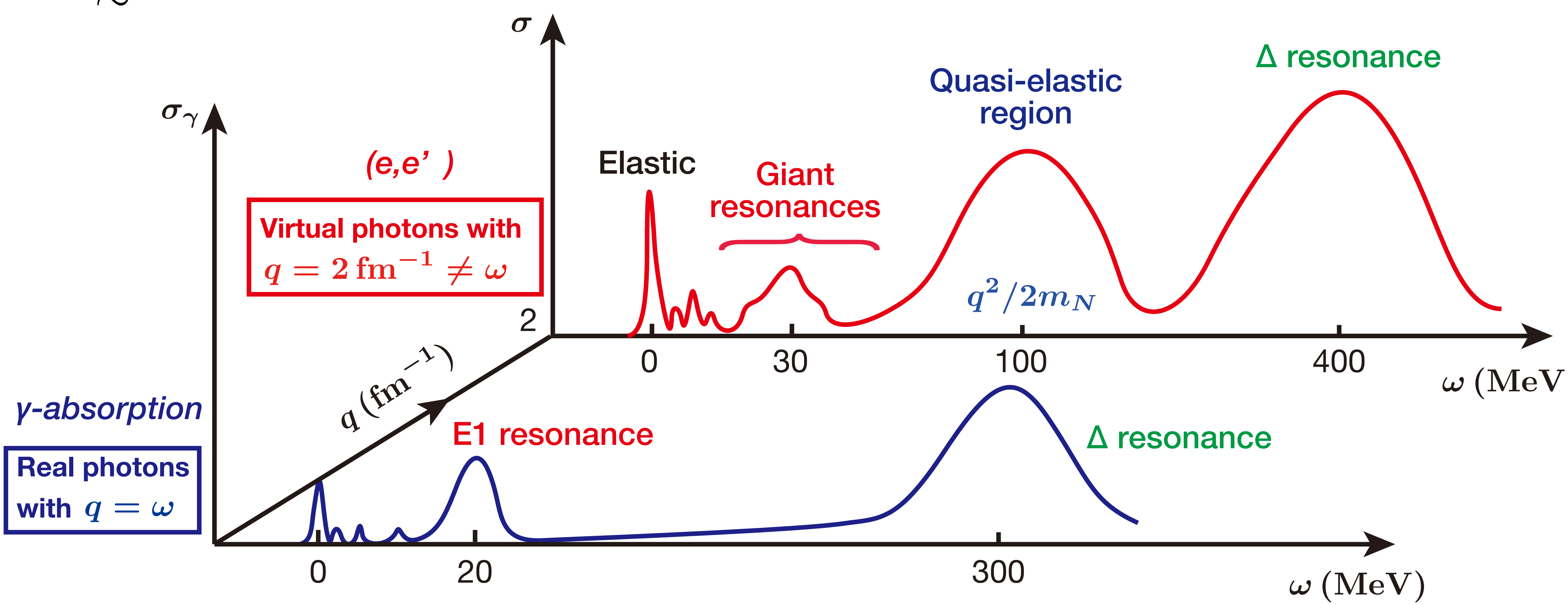
M.N.Harakeh and A. van der Wunde, "Giant Resonances".

A powerful method to study the properties of a system

= Measure its response to the external perturbation/impact

Nuclear responses to real/virtual photons

- $\omega \lesssim 10\,\mathrm{MeV}$: Excitations including one or a few particles
- $\omega \simeq 10-30\,\mathrm{MeV}$: Broad resonances involving many particles
- $\omega \simeq 100\,\mathrm{MeV}$ for (e,e') and (p,n) : Quasi-elastic scattering (scattering with a target nucleon)
- $\omega \gtrsim 300\,\mathrm{MeV}$: Δ resonance due to nucleon excitation



What is a Resonance

Resonance = Fundamental modes of nuclear vibration (in macroscopic view).

- shape oscillation (compression, dipole, …)
- spin oscillation (out-of-phase oscillation between ↑ and ↓)
- isospin oscillation (out-of-phase oscillation between p and n)

Resonances can be classified by:

multipolarity L

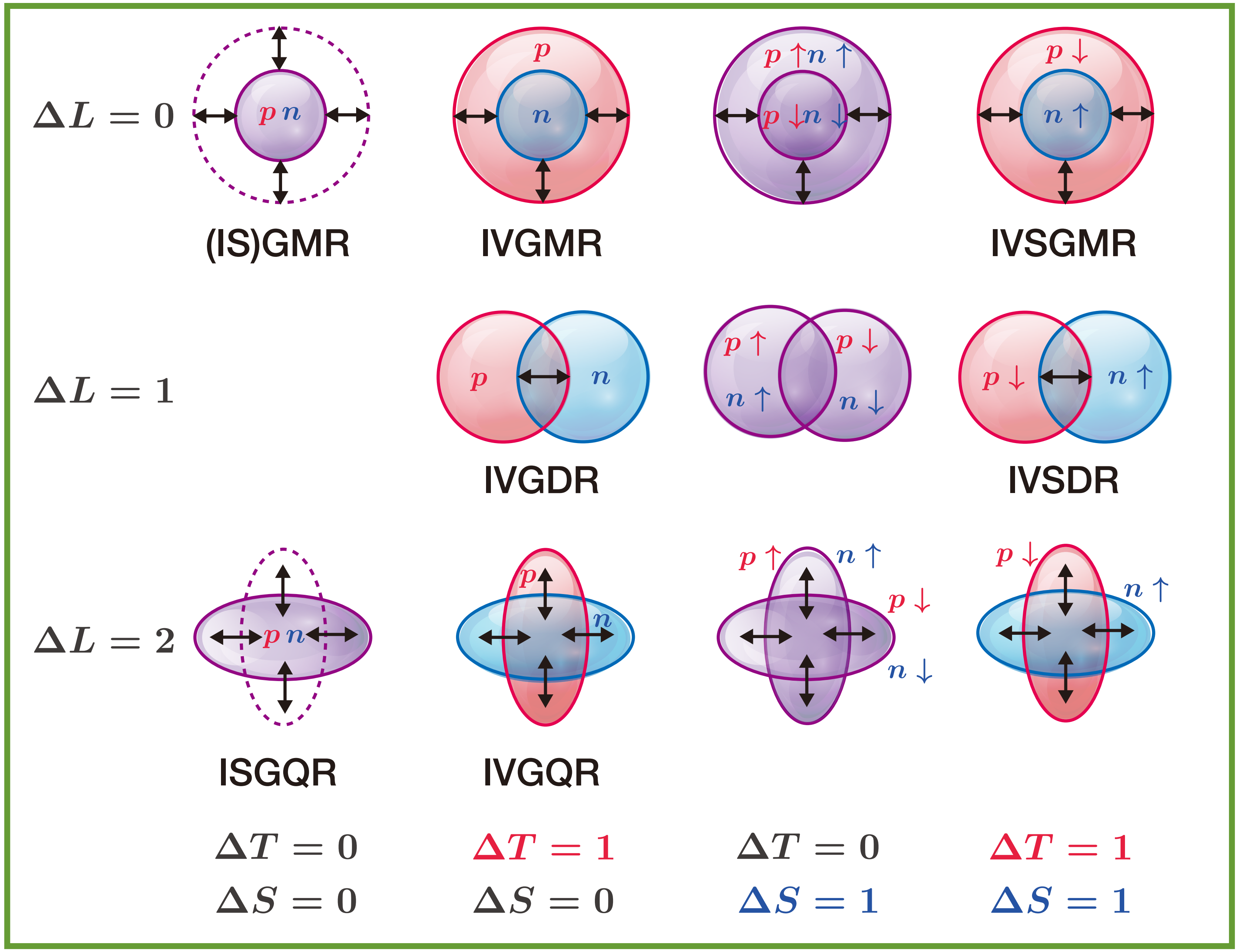
- ΔL=0 : monopole
- ΔL=1 : dipole
- ΔL=2 : quadrupole

spin S

- ΔS=0 : spin-scalar
(in-phase osc. b/w ↑ and ↓)
- ΔS=1 : spin-vector
(out-of-phase osc. b/w ↑ and ↓)

isospin T

- ΔT=0 : iso-scalar (IS)
(in-phase osc. b/w p and n)
- ΔT=1 : iso-vector (IV)
(out-of-phase osc. b/w p and n)



M.N.Harakeh and A. van der Wunde, “Giant Resonances”.

Graphical images of Resonances

Animations: taken from presentation by T. Aumann @ INPC2007

Electric giant resonance (spin transfer ΔS=0)

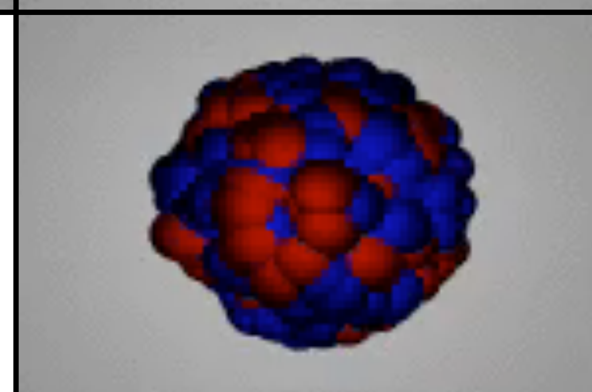
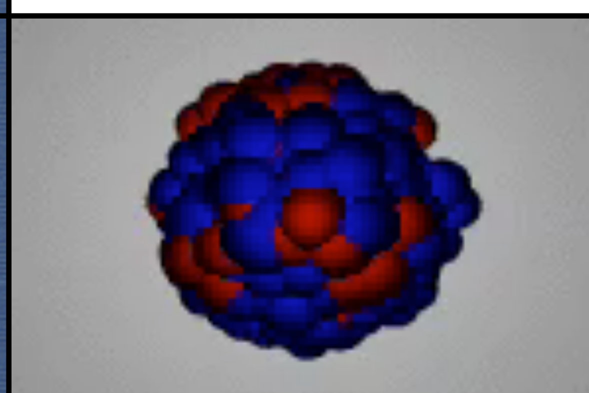
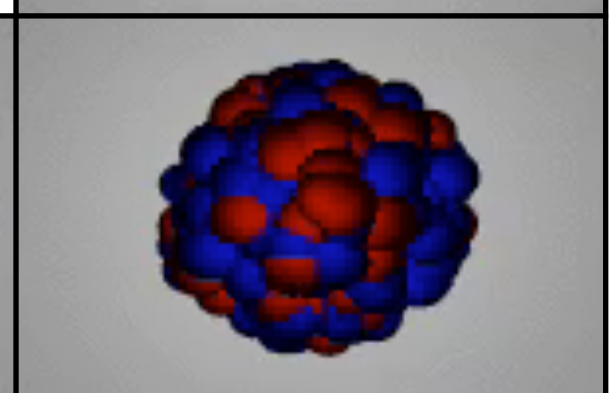
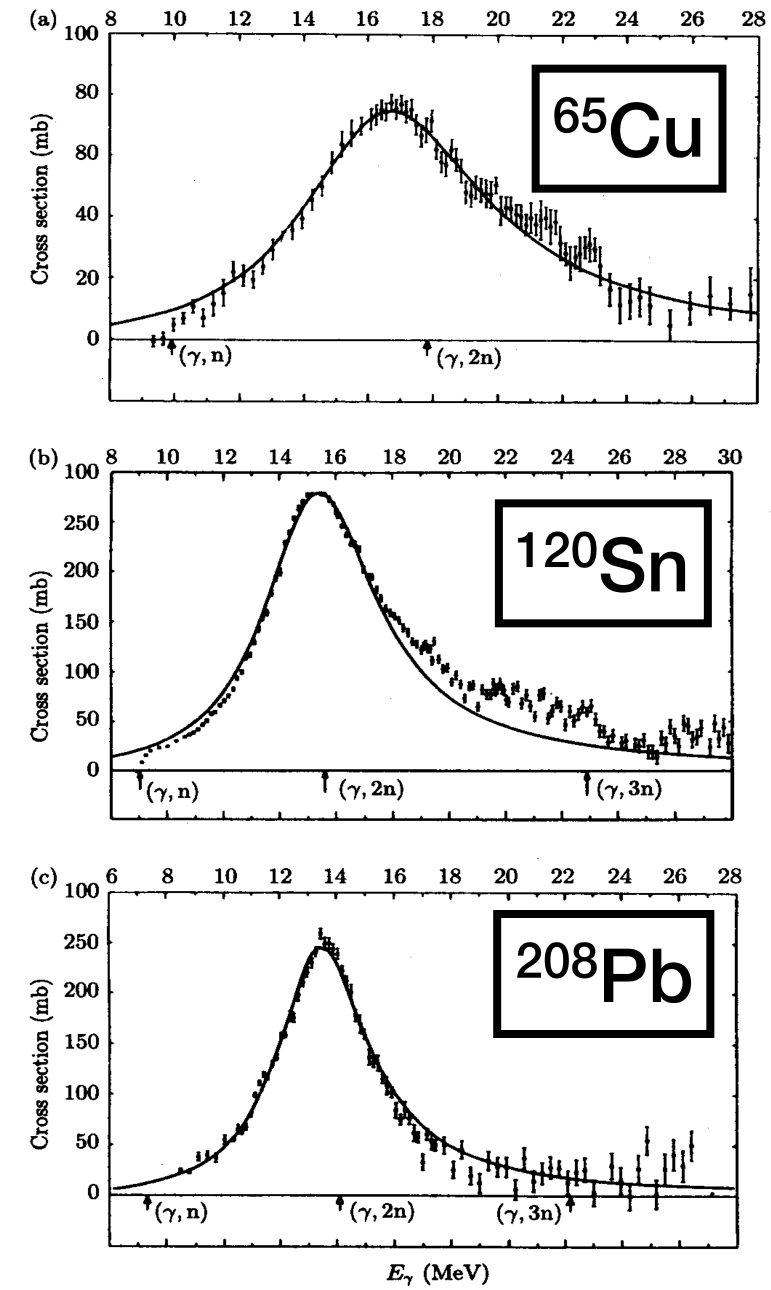
	Isoscalar $\Delta T = 0$	Isovector $\Delta T = 1$
Monopole (GMR) $\Delta L = 0$		
Dipole (GDR) $\Delta L = 1$		
Quadrupole (GQR) $\Delta L = 2$		

Photo-neutron c.s.



In-Phase/Out-phase changes of spatial w.f of neutrons and protons for isoscalar/isovector GR

B.L. Berman and S.C. Fultz, RMP 47,713(1975).

Quantum mechanical description of Resonances

Resonance = collective motion involving many if not all the particles in the nucleus

In quantum mechanics:

- The resonance is a transition between *the ground state* and *the collective state*
- Its strength is described by a transition amplitude

The transition strength depends on the basic properties of the system:

- *The number of particles* participating in the transition } *of the ground state*
- *The size of the system*

The total transition strength should be limited by *a sum rule*:

- The sum rule depends *only on ground-state properties*

The "Giant" resonance and the sum rule:

- A giant resonance (GT) = resonance exhausting a major part of the sum rule
  - Typically more than 50%

What is the specific form of the sum rule (E1, GT, etc.)?

