High Spin States in Nuclei:

Exotic Quantal Rotation

III

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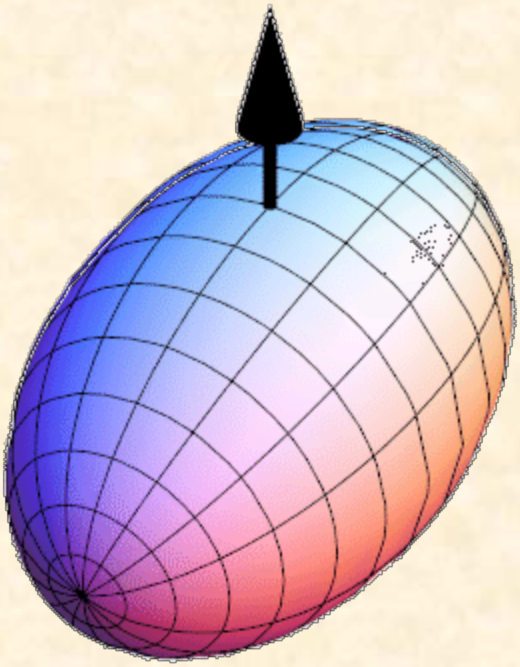
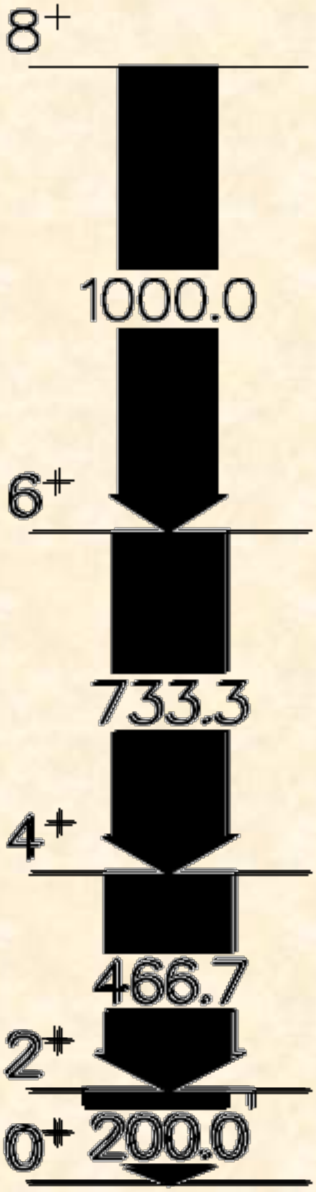
- “normal” collective rotation
  - principal axis; prolate deformation
- magnetic rotation
  - planar tilted axis; weak deformation
- chiral rotation
  - aplanar tilted axis; triaxial shape
- tidal waves
  - rotating condensate of phonons





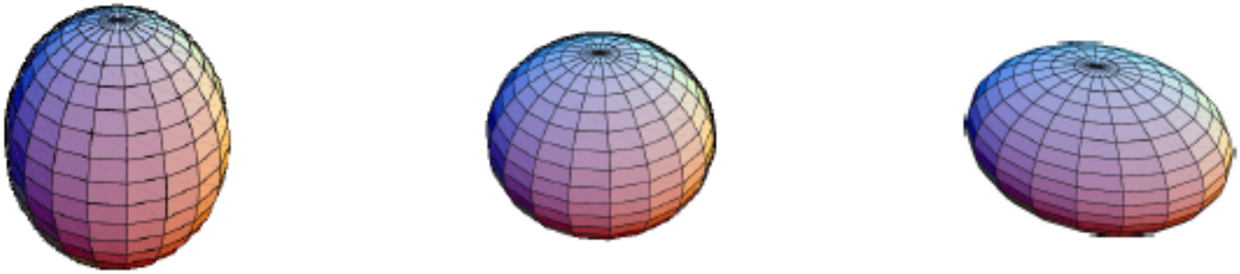
E2 transitions

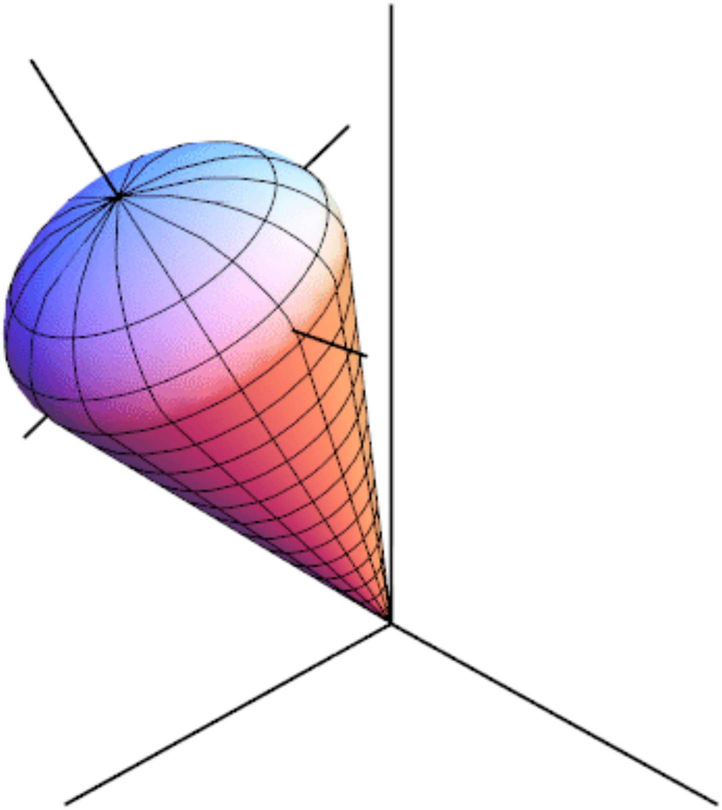
$E_I = (\hbar^2/2\mathcal{J})\ I(I+1);\ E_\gamma = [\ ](4I - 2)$