Sum rule

Sum-rule in quantum mechanics

Assume that the Hamiltonian $\hat{H}$ has a complete set of:

- eigenfunctions $|n\rangle$ with
- eigenvalues E_n

For the Hermitian operator $\hat{A}$, we define the operator (commutator) $\hat{C}$:

$$\hat{C} \equiv [\hat{H}, \hat{A}] = \hat{H}\hat{A} - \hat{A}\hat{H}$$

- The operator $\hat{C}$ is anti-Hermitian:

$$\hat{C}^\dagger = (\hat{H}\hat{A})^\dagger - (\hat{A}\hat{H})^\dagger = \hat{A}\hat{H} - \hat{H}\hat{A} = -\hat{C}$$

For $\hat{C}$, we find:

$$\begin{aligned} \langle n|\hat{C}|m\rangle &= \langle n|(\hat{H}\hat{A} - \hat{A}\hat{H})|m\rangle \\ &= \langle n|(E_n\hat{A} - \hat{A}E_m)|m\rangle \quad (\because \hat{H}\,|m\rangle = E_m\,|m\rangle\,,\ \hat{H}\,|n\rangle = E_n\,|n\rangle) \\ &= (E_n - E_m)\langle n|\hat{A}|m\rangle \end{aligned}$$

and also:

$$|\langle n|\hat{A}|m\rangle|^2 = \langle n|\hat{A}|m\rangle^*\langle n|\hat{A}|m\rangle = \langle m|\hat{A}|n\rangle\langle n|\hat{A}|m\rangle \quad (\because\ \hat{A}^\dagger = \hat{A})$$

Sum-rule in quantum mechanics

$$\left.\begin{aligned} \langle n|\hat{C}|m\rangle &= (E_n - E_m)\langle n|\hat{A}|m\rangle \\ |\langle n|\hat{A}|m\rangle|^2 &= \langle m|\hat{A}|n\rangle\langle n|\hat{A}|m\rangle \end{aligned}\right\} \cdots (\bigstar)$$

Using these relations, we can derive:

$$\langle m|[\hat{A},\hat{C}]|m\rangle$$

$$= \langle m|\hat{A}\hat{C}|m\rangle - \langle m|\hat{C}\hat{A}|m\rangle$$

$$= \sum_n \left[\langle m|\hat{A}|n\rangle\langle n|\hat{C}|m\rangle - \langle m|\hat{C}|n\rangle\langle n|\hat{A}|m\rangle\right] \qquad (\because \sum_n |n\rangle\langle n| = 1)$$

$$= \sum_n \left[\langle m|\hat{A}|n\rangle\langle n|\hat{A}|m\rangle(E_n - E_m) - (E_m - E_n)\langle m|\hat{A}|n\rangle\langle n|\hat{A}|m\rangle\right] \qquad (\because \bigstar)$$

$$= 2\sum_n (E_n - E_m)|\langle n|\hat{A}|m\rangle|^2$$

Since $\hat{C} \equiv [\hat{H},\hat{A}]$ **, we find:**

$$\langle m|[\hat{A},[\hat{H},\hat{A}]]|m\rangle = 2\sum_n (E_n - E_m)|\langle n|\hat{A}|m\rangle|^2$$

Sum-rule in quantum mechanics

$$\langle m|[\hat{A},[\hat{H},\hat{A}]]|m\rangle = 2\sum_n (E_n - E_m)|\langle n|\hat{A}|m\rangle|^2$$

This relation can be also described as:

$$\langle m|[\hat{A},[\hat{H},\hat{A}]]|m\rangle = 2\langle m|\hat{A}(\hat{H}-E_m)\hat{A}|m\rangle$$

since

$$\hat{H}|n\rangle = E_n|n\rangle$$

and the completeness relation:

$$\sum_n |n\rangle\langle n| = 1$$

GDR (TRK) sum rule

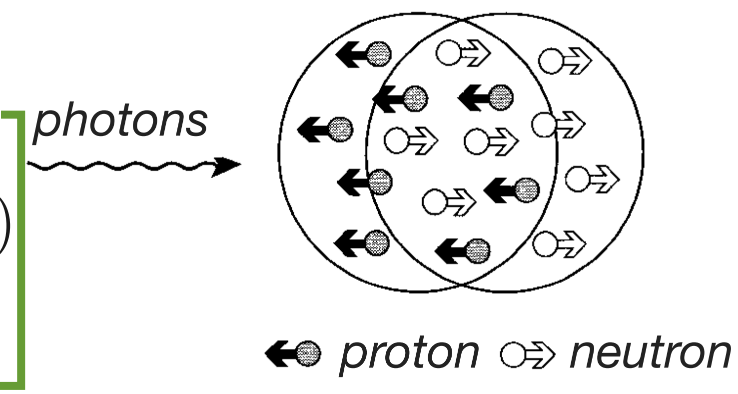
The Thomas-Reich-Kuhn (TRK) sum rule shows:

For the E1 operator:

$$\hat{O}(\mathrm{E1}) = -\sqrt{\frac{3}{4\pi}} \sum_{i=1}^{A} t_3(i) z_i$$

Nuclear physics convention

$$\begin{cases} t_3(i) = +\frac{1}{2} \ (\text{neutron}) \\ t_3(i) = -\frac{1}{2} \ (\text{proton}) \end{cases}$$



The energy-weighted sum of the response function (transition strength):

$$R_{\mathrm{E1}}(\omega) = \sum_n |\langle n | \hat{O}(\mathrm{E1}) | 0 \rangle|^2 \delta(\omega - (E_n - E_0))$$

should be equal to the sum rule value

$$S_1(\mathrm{E1}) \equiv \int_0^{\infty} R_{\mathrm{E1}}(\omega) \omega \, d\omega = \frac{1}{2} \langle 0 | [\hat{O}(\mathrm{E1}), [\hat{H}, \hat{O}(\mathrm{E1})]] | 0 \rangle = \boxed{\frac{3A\hbar^2}{32\pi M}}$$

irrespective to the potential V (interaction)

- TRK sum rule = model-independent sum rule

Exercise: Derive the TRK sum rule referring Appendix B of this lecture.

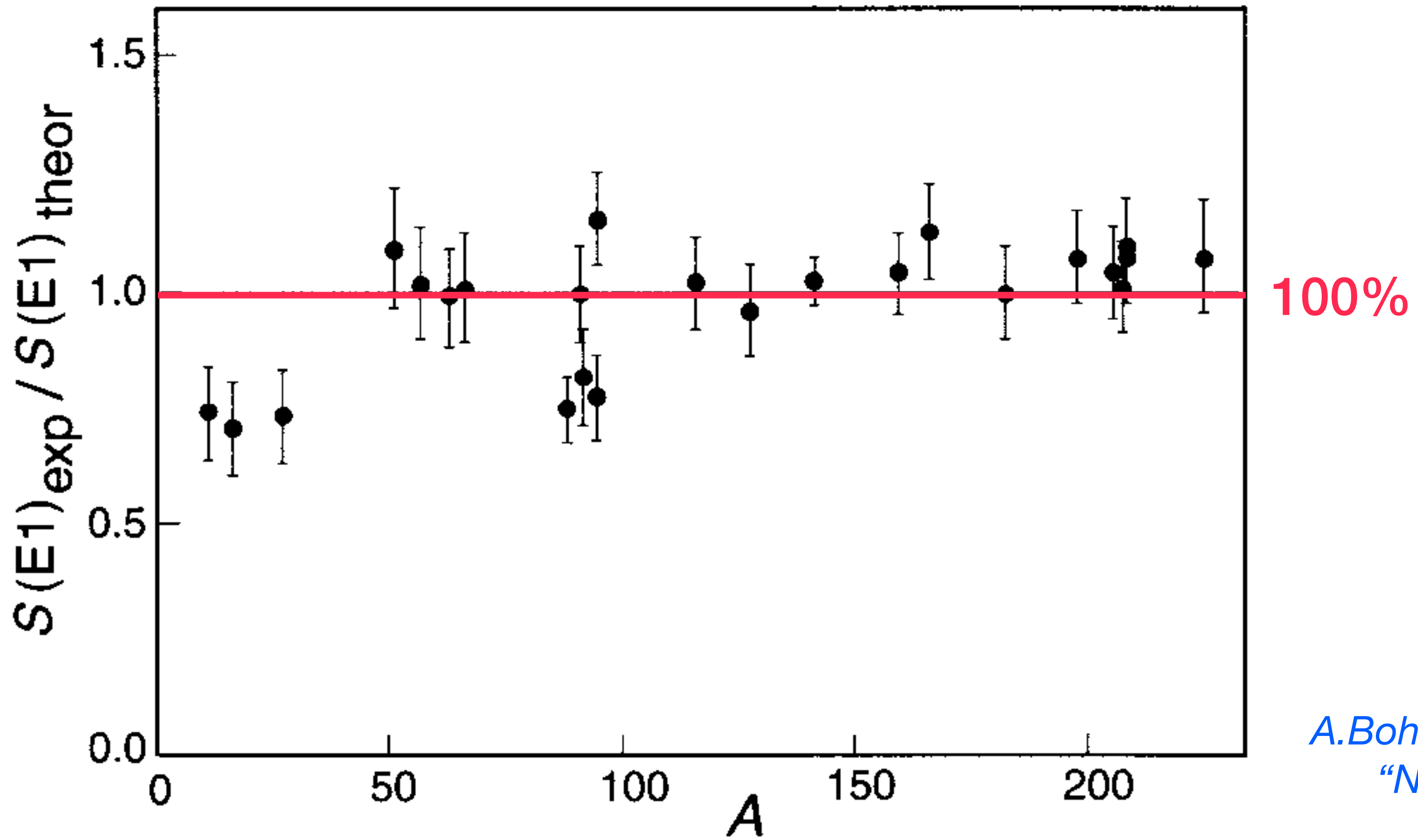
Giant resonance

- A resonance which exhausts a major part of the sum-rule strength (> 50%).
  → Observed GDR's exhaust their sum-rule strengths?

Experimental evaluation of TRK sum rule

Ratios of the experimental $S_1(E1)$ up to 30 MeV (=GDR) and the TRK sum rule:

- The GDR strength corresponds to ～100% of the TRK sum rule.
- The GDR is a collective nuclear vibration in which all the protons move collectively against all the neutrons.



A.Bohr and B.R.Mottelson, "Nuclear Structure"

In general, there is the appropriate sum rule depending on ΔL, ΔS, and ΔT.
✤ If the total strength in a limited energy region (< E) does not satisfy the sum rule.
→ Further strengths exist beyond E.

What can we learn from GRs?

For each mode specified by ΔL, ΔS, and ΔT, the relevant sum-rule exists:

- e.g., E1 (TRK) sum rule for ΔL=1 (dipole), ΔT=1 (isovector), and ΔS=0
- A giant resonance (GT) = resonance exhausting a major part of the sum rule
    - Typically more than 50%

If the total strengths including the GR do not exhaust the sum rule:

- Missing strengths should exists beyond the GR excitation energy
- Some basic assumptions for the sum-rule might be wrong
    - e.g., a nuclei consists of point-like protons and neutrons

What can we lean from the strength and position of GR

- peak position
- fraction to the sum-rule

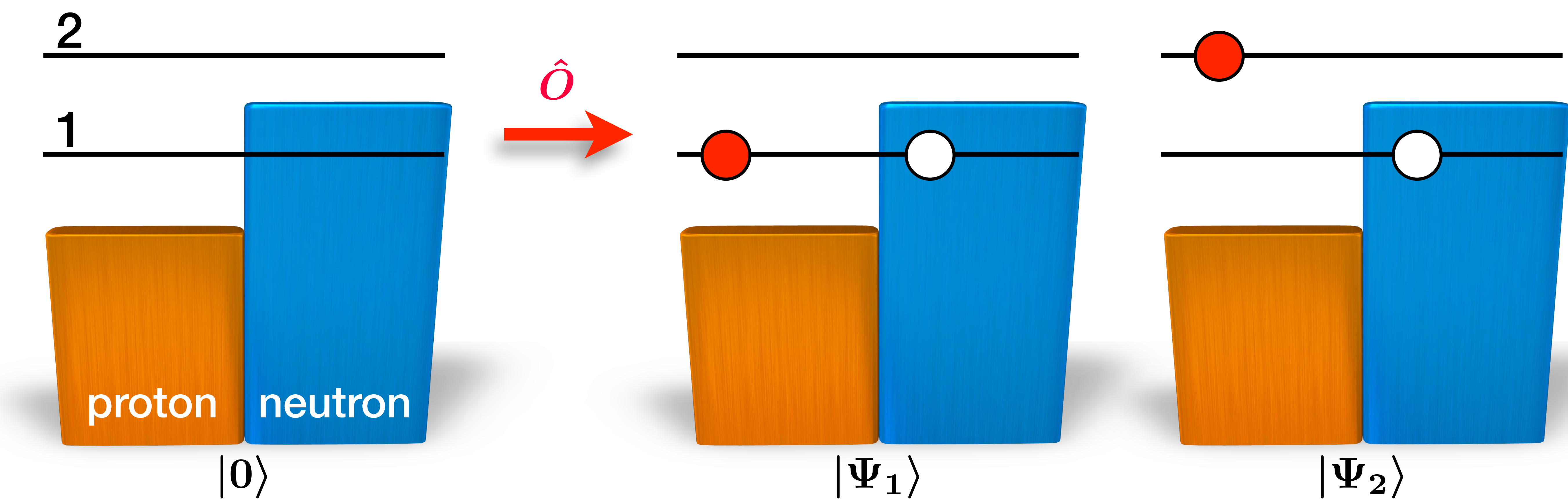
} *Both depend on the residual interaction in nuclei*

The residual interaction dependence can be understood easily

Simple 2-states model w/o residual int.

G.E.Brown, "Unified theory of nuclear models and forces"
K.Yako, Private communication.

Excite two p-h states via $\hat{O}$ (neutron → proton)



Hamiltonian of daughter nucleus w/o residual interaction

- $H = H_0$ → $\begin{cases} H_0|\Psi_1\rangle = \varepsilon_1|\Psi_1\rangle \\ H_0|\Psi_2\rangle = \varepsilon_2|\Psi_2\rangle \end{cases}$

$|\Psi_1\rangle$ and $|\Psi_2\rangle$ are orthogonal

Transition matrixes via $\hat{O}$ operator

- to state 1 : $\langle\Psi_1|\hat{O}|0\rangle \equiv X_1$
- to state 2 : $\langle\Psi_2|\hat{O}|0\rangle \equiv X_2$

Transition strengths (probabilities) become

$$|X_1|^2 = |X_2|^2$$

(forgetting about CG coefficients, neutron excess, etc.)

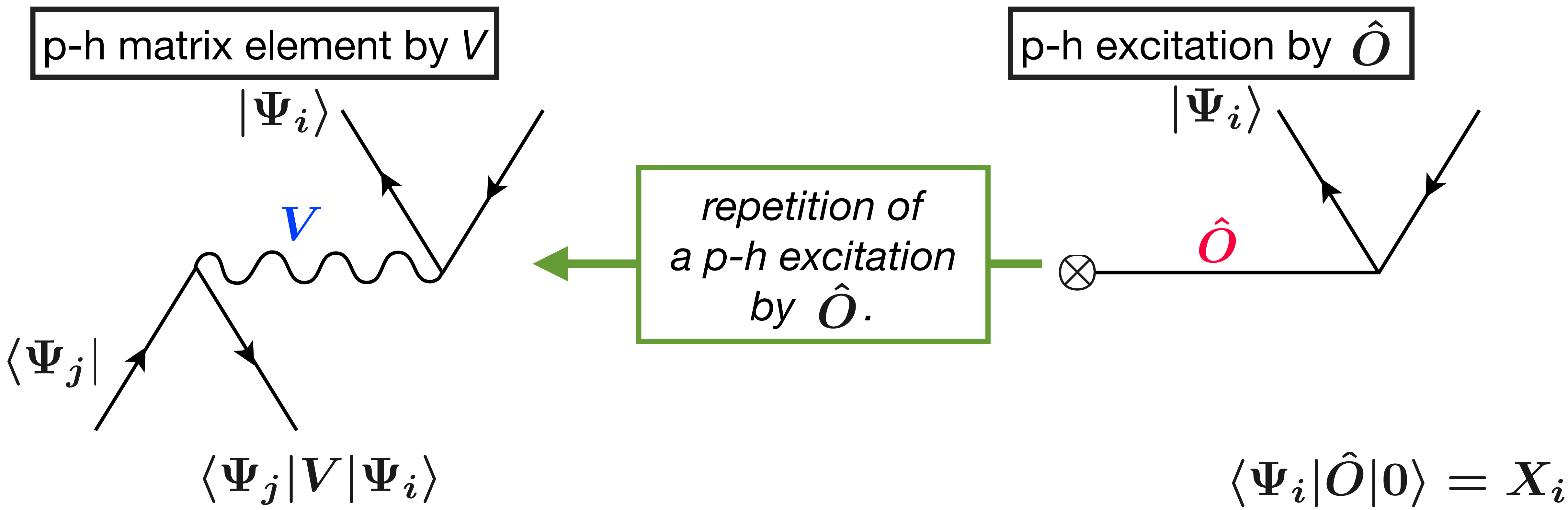
No GR w/o residual interaction ($|\Psi_1\rangle$ and $|\Psi_2\rangle$ are equally excited)

Simple 2-states model with residual int.

Add residual interaction: V

- Hamiltonian and Schrödinger eq.: $H = H_0 + V$ $\quad (H_0 + V)|\Psi\rangle = E|\Psi\rangle$
- Eigenstate: $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$ ← Mixing $|\Psi_1\rangle$ and $|\Psi_2\rangle$ with V

Similarity between residual interaction and p-h excitation by $\hat{O}$



Since the p-h matrix element by V is a repetition of a p-h excitation by $\hat{O}$, the matrix element can be expressed as:

$$\langle\Psi_j|V|\Psi_i\rangle \simeq \lambda X_i X_j$$ (λ : strength of the residual interaction)

Simple 2-states model with residual int.

- Hamiltonian and Schrödinger eq.: $H = H_0 + V \qquad (H_0 + V)|\Psi\rangle = E|\Psi\rangle$
- Eigenstate: $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$
- Matrix elements : $\langle\Psi_j|V|\Psi_i\rangle \simeq \lambda X_i X_j$

The secular equation/problem becomes

$$\begin{pmatrix} \varepsilon_1 + \lambda X_1^2 & \lambda X_1 X_2 \\ \lambda X_1 X_2 & \varepsilon_2 + \lambda X_2^2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = E \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \varepsilon_1 + \lambda X_1^2 - E & \lambda X_1 X_2 \\ \lambda X_1 X_2 & \varepsilon_2 + \lambda X_2^2 - E \end{pmatrix}}_{M} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- $\det M = 0$ for $(c_1, c_2) \neq (0, 0)$

$$\frac{1}{\lambda} = \frac{X_1^2}{E - \varepsilon_1} + \frac{X_2^2}{E - \varepsilon_2}$$

Solution

Assumption for simplicity

- $\varepsilon_1 = \varepsilon_2 \equiv \varepsilon_0$(two states are degenerate)
- $\lambda > 0$ (repulsive)

$$\frac{1}{\lambda} = \frac{X_1^2}{E - \varepsilon_1} + \frac{X_2^2}{E - \varepsilon_2}$$

Solution #1 (Low-lying state)

- $E = \varepsilon_0$ (not changed)
- $c_1 X_1 + c_2 X_2 = 0$

Transition matrix: $\mathcal{D} \equiv \langle \Psi | \hat{O} | 0 \rangle = c_1 X_1 + c_2 X_2 = 0$

Transition probability : $\mathcal{D}^2 = 0$ (zero probability)

Solution #2 (High-lying collective state)

- $E = \epsilon_0 + \lambda(X_1^2 + X_2^2)$ (shifted to higher energy by $\lambda(X_1^2 + X_2^2)$)
- $c_2 X_1 = c_1 X_2$

Transition matrix : $\mathcal{D} \equiv \langle \Psi | \hat{O} | 0 \rangle = c_1 X_1 + c_2 X_2 = \sqrt{X_1^2 + X_2^2}$

Transition probability : $\mathcal{D}^2 = X_1^2 + X_2^2$ (sum of all the transition probabilities)

Both $|\Psi_1\rangle$ and $|\Psi_2\rangle$ contribute constructively. → "Coherent"

Summary of simple model

Inputs

- Structure: two “**unperturbed**” states
- Interaction: “**repulsive**” residual interaction λ

Outputs

Low-lying state

- Similar excitation energy
- Almost zero strength

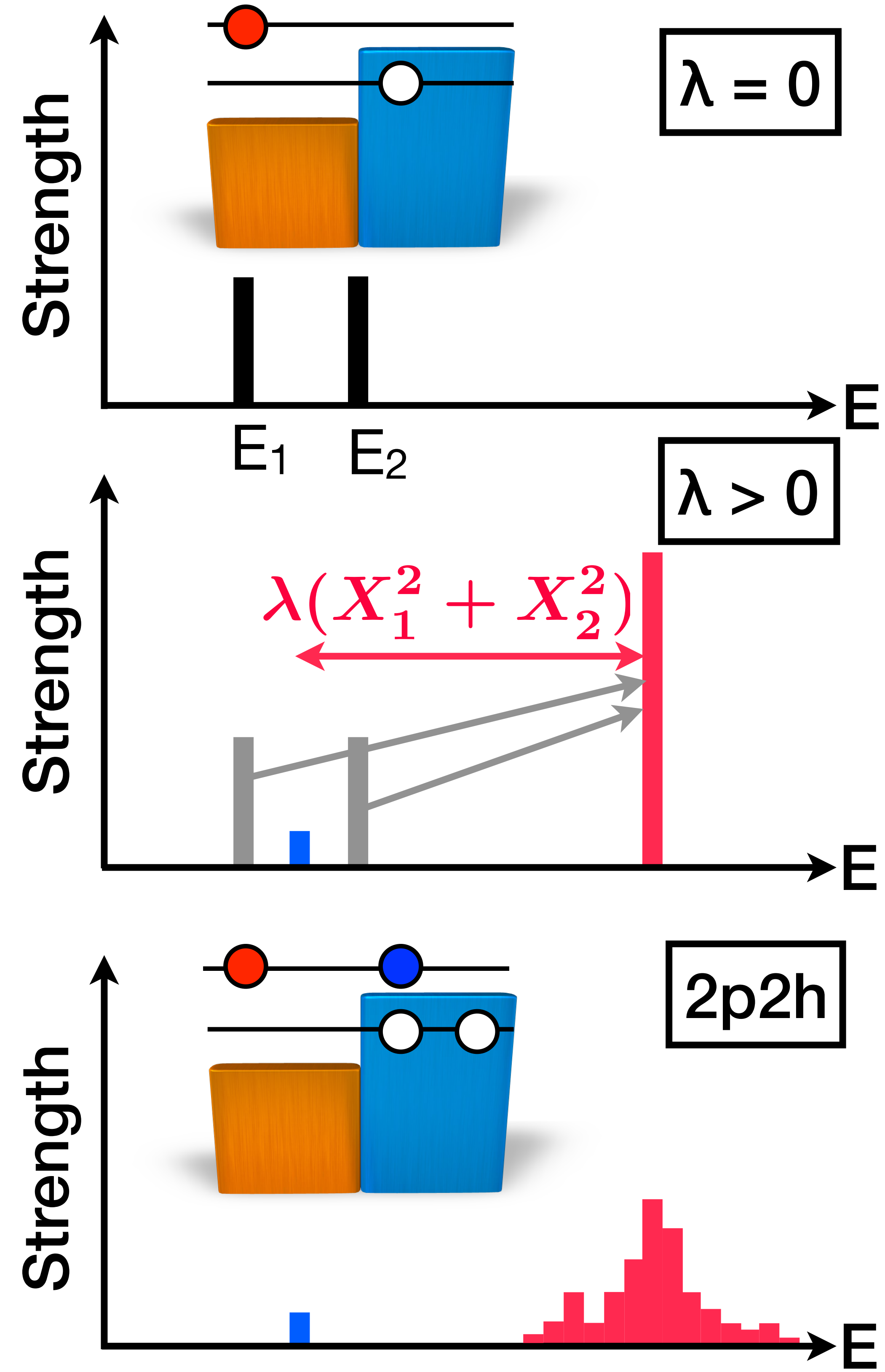
High-lying state

- Higher excitation energy by $\lambda(X_1^2 + X_2^2)$
- Almost all strength (collective state)
- Oscillating between $|\Psi_1\rangle$ and $|\Psi_2\rangle$

Real width of GR

Coupling with more complicated states (2p2h)

- Fragmentation of strength



The Landau-Migdal interaction

As an effective interaction V (λ), the Landau-Migdal interaction V_{LM} is often used

$$V_{\rm LM} = C_0 \left[f_0 + \underbrace{f_0'(\tau_1\cdot\tau_2)}_{} + g_0(\sigma_1\cdot\sigma_2) + \underbrace{g_0'(\sigma_1\cdot\sigma_2)(\tau_1\cdot\tau_2)}_{} \right]$$

$\Delta T = 0$	$\Delta T = 1$	$\Delta T = 0$	$\Delta T = 1$
$\Delta S = 0$	$\Delta S = 0$	$\Delta S = 1$	$\Delta S = 1$

For isovector ΔT=1 excitations, the following two interactions contribute:

- spin-scalar (ΔS=0) : $V_{\rm LM}^{\tau} = C_0 f_0'(\tau_1\cdot\tau_2)$
- spin-vector (ΔS=1) : $V_{\rm LM}^{\sigma\tau} = C_0 g_0'(\tau_1\cdot\tau_2)(\sigma_1\cdot\sigma_2)$

There are several choices for the strength C_0:

- pionic unit : $C_0 = \dfrac{f_{\pi NN}^2}{m_\pi^2} \simeq 400\,{\rm MeV\,fm^3}$
- Julich unit : $C_0 = \dfrac{1}{2}\dfrac{2\pi^2}{m^* k_F} \simeq 300\dfrac{m_N}{m^*}\,{\rm MeV\,fm^3}$
- Osterfeld, etc. : $C_0 = \dfrac{1}{4}\dfrac{2\pi^2}{m^* k_F} \simeq 150\dfrac{m_N}{m^*}\,{\rm MeV\,fm^3}$

$f_{\pi NN}$	*: πNN coupling const.*
m^*	*: effective nucleon mass*
k_F	*: Fermi momentum*

In the following, we set $f_0' \equiv f'$ *and* $g_0' \equiv g'$ *for simplicity.*

A.B.Migdal, "Theory of Finite Systems and Application to Atomic Nuclei" (1967).

Landau-Migdal parameter *f'* and GDR

Theoretical calculations

- respq by Ichimura-san
- available from RIKEN Nishina HP

Without residual interaction (f'=0)

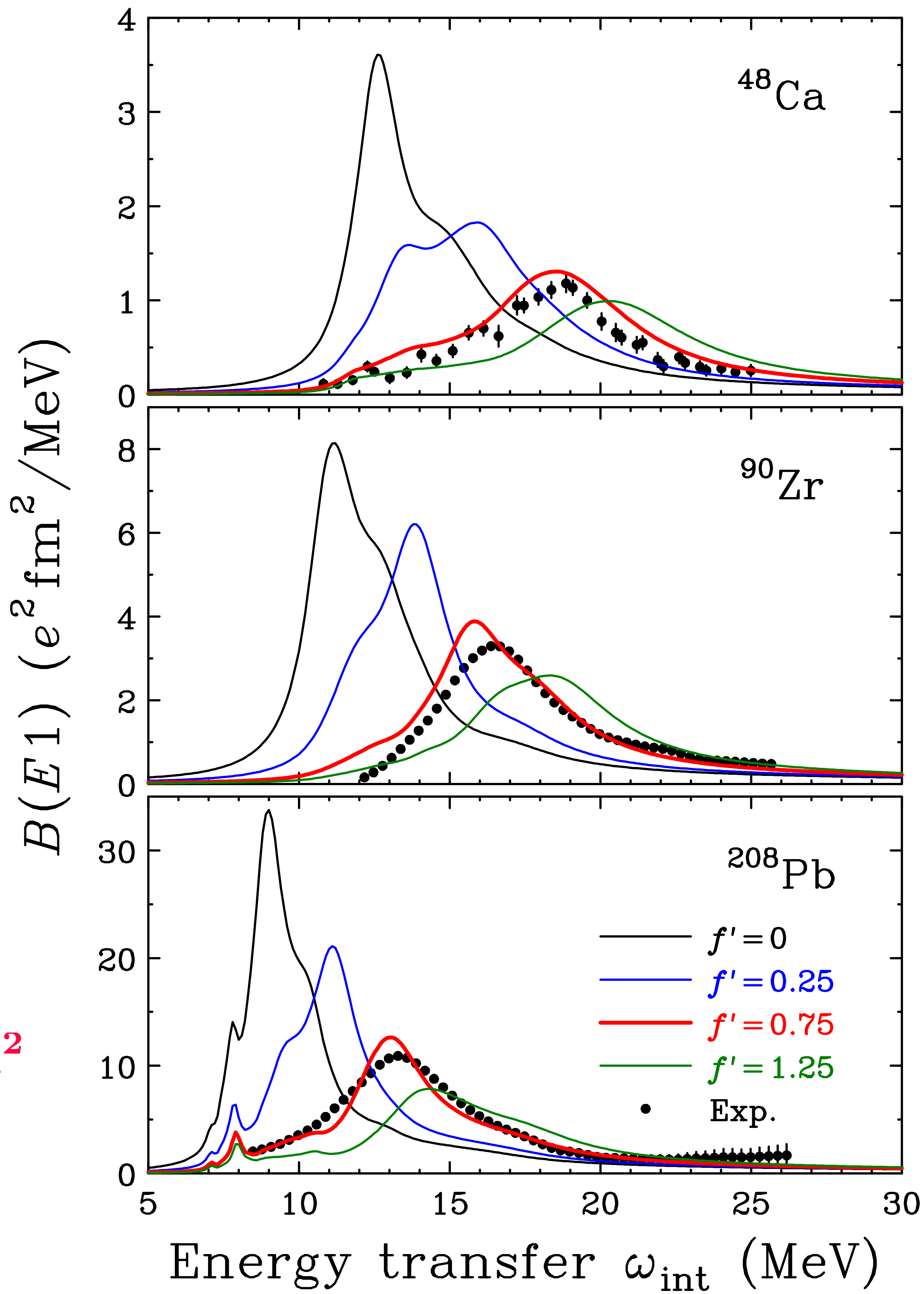
- Many $\Delta J^{\pi}=1^{-}$ 1p-1h states
- Significant widths
- Significantly lower than the exp. data