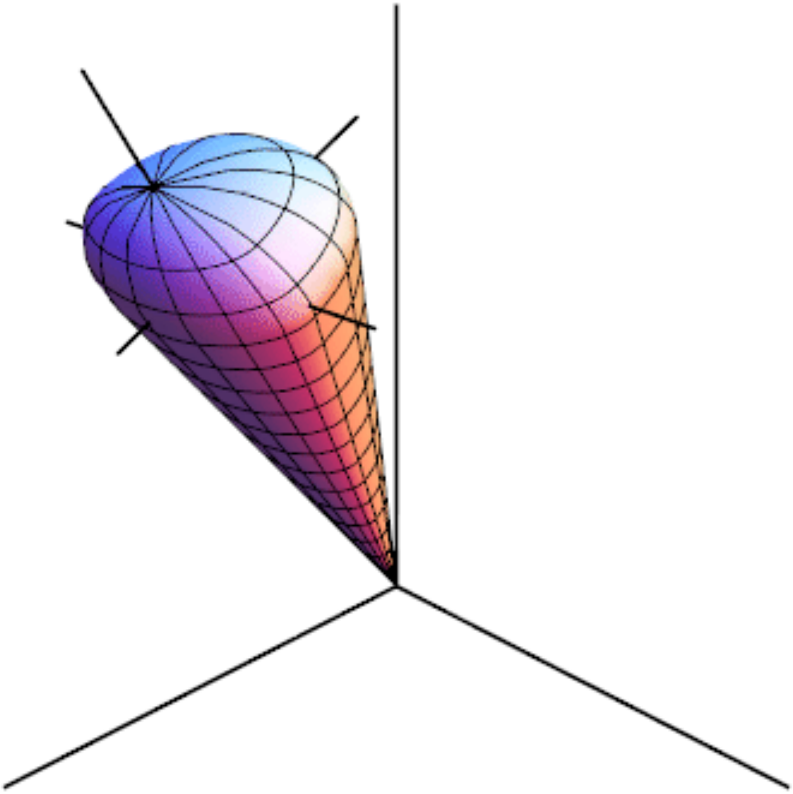
ΔE_γ = Constant → "picket fence"