With residual interaction (f'>0)

- Strengths concentrate to the high-ω state
- The peak also shifts to higher ω
- A relatively strong ***f'=0.75*** (repulsive) reproduces the exp. data reasonably well

$$f' = 0.75 \longrightarrow V^{\tau} \simeq 300(\tau_1 \cdot \tau_2)\,\mathrm{MeV\ fm^2}$$

GR distributions provide important information on the interaction



Fermi and Gamow-Teller transitions

The GDR is a isovector (ΔT=1) multipole (ΔL=1) mode:

- Dipole mode with ΔL=1 and ΔS=0 ($\Delta J^{\pi}=1^{-}$)
- Dipole oscillation in the nuclear shape (anti-phase oscillations between p and n)

Here we concentrate on the “simplest” isovector (ΔT=1) modes:

Simple = no change in the nuclear shape

- No-change in angular momentum (ΔL=0)
- Experimentally, dominant at q=0 for ΔL=0

Spin-vector mode with ΔS=1 ($\Delta J^{\pi}=1^{+}$)

- *Gamow-Teller (GT) by (p,n), etc.*
- Magnetic dipole (M1) by (p,p’)

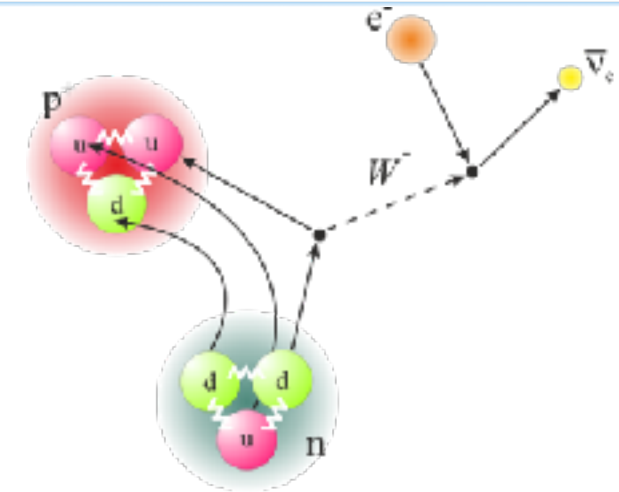
Spin-scalar mode with ΔS=0 ($\Delta J^{\pi}=0^{+}$)

- *Fermi (F) by (p,n), etc.*

Both Fermi and Gamow-Teller transitions/modes are closely related to beta decays
→ *Briefly overview the delay/quenching problem for beta decays*

Beta decays

Beta decays and electron capture (EC) are symbolically written as:



- β⁻ decay : $n \rightarrow p + e^- + \bar{\nu}_e$
- β⁺ decay : $p \rightarrow n + e^+ + \nu_e$
- EC : $e^- + p \rightarrow n + \nu_e$

The orbital angular momentum, L, carried away by the leptons is small ($L \ll 1\hbar$

- $L = 0$ for allowed transitions

Since leptons, e and ν_e, have spin $\frac{1}{2}\hbar$:

The total spin S of the leptons (= spin change b/w initial and final states) is 0 or $1\hbar$:

- S=0 ($\Delta J^\pi = 0^+$) → Fermi : $\hat{O}(\mathrm{F}^\pm) = g_V \sum_k t_{k,\pm}$
- S=1 ($\Delta J^\pi = 1^+$) → Gamow-Teller (GT) : $\hat{O}(\mathrm{GT}^\pm) = g_V \sum_k t_{k,\pm}\sigma_k$
  - σ : Pauli spin matrix of a nucleon
  - $t_\pm = \frac{1}{2}(\tau_x \pm i\tau_y)$: isospin ladder operator
  - g_V, g_A : vector and axial-vector weak coupling constants

Beta decay illustration :https://ati.tuwien.ac.at/

Definitions of Fermi and GT transition strengths

The Fermi and GT transition strengths, B(F$^\pm$) and B(GT$^\pm$), are defined as:

Fermi : $$B(\mathrm{F}^{\pm}) = \frac{1}{2J_i+1}\left|\left\langle f \left\| \sum_k t_{k,\pm} \right\| i \right\rangle\right|^2$$

GT : $$B(\mathrm{GT}^{\pm}) = \frac{1}{2J_i+1}\left|\left\langle f \left\| \sum_k t_{k,\pm}\sigma_k \right\| i \right\rangle\right|^2$$

Definition of reduced matrix elements

$$\langle j'm'|T(k,q)|jm\rangle = (-1)^{k-j+j'}\frac{\langle kqjm|j'm'\rangle}{\sqrt{2j'+1}} \times \langle j'||T(k)||j\rangle$$

- $|i\rangle$ and $|f\rangle$: parent and daughter states
- J_i : initial spin
- $\langle f||\hat{O}||i\rangle$: denote reduced matrix elements with respect to the spin and coordinate space

Exercise

Determine $\left\langle \frac{1}{2} \left\| \sigma \right\| \frac{1}{2} \right\rangle$.

Hints: $s = \frac{\hbar}{2}\sigma$ and $\left\langle 10\frac{1}{2}\frac{1}{2} \middle| \frac{1}{2}\frac{1}{2} \right\rangle = -\sqrt{\frac{1}{3}}$.

Beta decay strengths and rates

The connection between the beta-decay rates and the F and GT transition strengths, B(F) and B(GT), is simple and given by

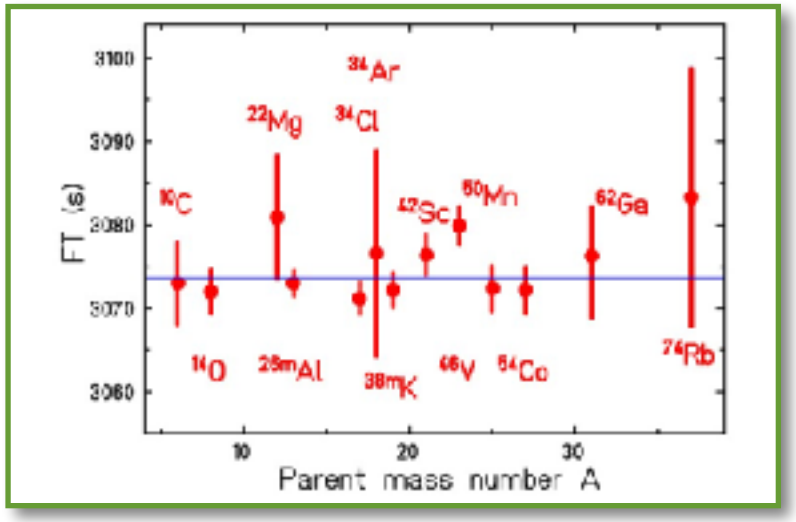
$$B(\mathrm{F}) + \left(\frac{g_A}{g_V}\right)^2 B(\mathrm{GT}) = \frac{K'}{ft}$$

- $B(\mathrm{F}),\ B(\mathrm{GT})$: Fermi and Gamow-Teller transition strengths
- t : half life
- f : phase space factor given by the total energy released
- $g_V,\ g_A$: vector and axial-vector coupling constants
- K' : empirically determined constant

***K'* is determined from pure Fermi transitions**

$$B(\mathrm{F}) = \frac{K'}{ft} \qquad B(\mathrm{F}) = N - Z$$

- *ft* = 3073.3±3.5 → $K' = 6147 \pm 7\,\mathrm{s}$



J.C.Hardy et al., Nucl. Phys. A 509, 249 (1990).

g_A/g_V is determined from neutron beta decay with B(F)=1 and B(GT)=3

- *t* = 623.6±6.2 s → $\left(\frac{g_A}{g_V}\right)^2 = (1.2605 \pm 0.0075)^2$

D.H.Wilkinson, Nucl. Phys. A 377, 474 (1982).

Fermi and Gamow-Teller sum rules

Fermi and Gamow-Teller sum rules

Fermi/Gamow-Teller β$^{\pm}$ operators exciting the Fermi/Gamow-Teller states are

$$\mathbf{F}_{\pm} = \sum_k t_{\pm,k}$$

$$\mathrm{GT}^{\pm}(\mu) = \sum_k t_{k,\pm}\sigma_{\mu,k}$$

Total F$_{\pm}$ and GT$_{\pm}$ strengths, S(F$_{\pm}$) and S(GT$_{\pm}$), are given by

$$S(\mathbf{F}_{\pm}) = \sum_f |\langle f|\mathbf{F}_{\pm}|i\rangle|^2 = \langle i|\mathbf{F}_{\pm}^{\dagger}\,\mathbf{F}_{\pm}|i\rangle$$

$$S(\mathrm{GT}_{\pm}) = \sum_{f,\mu} |\langle f|\mathrm{GT}_{\pm}(\mu)|i\rangle|^2 = \sum_{\mu}\langle i|\mathrm{GT}_{\pm}^{\dagger}(\mu)\,\mathrm{GT}_{\pm}(\mu)|i\rangle$$

(completeness of $\sum |f\rangle\langle f| = 1$ *)*

Separate sums are model dependent (shell-model, RPA, etc.).

But the difference is model independent → ***Only a function the neutron excess (N-Z).***

$$S(\mathbf{F}_{-}) - S(\mathbf{F}_{+}) = \langle i|\sum_k [t_{+,k}t_{-,k} - t_{-,k}t_{+,k}]|i\rangle = (N-Z)$$

$$S(\mathrm{GT}_{-}) - S(\mathrm{GT}_{+}) = 3\langle i|\sum_k [t_{+,k}t_{-,k} - t_{-,k}t_{+,k}]|i\rangle = 3(N-Z)$$

Note: $t_{+}|p\rangle = |n\rangle \quad t_{-}|n\rangle = |p\rangle$

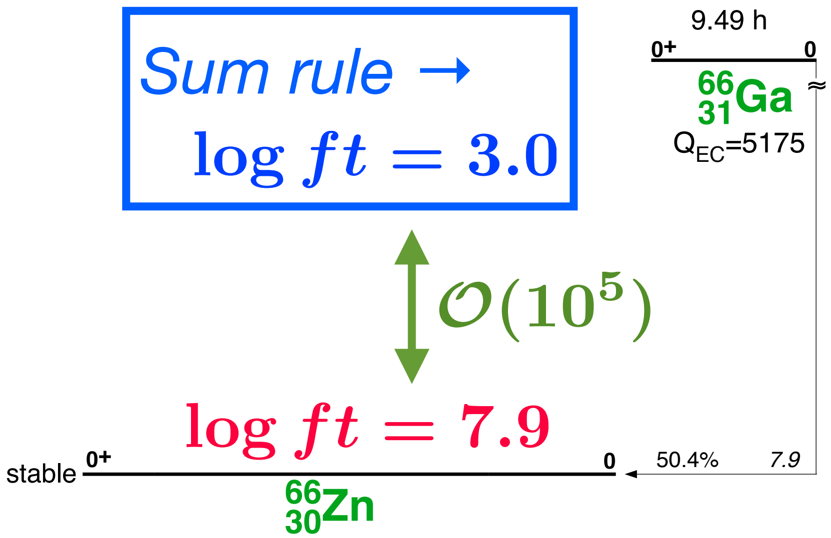
Exercise: Derive these sum-rules referring Appendix D of this lecture.

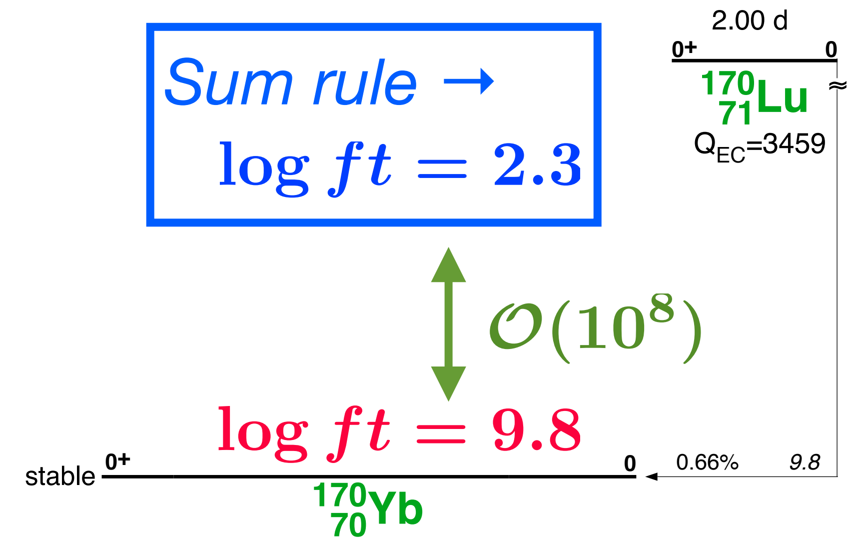
Delay/quenching of Fermi transitions

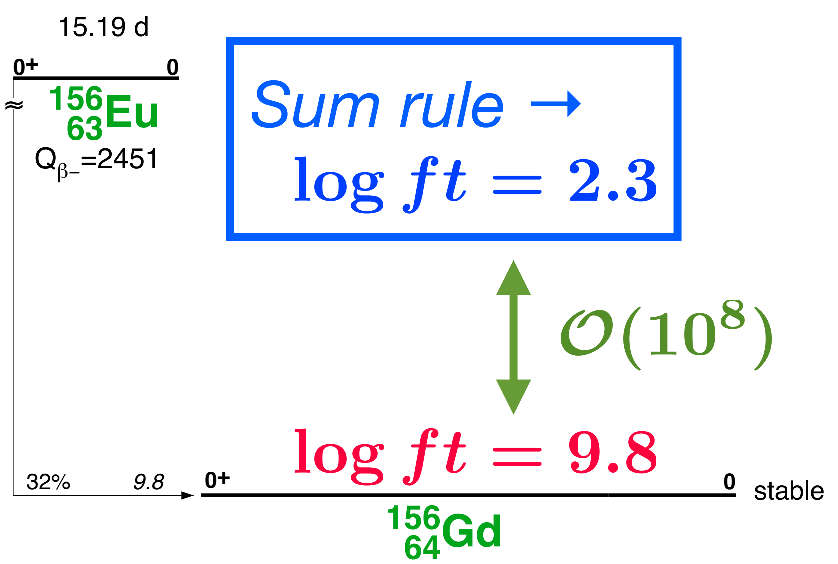
If the Fermi transition strengths are concentrated to a transition:

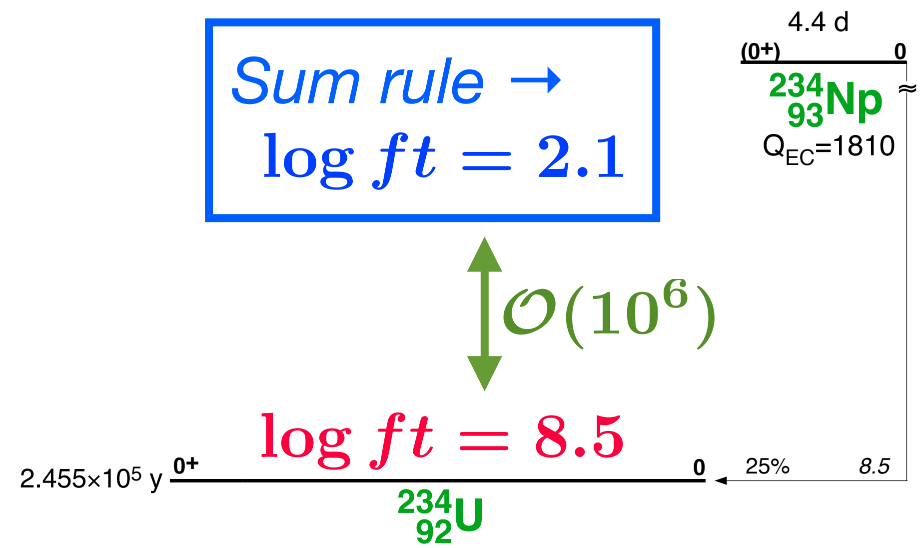
$$B(\mathrm{F}) = (N - Z) = \frac{6147\,\mathrm{s}}{ft}$$

- Its log(ft) value should be log[(6147 s)/(N-Z)] from the sum rule.