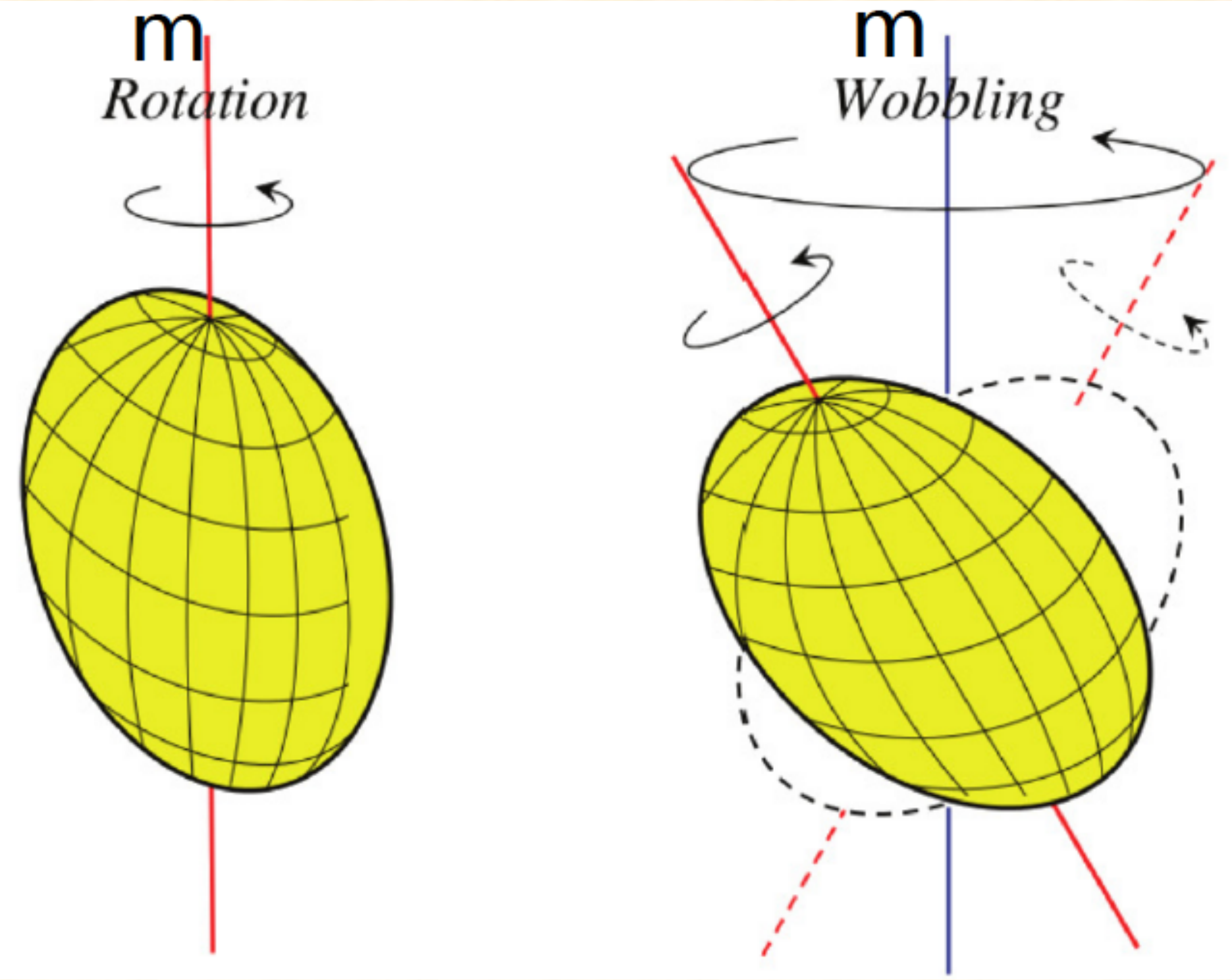
animation courtesy of J. T. Matta

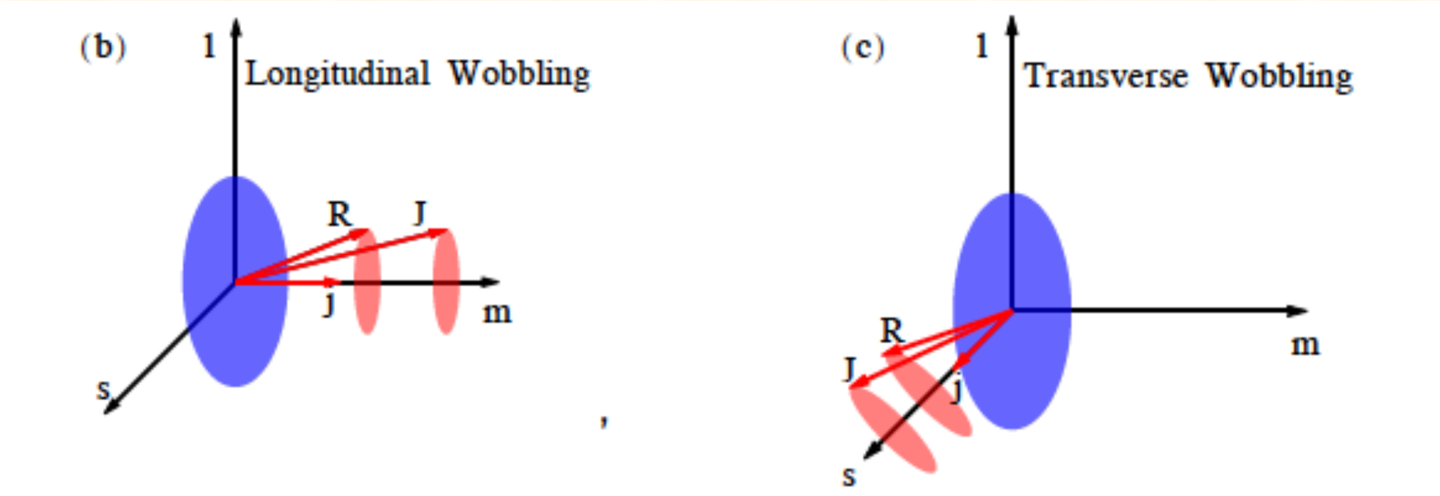
Nuclear shapes: prolate, oblate and triaxial

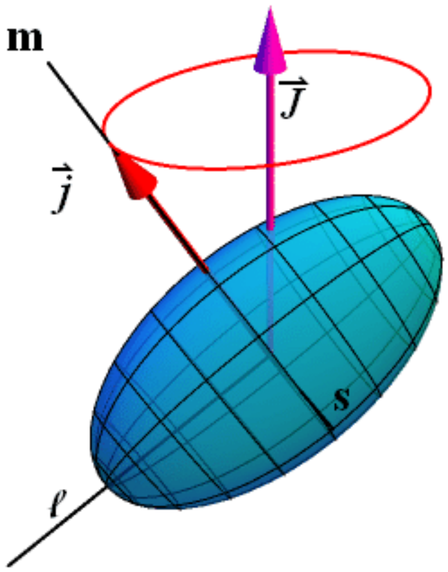


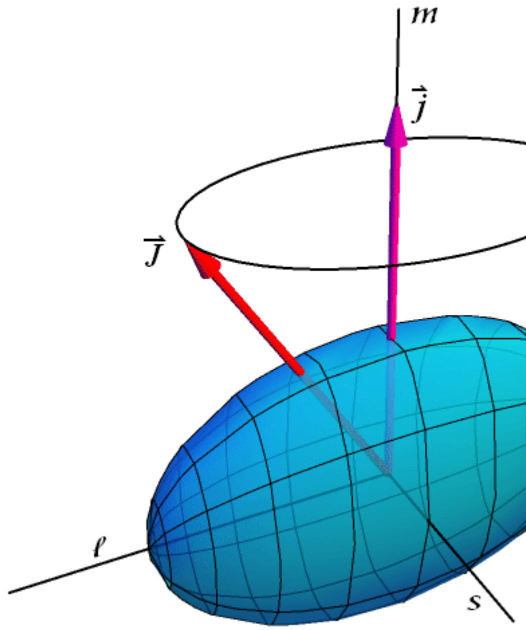


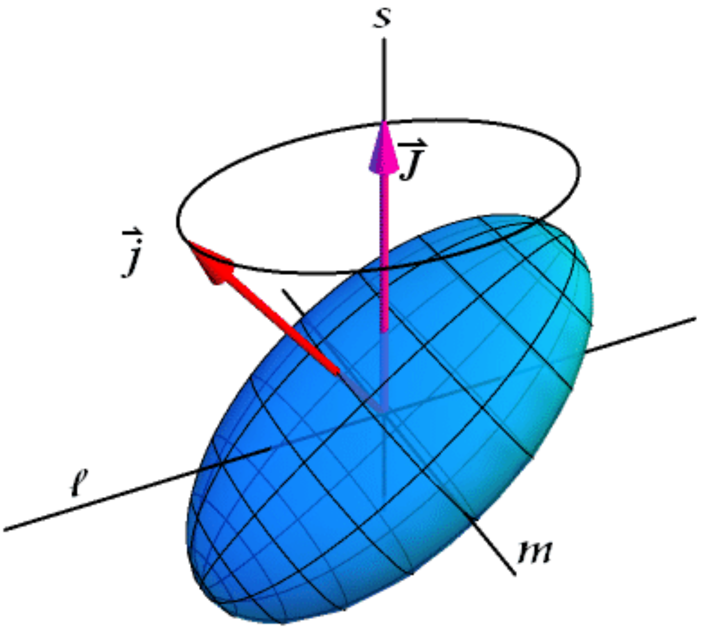


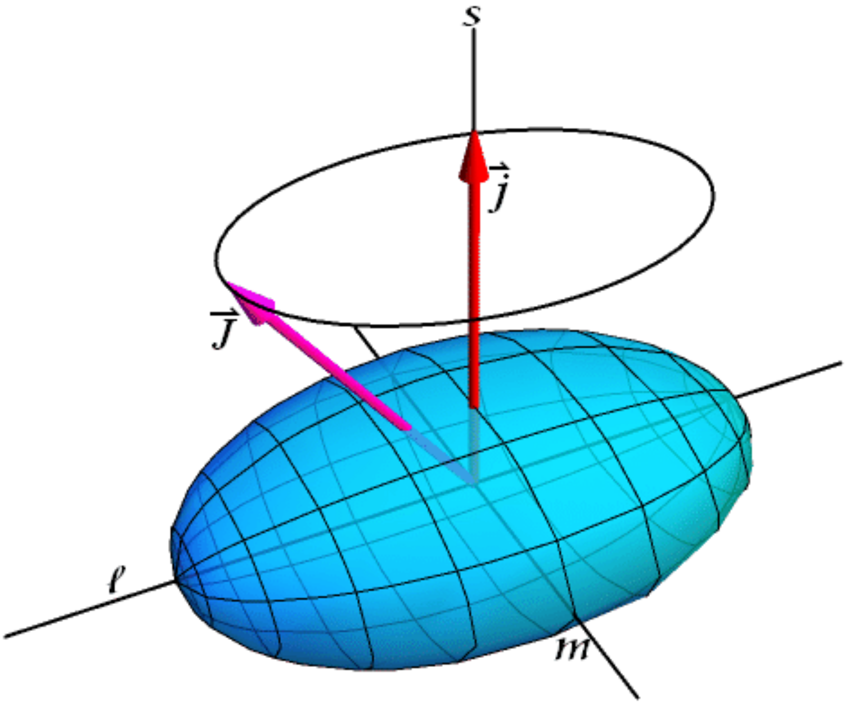

m
Rotation
m
Wobbling


(b)
1
Longitudinal Wobbling
R
J
j
m
s
,
(c)
1
Transverse Wobbling
R
J
j
m
s


m
$\vec{J}$
$\vec{j}$
s
ℓ


m
$\vec{j}$
$\vec{J}$
ℓ
s


s
$\vec{J}$
$\vec{j}$
ℓ
m



s
$\vec{j}$
$\vec{J}$
ℓ
m

Wobbling bands (TSD) are generally considered as one of the best signatures of nuclear triaxiality. Another is, of course, chirality.

Triaxiality in nuclei had been a longstanding prediction of theory, but had proved very difficult to establish experimentally.

The best example of wobbling has been seen in the Lu nuclei.

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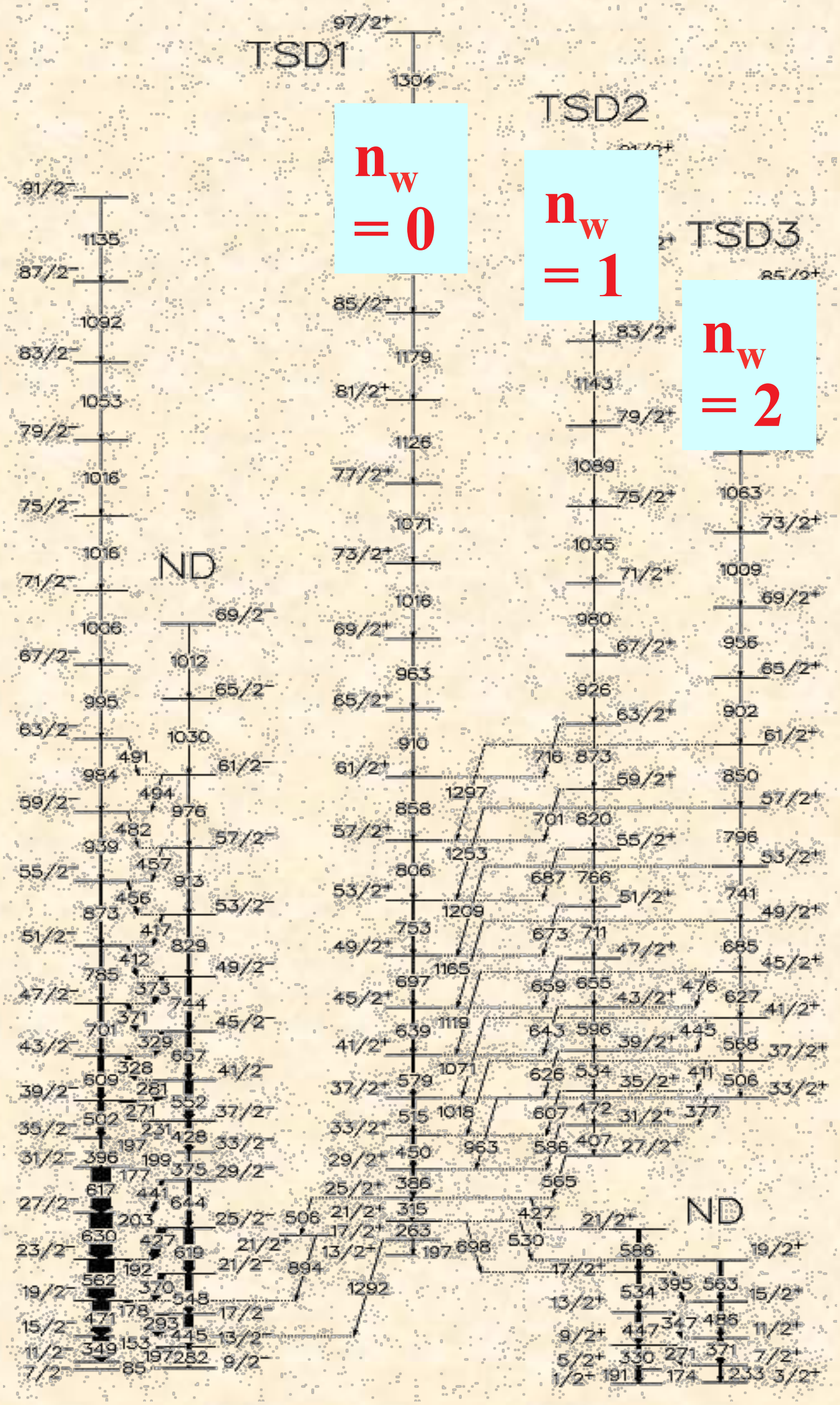