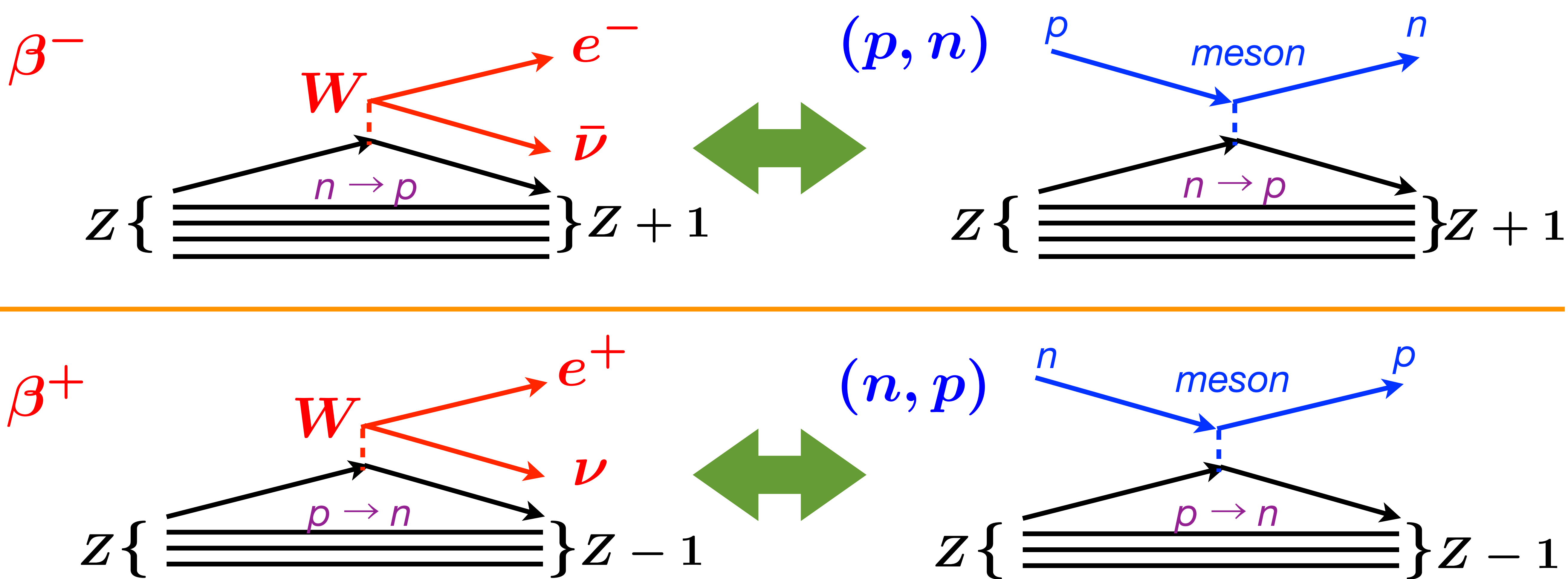




The Fermi transitions are hindered (delayed) by factors of the order of 10^4 to 10^8
→*Missing strengths should exist beyond the beta-decay energy window.*

Beta decays and charge exchange reactions

Beta decays (weak int.) | **charge-exchange (strong int.)**



weak int. | *strong (nuclear) int.*

weak int.		strong (nuclear) int.
$\sum g_A \sigma t_\pm$	Gamow-Teller (ΔS=1)	$\sum V_{\sigma\tau} \sigma t_\pm$
$\sum g_V t_\pm$	Fermi (ΔS=0)	$\sum V_\tau t_\pm$

$p \leftrightarrow n$ (in $t_\pm$)

Charge exchange reactions ↔ Information on beta-decays (except for coupling const.)

Energy transfers by reactions → can access the highly-excited states.

Observation of IAS (Fermi resonance) by (p,n)

Fermi strengths:

- Sum rule : summed to whole ω region
- Beta decay : limited by Q value

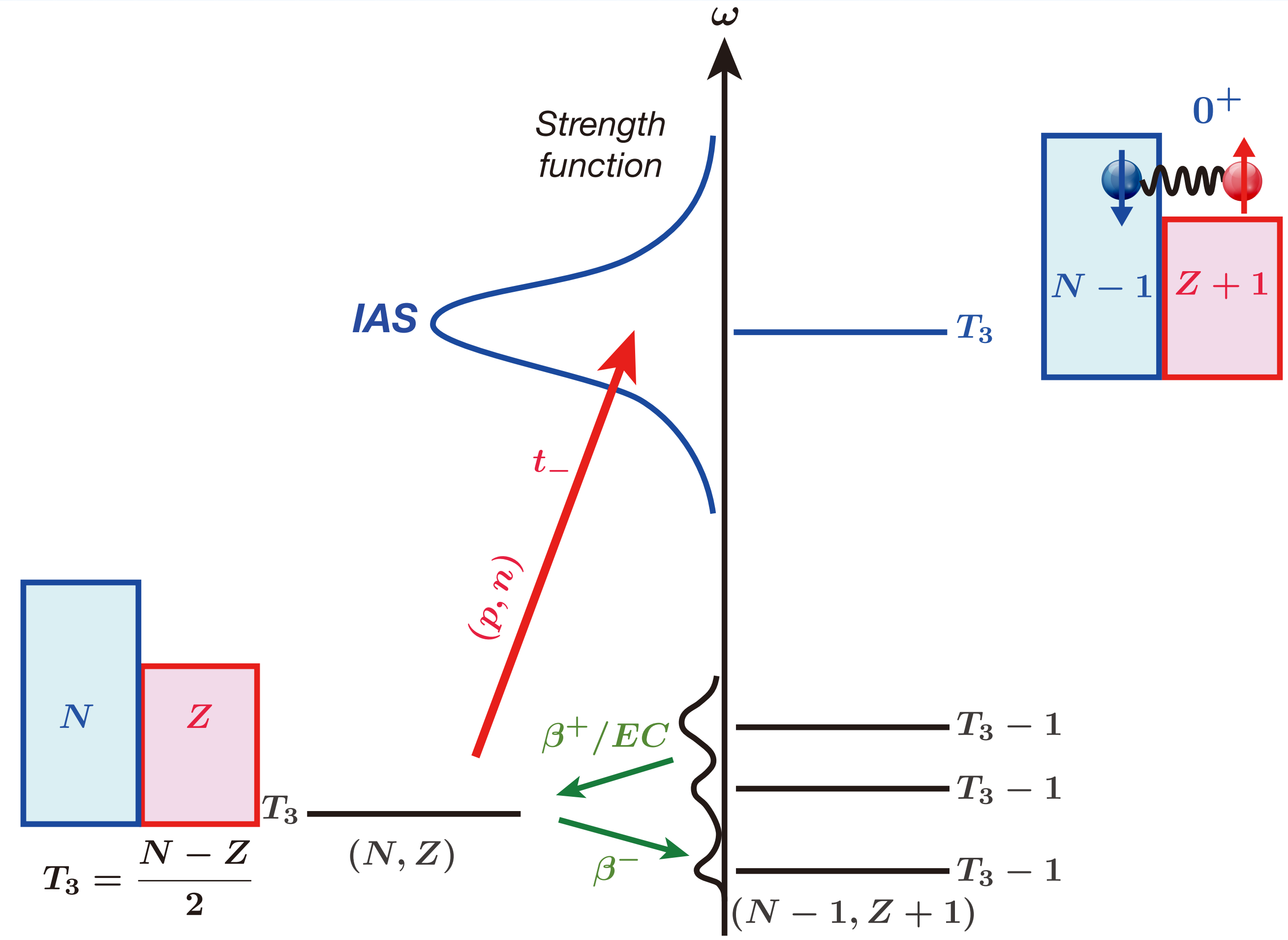
(p,n) reaction

- can excite the 0^+ (Fermi) states
  by charge-exchange t_- operator.
- can populated the 0^+ states
  beyond the beta-decay energy window

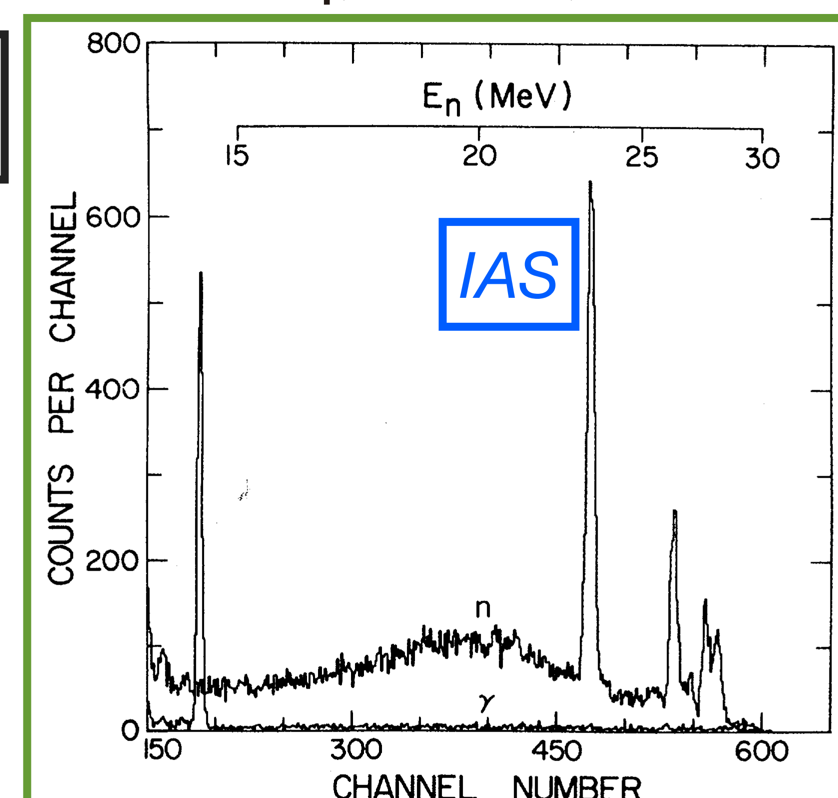
The IAS (0^+) are clearly observed.

- Isospin is a good quantum number
  for N>Z
- Almost all strengths are concentrated
  to the IAS with T_3

The IAS almost exhausts
the sum-rule strength of (N-Z)



^{90}Zr(p,n)
at 35 MeV and 0°



J.D.Anderson and C.Wong, Phys. Rev. Lett. 7, 250 (1961).
R.R.Doering et al., Phys. Rev. C 12, 378 (1975).

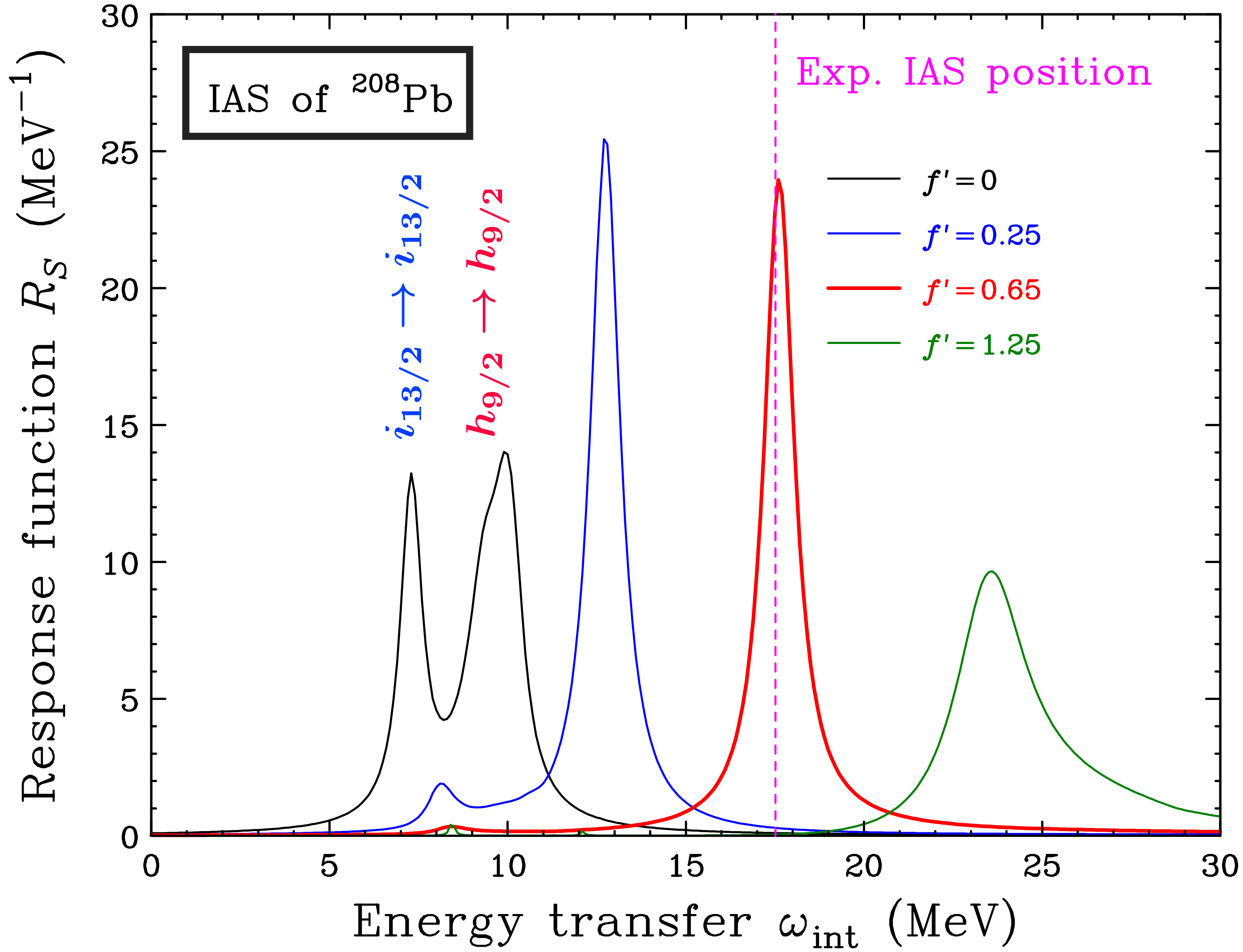
Landau-Migdal parameter *f'* and IAS

Without residual interaction (f'=0)