$n_w = 0$

$n_w = 1$

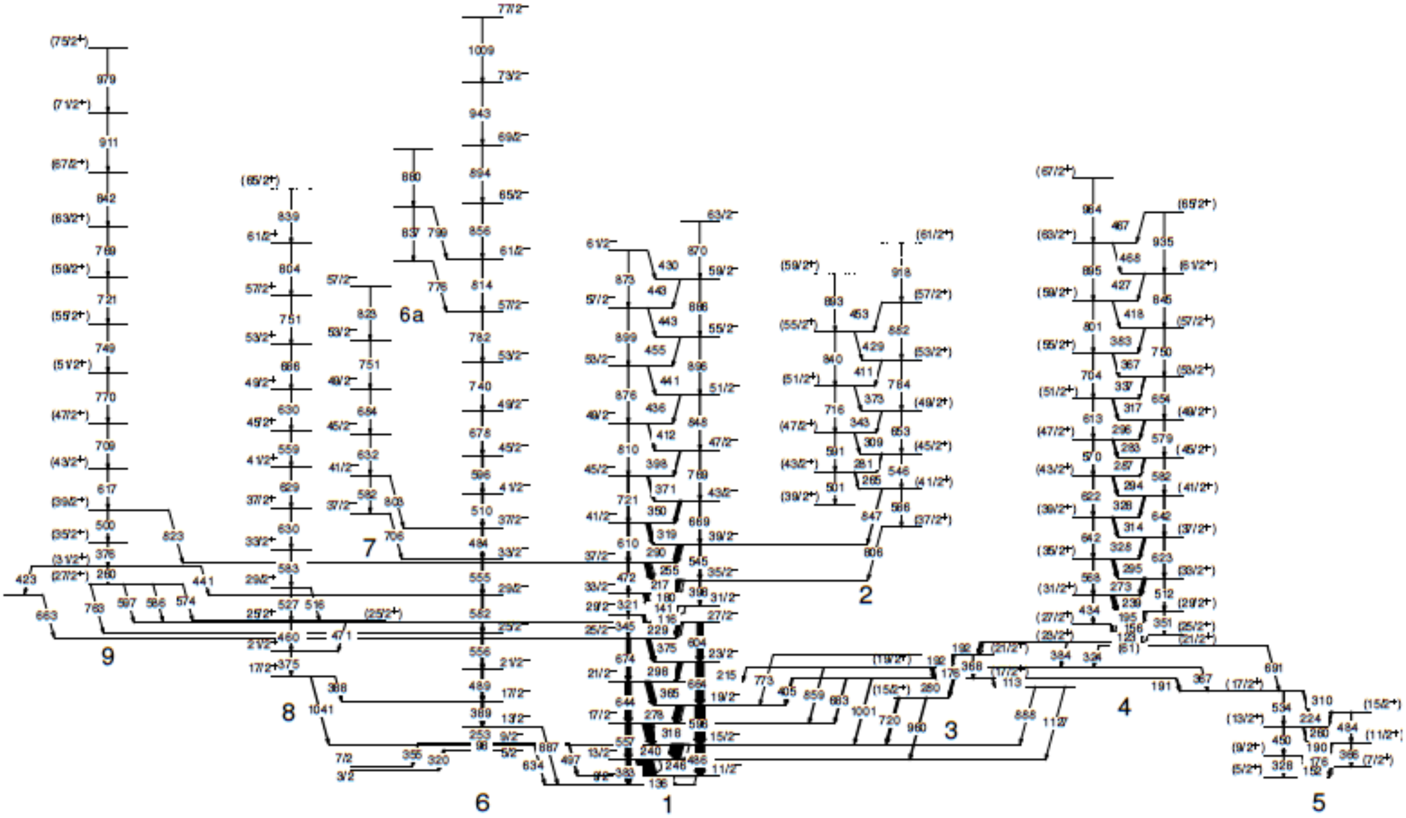
$n_w = 2$

^{163}Lu

“Wobbler Bands”

- Rotational bands corresponding to $n_\omega = 0, 1, 2,\ldots\ldots$
- Transitions from $n_{\omega+1} \rightarrow n_\omega$ [“one way” and $\Delta n_\omega = +1$]
- Interband transitions are $\Delta J = 1$, $E2$

Observed only in ^{161,163,165,167}Lu and ^{167}Ta



Nine decay sequences are now known in ^{169}Re including the $i_{13/2}$ band that is the basis for wobbling structures in nearby nuclei.
Wobbling could not be identified in this nucleus as the $i_{13/2}$ band was relatively weakly populated due to its location high in energy relative to the yrast structures.

S. D. J. Hartley *et* al., Phys. Rev. C **87**, 024315 (2013)

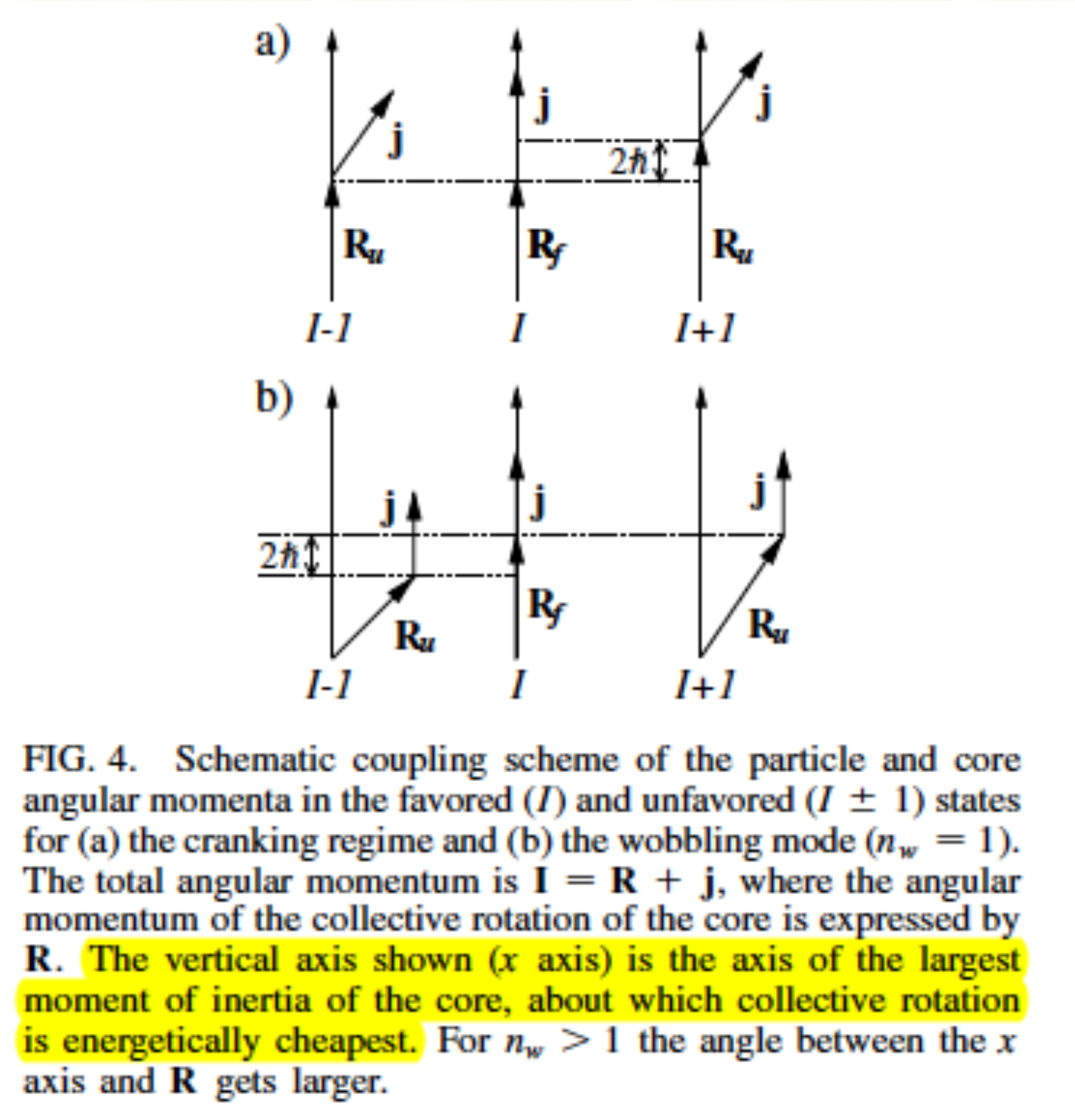


FIG. 4. Schematic coupling scheme of the particle and core angular momenta in the favored (I) and unfavored ($I \pm 1$) states for (a) the cranking regime and (b) the wobbling mode ($n_w = 1$). The total angular momentum is $\mathbf{I} = \mathbf{R} + \mathbf{j}$, where the angular momentum of the collective rotation of the core is expressed by $\mathbf{R}$. The vertical axis shown (x axis) is the axis of the largest moment of inertia of the core, about which collective rotation is energetically cheapest. For $n_w > 1$ the angle between the x axis and $\mathbf{R}$ gets larger.

S.W. Odegard *et al.*, Phys. Rev. Lett. **86**, 5866 (2001)

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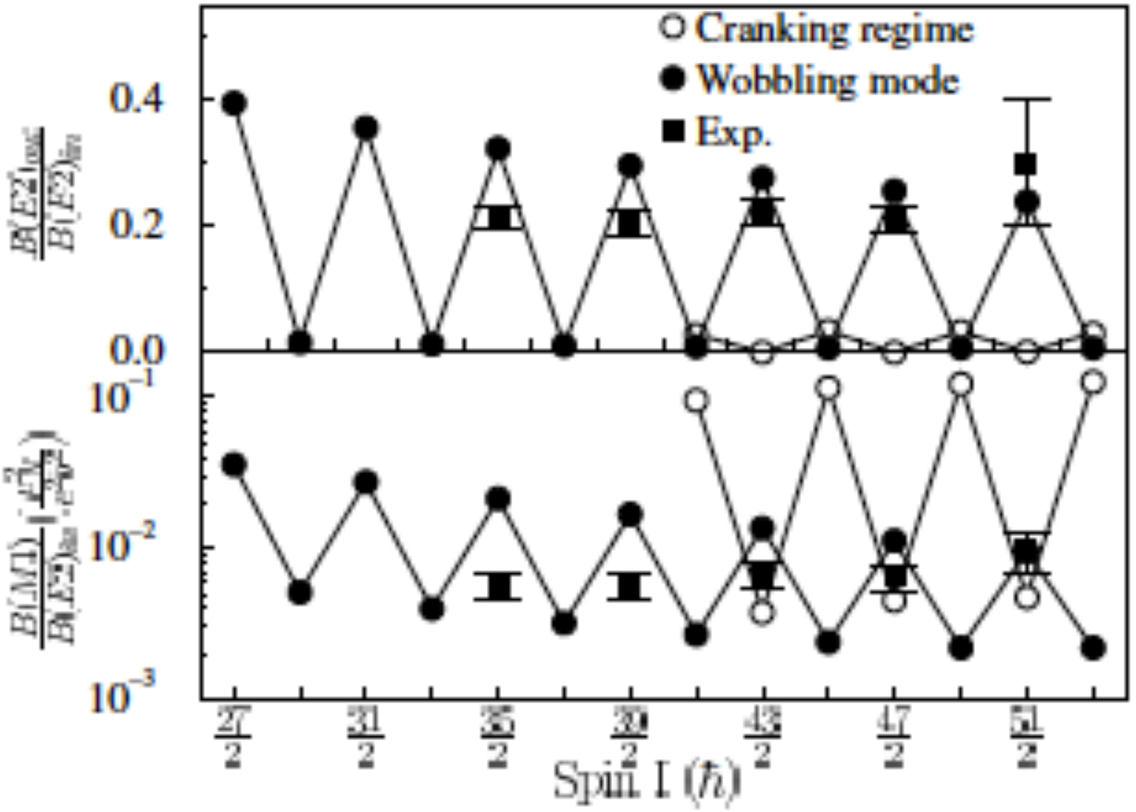
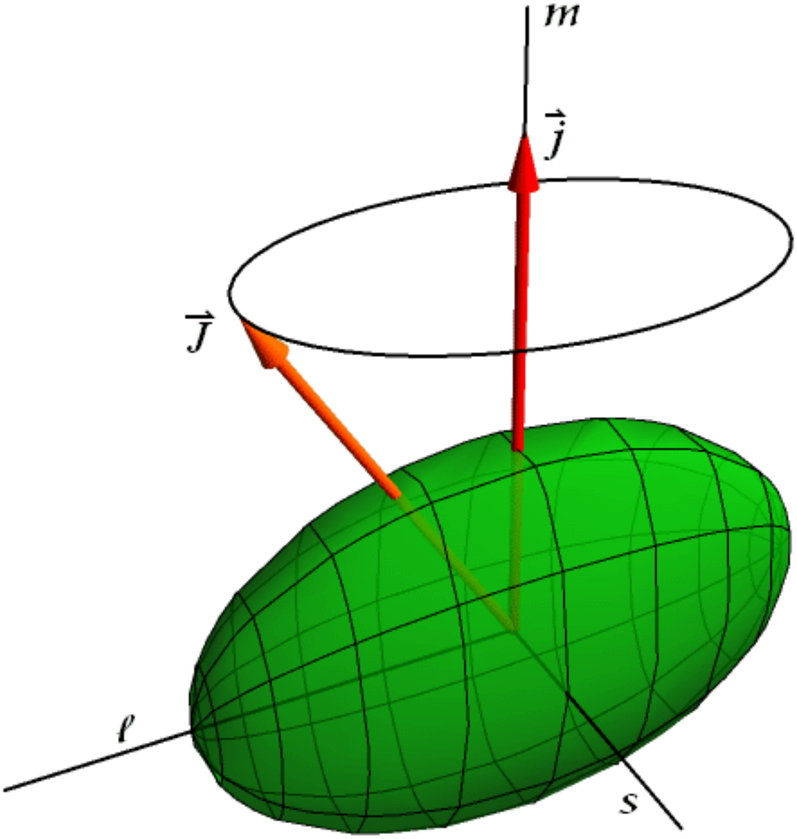