- Many $\Delta J^{\pi}=0^{+}$ 1p-1h states → *Strengths are fragmented and lower than exp. data*

With residual interaction (f'>0)

- Strengths concentrate to the high-ω state & The peak also shifts to higher ω
- A relatively strong ***f'=0.65*** (repulsive) reproduces the exp. IAS position
  - This f'～0.65 is roughly consistent with the value of f'～0.75 determined from GDRs

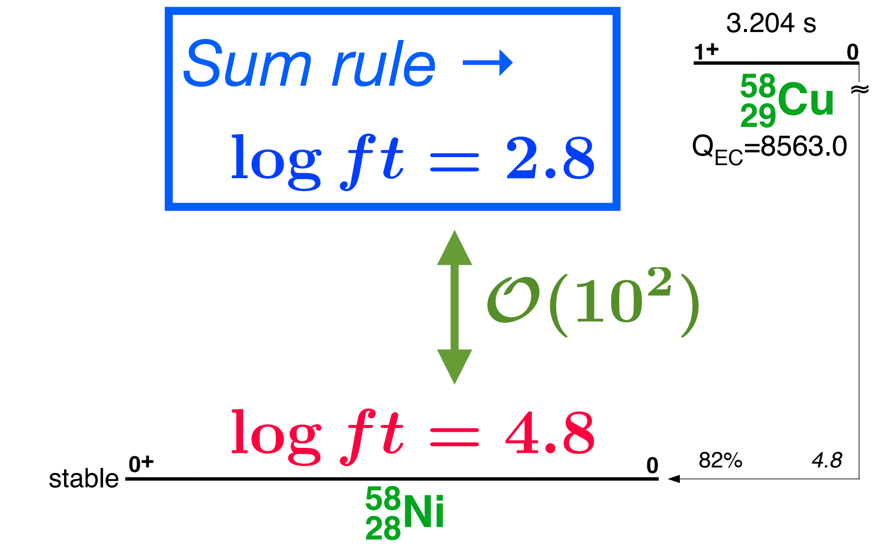


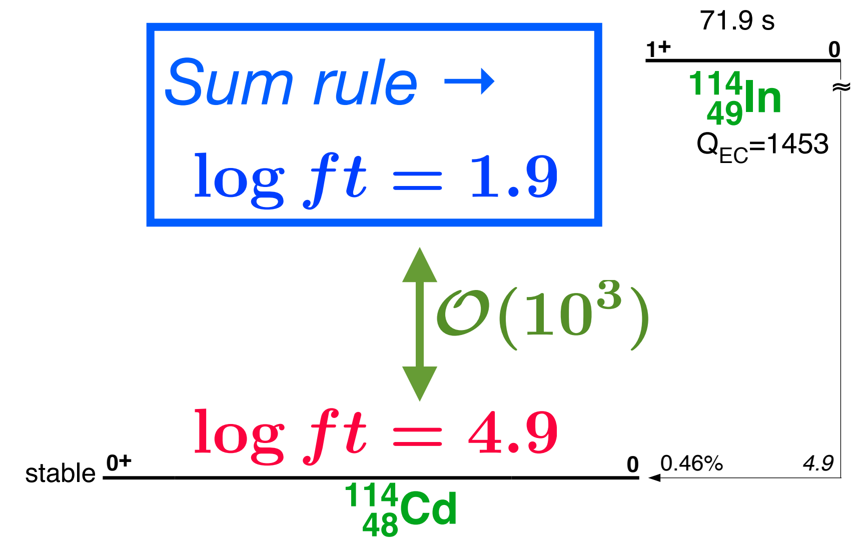
Theoretical calculations were performed with the computer code respq by Ichimura-san.

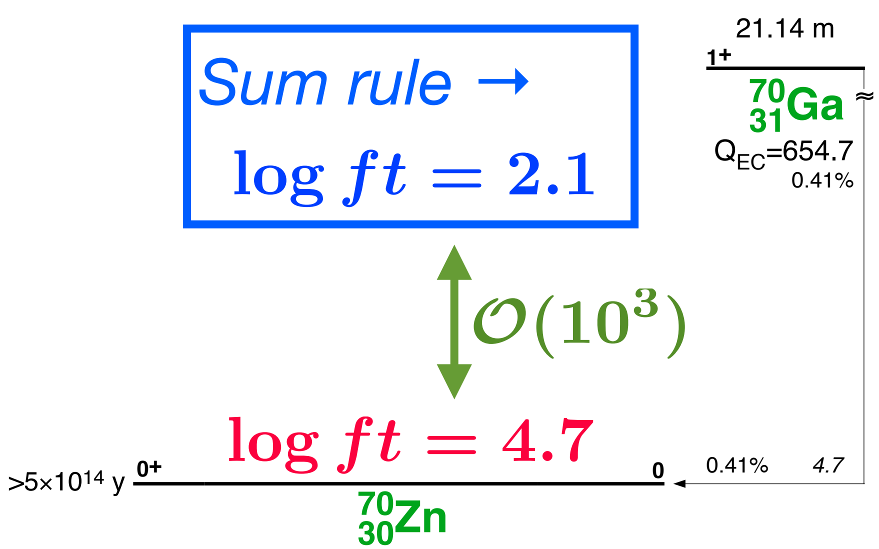
Delay/quenching of GT transitions

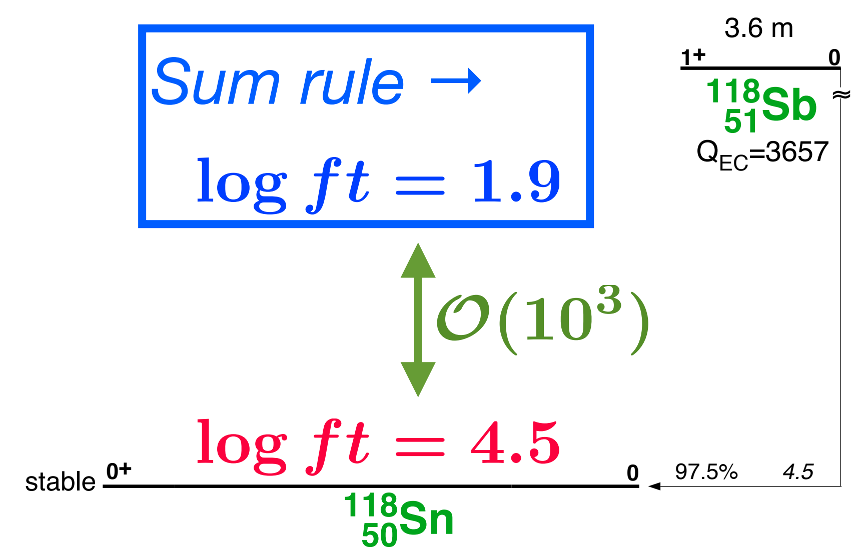
If the GT transition strengths are concentrated to a transition:

- Its log(ft) value should be $\log\left[\frac{K'}{(g_A/g_V)^2 B(\mathrm{GT})}\right] = \log\left[\frac{(6147\,\mathrm{s})}{(1.261)^2 3(N-Z)}\right]$ from the sum rule.









The GT transitions are hindered (delayed) by factors of the order of 10^2 to 10^3
→*Missing strengths should exist beyond the beta-decay energy window.*

Observation of GTR by (p,n)

GT strengths:

- Sum rule : summed to whole ω region
- Beta decay : limited by Q value

Quenching of B(GT) (delay of GT β-decay)

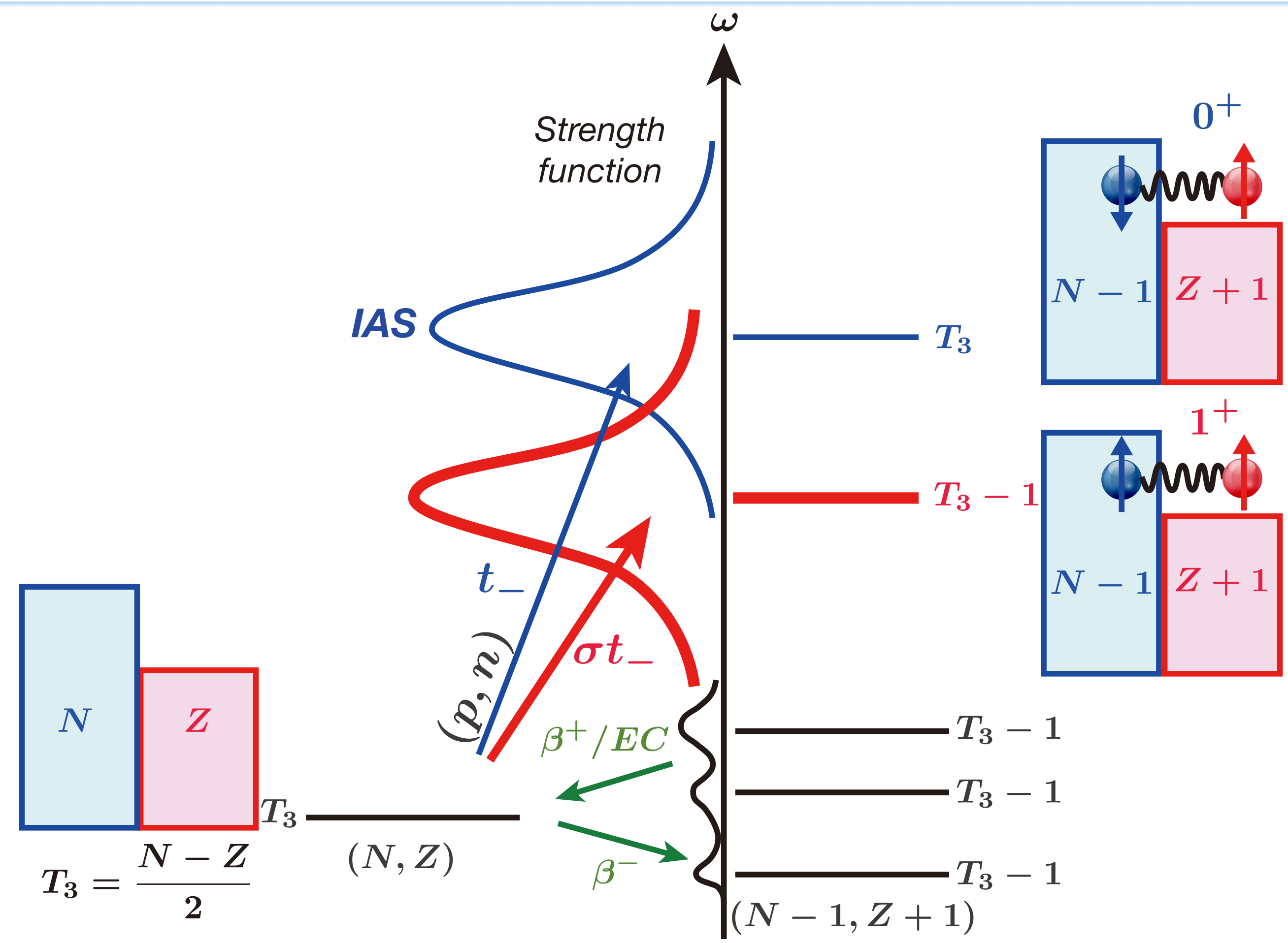
- suggests the GTR beyond the β-decay window

(p,n) reaction

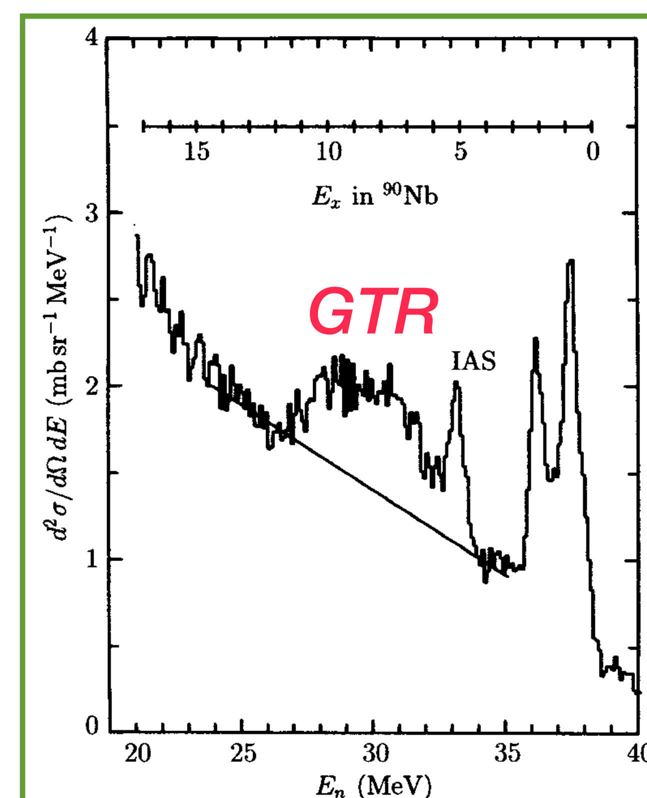
- can excite the 1^+ (GT) states by charge-exchange σt_- operator.
- can populated the 1^+ states beyond the beta-decay energy window

The GTR (1^+) are observed.

- *GTR takes a major part of the GT strength*
- Low-energy GT strength is quenched



^{90}Zr(p,n)
at 45 MeV and 0°

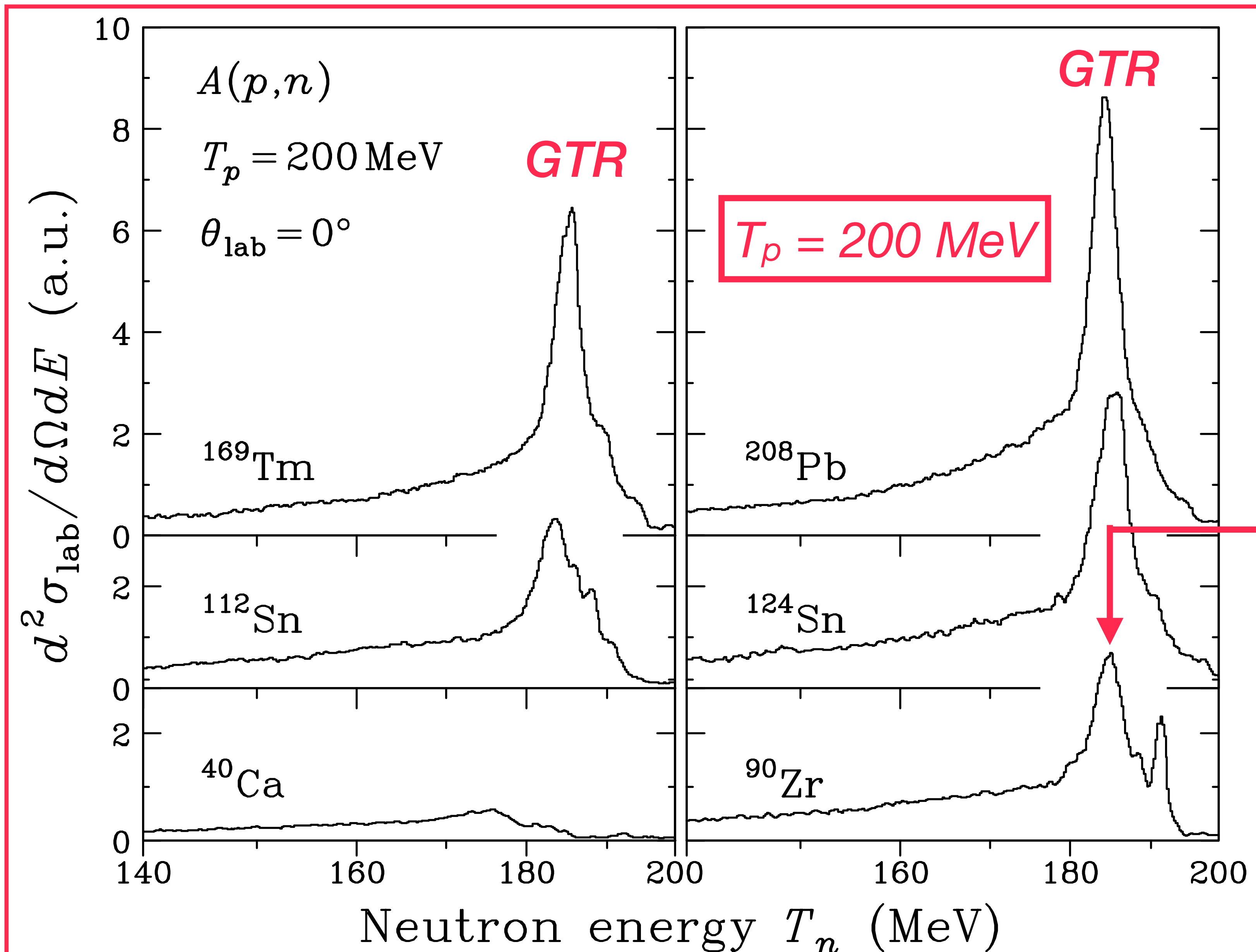


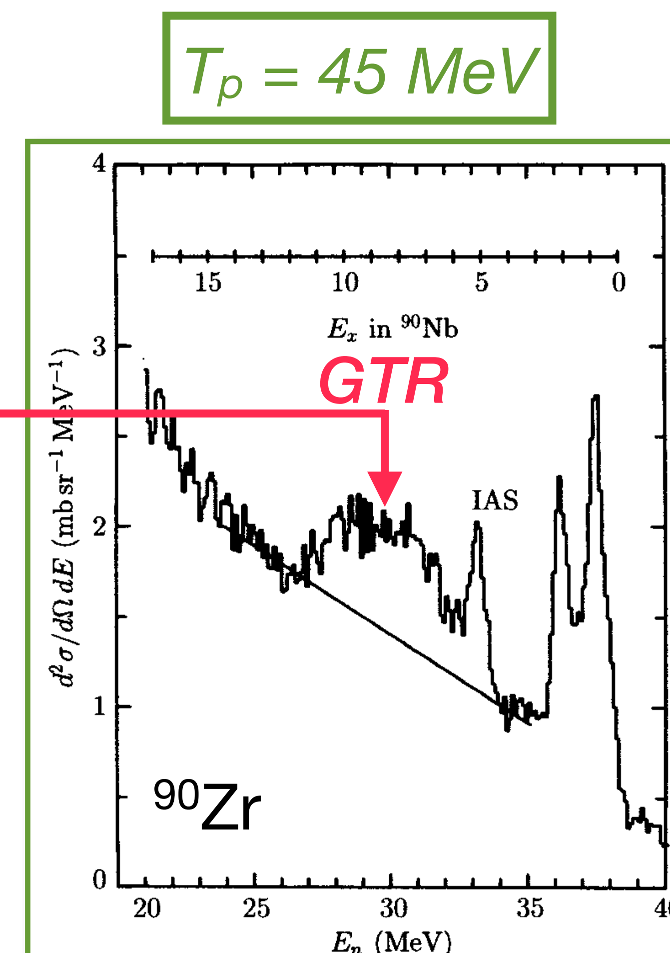
R.R.Doering et al., Phys. Rev. Lett. 35, 1691 (1975).

Systematic studies at IUCF

The GT resonances were observed for medium-heavy N>Z nuclei.

- With increasing neutron excess (N-Z), the GTR becomes more pronounced.
- With increasing incident energy T_p, the GTR becomes more pronounced
    - The IAS is only weakly excited.





C. Gaarde, Nucl. Phys. A 369, 258 (1981).

R.R.Doering et al., Phys. Rev. lett. 35, 1691 (1975).

Missing GT strength

In the 0° (p,n) reaction:

GT ΔL=0 strength is predominantly excited (GT resonances have been observed)

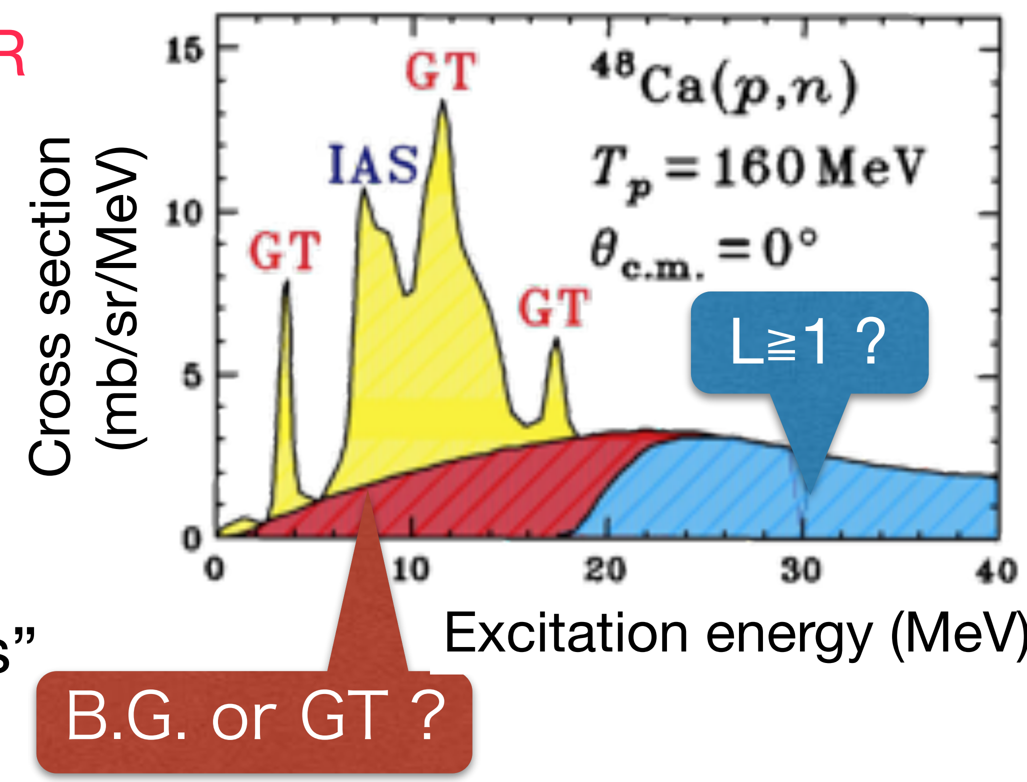
But the extraction of GT strength from σ(0°) has some problems:

- The strength of L≧1 would contribute beyond the GTR
  → Ex is limited up to GTR (E_x～20 MeV)

- The GTR bump is located on top of a continuum
  → This continuum is B.G. or GT ?

Minimum GT strengths have been obtained
by *subtracting the continuum as B.G.*

- Continuum contributions are treated as “uncertainties”



The summed total strength is compared with the sum rule (Ikeda sum rule)

- For N>Z nuclei, $S_{\beta+} \simeq 0$ due to Pauli blocking
- $S_{\beta-}$ is compared to the sum rule value of 3(N-Z)

F. Osterfeld, Rev. Mod. Phys. 64, 491 (1992).

Missing GT strengths

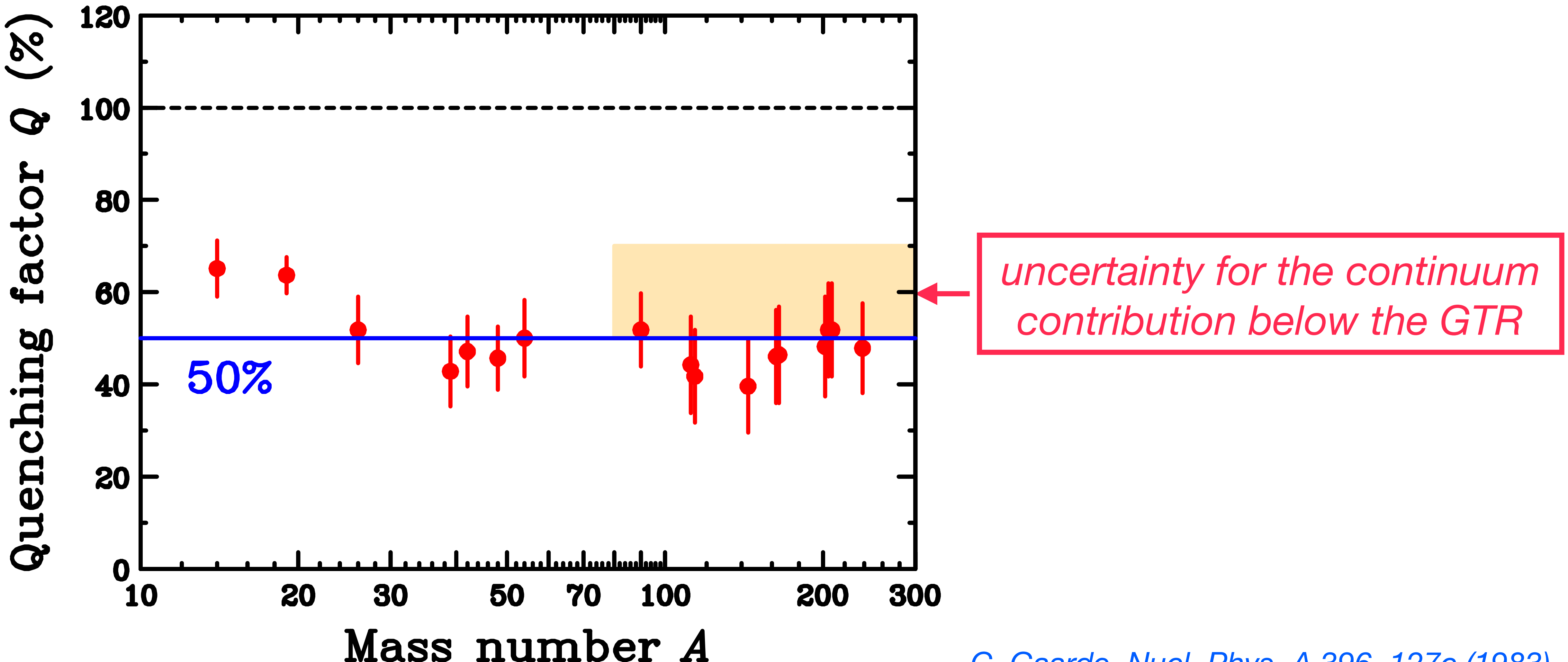
Fraction of GT sum-rule strength observed in (p,n) up to 20 MeV

Only about 50-60% of the sum-rule value is found up to GTR (≦20 MeV)

Uncertainties for ^{90}Zr

- $(52 \pm 9)\%$ [minimum] $\sim$ $(67^{+7}_{-10})\%$ [maximum]
- The maximum value → The continuum under the GTR is also the GT contribution.

About 40% strength is missing in the GTR region.



C. Gaarde, Nucl. Phys. A 396, 127c (1983).

Questions to be solved in the following lectures

Lecture 2

How can we identify the resonance as Fermi (ΔS=0) or Gamow-Teller (ΔS=1) ?

- How to identify the resonance as GT 1^+ (not 0^+) ?

What is the best energy for studying the GT strength by (p,n) ?

- The IAS was found at 35 MeV whereas the GT was found at 45 MeV.

Lecture 3

How were the (p,n) data obtained ?

- How was the neutron measured ?

How is the (p,n) cross section converted to the GT strength ?

- The relation between $\sigma(0°)$ and B(GT)

Is the continuum below the GTR really B.G. ?

- The continuum should be subtracted? or added ?

Is there any GT strength beyond the GTR ($E_x \geqq 20$ MeV) ?

- How to identify the GT L=0 strength in the continuum ?

Homework #1

1. Determine the reduced matrix element $\left\langle \frac{1}{2} \middle\| \sigma \middle\| \frac{1}{2} \right\rangle$.

2. The spherical components of a unit vector $\vec{r}$ are defined as

$$r_{\pm 1} \equiv \mp \frac{1}{2}(x \pm iy)\,; \quad r_0 \equiv z$$

Express the spherical harmonics Y_1^m ($m = \pm 1\,, 0$) using $r_{\pm 1}$ and r_0 .

3. In the γ-absorption measurement, the total absorption cross section is obtained by measuring all possible partial cross sections as

$$\sigma(\gamma) = \sigma(\gamma, \gamma') + \sigma(\gamma, p) + \sigma(\gamma, n) + \cdots$$

In practice, the total cross sections for heavy nuclei (A≧90) are approximately obtained as

$$\sigma(\gamma) \simeq \sum_x \sigma(\gamma, xn)$$

Explain the validity of this approximation referring the Appendix A of this lecture.