FIG. 5. Experimental and calculated electromagnetic properties of the connecting transitions.


m
$\vec{j}$
$\vec{J}$
ℓ
s

Wobbling frequency, defined by:

$E_{\text{wobb}} = E(I, n_\omega=1) - [E(I+1, n_\omega=0) + E(I-1, n_\omega=0)]/2$

$$\hbar\omega_w = \frac{j}{\mathcal{J}_3}\left[\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_1}-1\right)\right)\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_2}-1\right)\right)\right]^{1/2}$$

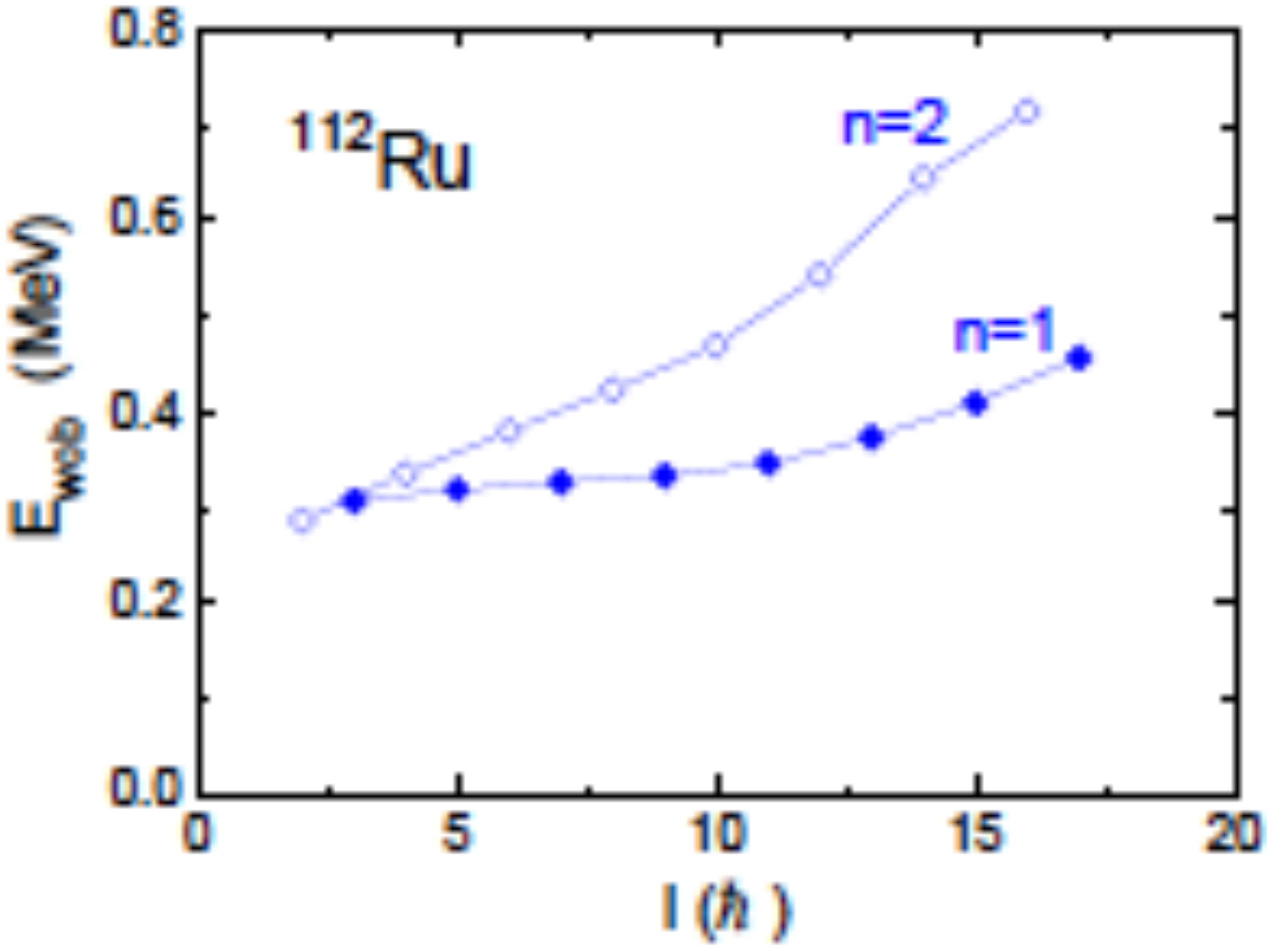
“Longitudinal “ wobbler:

Odd-particle aligned with the axis with maximum moment of inertia (the “medium axis)