Appendix A

γ-absorption cross section

γ absorption

γ-absorption

A selective tool for excitation of GDR (ΔL=1, ΔT=1)

In order to distinguish nuclear from atomic processes, total absorption cross section is obtained by measuring all possible partial cross sections:

$$\sigma(\gamma) = \sigma(\gamma, \gamma') + \sigma(\gamma, p) + \sigma(\gamma, n) + \cdots$$

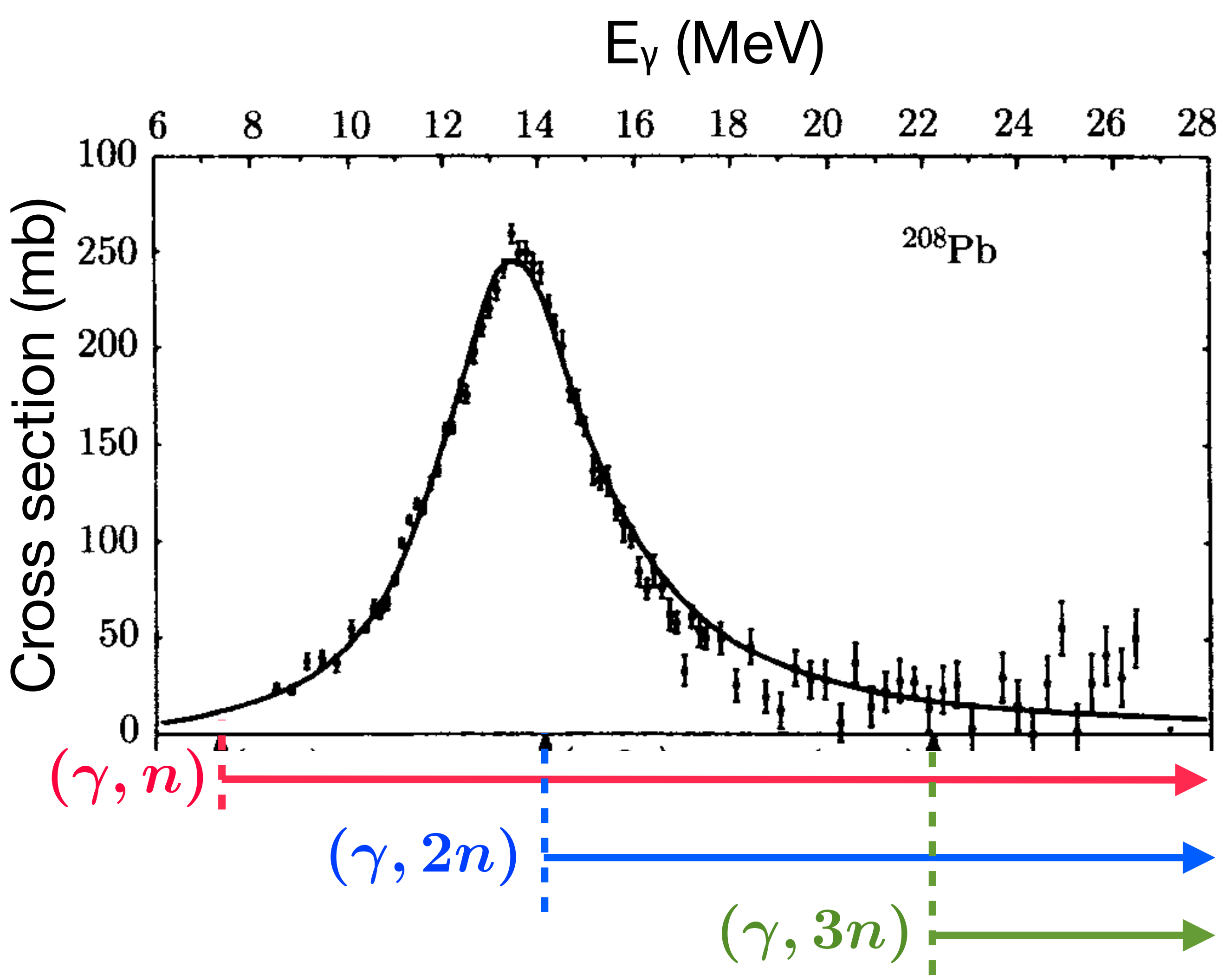
In almost all nuclei, E_x(GDR) > particle decay threshold

- $\sigma(\gamma, \gamma')$ is low.

In heavy nuclei (A ≧ 90), proton-emission is hindered by the coulomb barrier.

- $\sigma(\gamma, p)$ is small

$$\rightarrow \sigma(\gamma) \simeq \sum_x \sigma(\gamma, xn)$$



Appendix B

TRK sum rule for GDR

GDR sum rule

$$\hat{O}(\mathrm{E1}) = -\sqrt{\frac{3}{4\pi}} \sum_{i=1}^{A} t_3(i) z_i$$

$$S_1(\mathrm{E1}) = \frac{1}{2} \langle 0 | [\hat{O}(\mathrm{E1}), [\hat{H}, \hat{O}(\mathrm{E1})]] | 0 \rangle$$

Since $\hat{O}(\mathrm{E1}) \propto z_i$, let's consider

$$\langle 0 | [z, [\hat{H}, z]] | 0 \rangle$$

(Here we omit "i" for simplicity)

In $\hat{H} = T + V$, the potential V is a function of $\vec{r} = (x, y, z)$. Thus

$$[V, z] = 0$$

Therefore, it is sufficient to consider:

$$\langle 0 | [z, [T, z]] | 0 \rangle$$

GDR sum rule

Since $T = \vec{p}^{\,2}/2M$, we find:

$$\langle 0|[z,[\hat{H},z]]|0\rangle = \langle 0|[z,[\vec{p}^{\,2}/2M,z]]|0\rangle$$

$$= \frac{1}{2M}\langle 0|[z,[p_z^2,z]]|0\rangle$$

$$= p_z[p_z,z] + [p_z,z]p_z = 2p_z\frac{\hbar}{i}$$

$$= \frac{1}{2M}\frac{2\hbar}{i}\langle 0|[z,p_z]|0\rangle = \frac{\hbar^2}{M}$$

Then, for the E1 operator:

$$\hat{O}(\mathrm{E1}) = -\sqrt{\frac{3}{4\pi}}\sum_{i=1}^{A} t_3(i) z_i$$

the 1st moment (sum-rule) becomes:

$$S_1(\mathrm{E1}) = \frac{1}{2}\langle 0|[\hat{O}(\mathrm{E1}),[\hat{H},\hat{O}(\mathrm{E1})]]|0\rangle$$

sum for i (1〜A)

$$= \frac{1}{2}\frac{3}{4\pi}\frac{1}{4}A\langle 0|[z,[\hat{H},z]]|0\rangle = \frac{3A\hbar^2}{32\pi M}$$

isospin

Thomas-Reich-Kuhn (TRK) sum rule

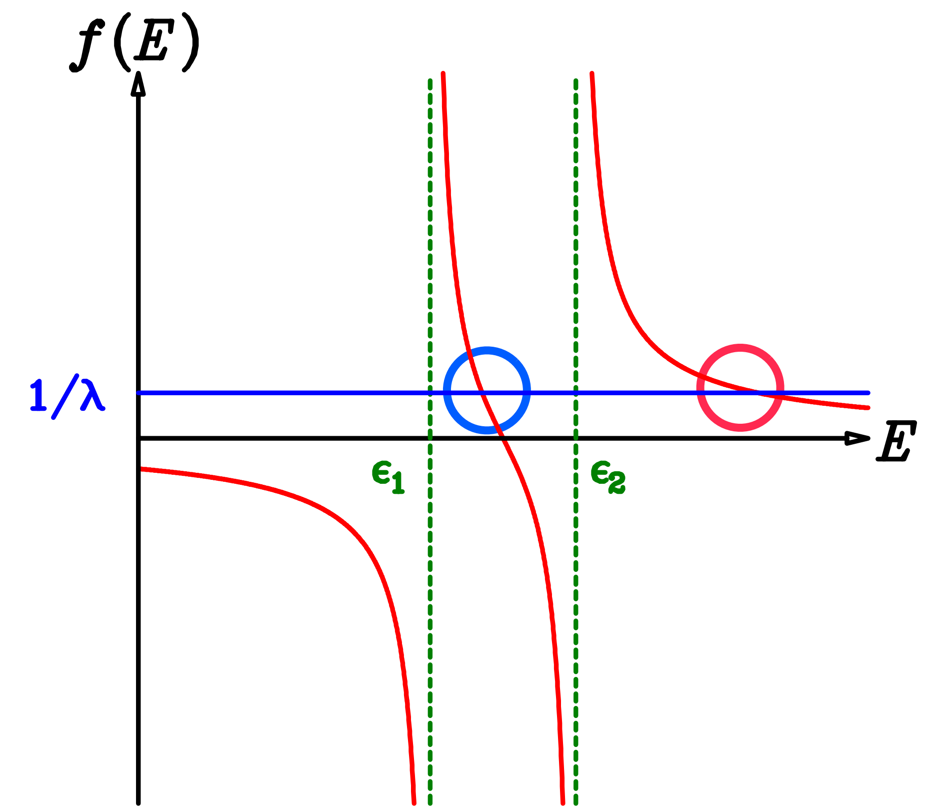
Appendix C

Graphical solution of the 2-state model

Graphical solution

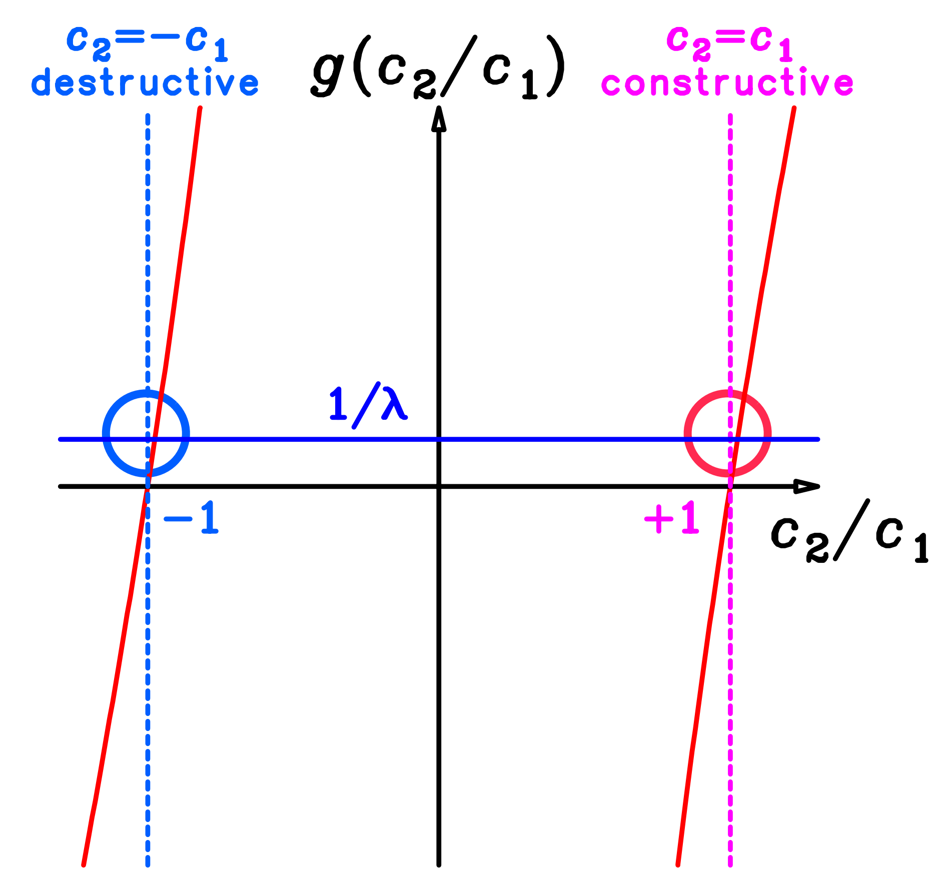
Equation of eigenvalue problem

$$\frac{1}{\lambda} = \underbrace{\frac{X_1^2}{E-\varepsilon_1} + \frac{X_2^2}{E-\varepsilon_2}}_{\equiv f(E)} = \underbrace{\frac{X_1 X_2}{\varepsilon_2 - \varepsilon_1}\left(\frac{c_2}{c_1} - \frac{c_1}{c_2}\right)}_{\equiv g(c_2/c_1)}$$



Energy: E

- Intersection of $f(E)$ and $1/\lambda$
- One state has a significantly high energy



Matrix element: c_2/c_1

- Intersection of $g(c_2/c_1)$ and $1/\lambda$
- One state has a constructive feature → GR
- Other state has a destructive feature → weak

Appendix D

Fermi and Gamow-Teller sum rules

Fermi operator and sum rule

Fermi β± operator exciting the Fermi state is

$$\mathbf{F}_\pm = \sum_k t_{\pm,k}$$

Nuclear Physics Convention

$$t_3|n\rangle = \frac{1}{2}|n\rangle \quad t_3|p\rangle = -\frac{1}{2}|p\rangle$$

- $t_-|n\rangle = |p\rangle$
- $t_+|p\rangle = |n\rangle$
- $t_-|p\rangle = t_+|n\rangle = 0$

Total F± strength, S(F±), is given by

$$S(\mathbf{F}_\pm) = \sum_f |\langle f|\mathbf{F}_\pm|i\rangle|^2$$

$$= \langle i|\mathbf{F}^\dagger_\pm \mathbf{F}_\pm|i\rangle \quad \text{(completeness of } \sum_f |f\rangle\langle f| \text{)}$$

Separate sums, S(F+) and S(F-), are model dependent (shell-model, RPA, etc.).

But the difference is model independent → Only a function the neutron excess (N-Z).

$$S(\mathbf{F}_-) - S(\mathbf{F}_+) = \langle i|\sum_k [t_{+,k}t_{-,k} - t_{-,k}t_{+,k}]|i\rangle \quad (\because t^\dagger_{\pm,k} = t_{\mp,k})$$

$$= (N - Z)$$

{ *model-independent*
only a function of (N-Z)

Total GT$^{\pm}$ strengths

Total GT$^{\pm}$ strength, S(GT$_{\pm}$), is given by

$$S(\mathrm{GT}_{\pm}) = \sum_{f,\mu} |\langle f|\mathrm{GT}_{\pm}(\mu)|i\rangle|^2$$

$$= \sum_{f,\mu} \langle f|\mathrm{GT}_{\pm}(\mu)|i\rangle^{\star} \langle f|\mathrm{GT}_{\pm}(\mu)|i\rangle$$

$$= \sum_{f,\mu} \langle i|\mathrm{GT}^{\dagger}_{\pm}(\mu)|f\rangle \langle f|\mathrm{GT}_{\pm}(\mu)|i\rangle$$

$$= \sum_{\mu} \langle i|\mathrm{GT}^{\dagger}_{\pm}(\mu)\,\mathrm{GT}_{\pm}(\mu)|i\rangle$$ *(completeness of $\sum_f |f\rangle\langle f|$)*

- $|i\rangle$, $|f\rangle$: initial and final states
- f : runs over all GT$^{\pm}$ states

In general, S(GT±) is model-dependent (shell-model, RPA, etc).

GT sum rule

Separate sums, S(GT$_+$) and S(GT$_-$), are model dependent

But the difference is model independent → Only a function the neutron excess (N-Z)

$$S(\mathrm{GT}_-) - S(\mathrm{GT}_+)$$

$$= \sum_\mu \langle i|[\mathrm{GT}_-^\dagger(\mu)\mathrm{GT}_-(\mu) - \mathrm{GT}_+^\dagger(\mu)\mathrm{GT}_+(\mu)]|i\rangle$$

$$= \langle i|\sum_k \sum_\mu [t_{+,k}\sigma_{\mu,k}^\dagger t_{-,k}\sigma_{\mu,k} - t_{-,k}\sigma_{\mu,k}^\dagger t_{+,k}\sigma_{\mu,k}]|i\rangle \quad (\because t_{\pm,k}^\dagger = t_{\mp,k})$$

$$= \langle i|\sum_k [\sigma_k^2 t_{+,k}t_{-,k} - \sigma_k^2 t_{-,k}t_{+,k}]|i\rangle \quad (\sigma^2 \equiv \sum_\mu \sigma_\mu^\dagger \sigma_\mu)$$

$$= 3\langle i|\sum_k [t_{+,k}t_{-,k} - t_{-,k}t_{+,k}]|i\rangle \quad (\because \sigma^2 = 3)$$

For the isospin-ladder operators:

$$t_{+,k}t_{-,k}|n\rangle = |n\rangle \qquad t_{-,k}t_{+,k}|p\rangle = |p\rangle \qquad t_{-,k}t_{+,k}|n\rangle = t_{+,k}t_{-,k}|p\rangle = 0$$

→ $S(\mathrm{GT}_-) - S(\mathrm{GT}_+) = 3(N-Z)$ { *model-independent*; *only a function of (N-Z)* }

- Assumption: Nucleons are structureless, point-like particles.