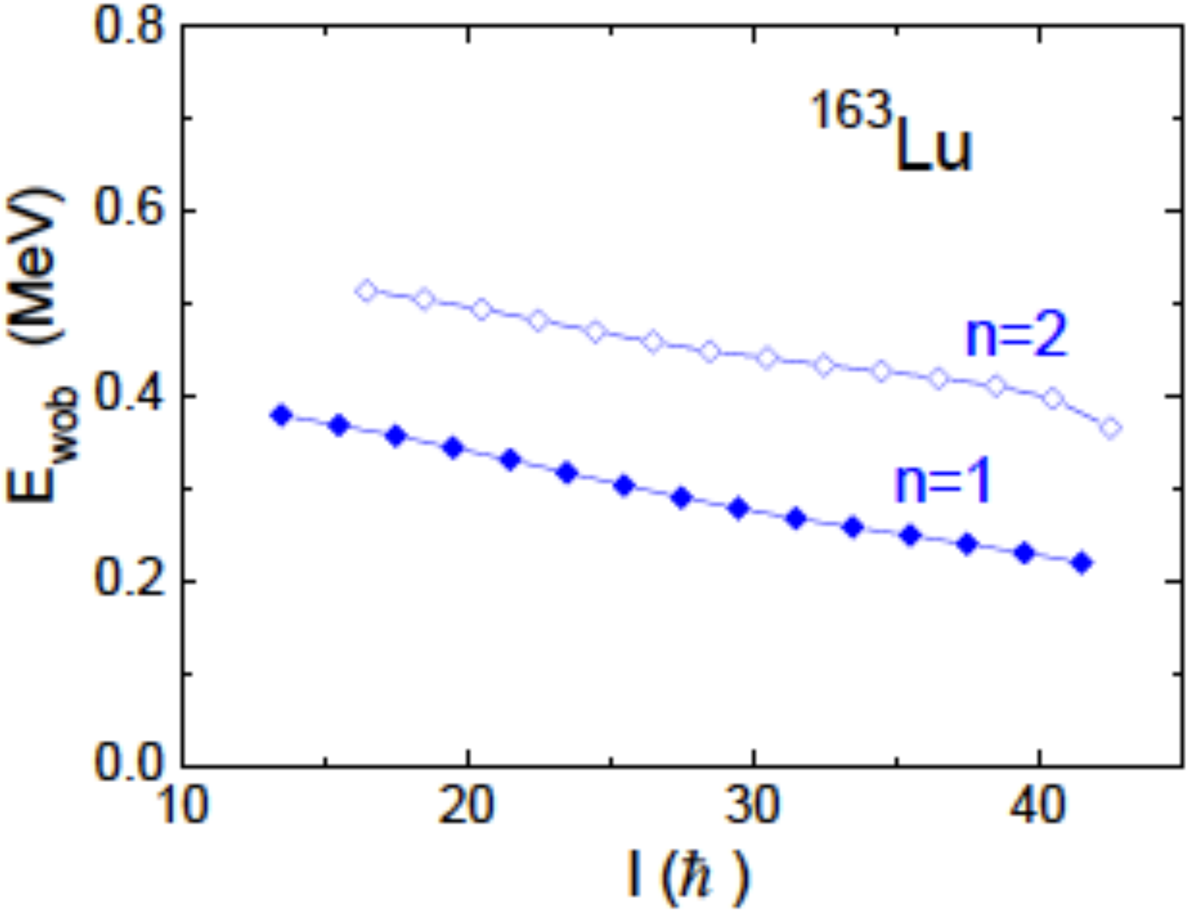
$\mathfrak{J}_3 > \mathfrak{J}_2$; $\mathfrak{J}_3 > \mathfrak{J}_1$

→ E_ω increases with J

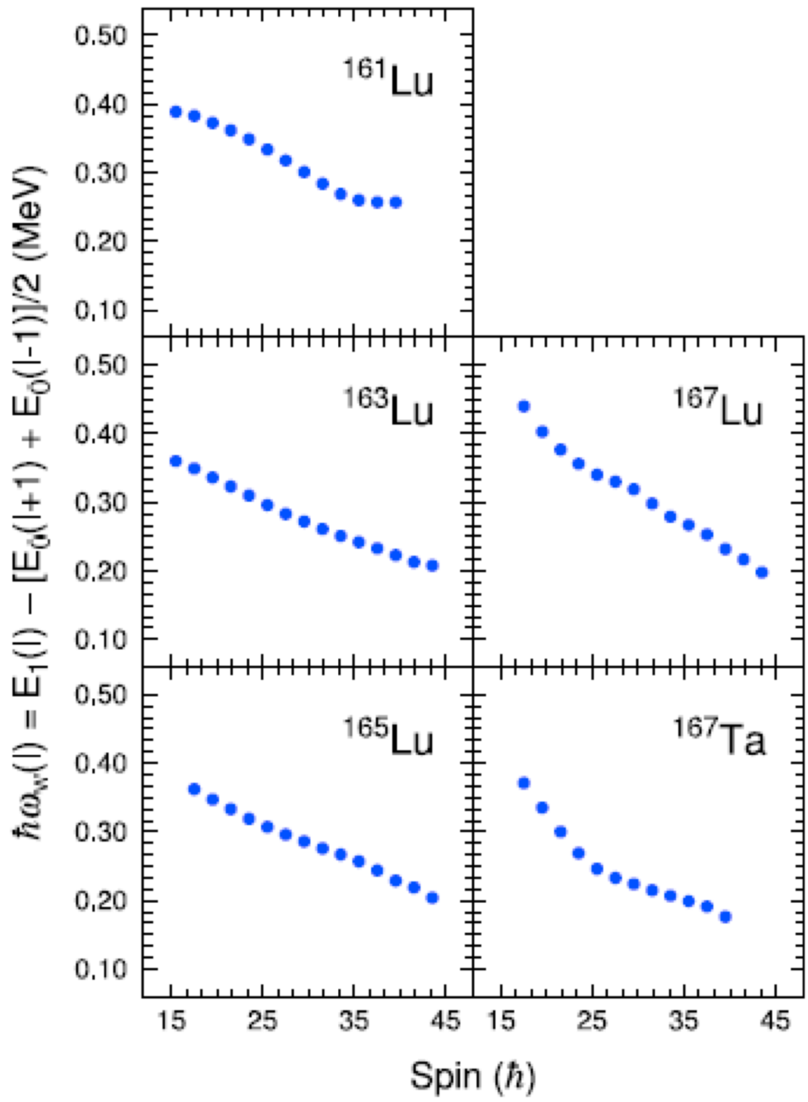
S. Frauendorf and F. Doenau, Phys. Rev. C **89**, 014322 (2014)



112Ru
n=2
n=1
E_wob (MeV)
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
I (ħ)



“Standard” wobbler would have increasing E_{wobb}!


$\hbar\omega_w(I) = E_1(I) - [E_0(I+1) + E_0(I-1)]/2$ (MeV)
0.50
0.40
0.30
0.20
0.10
^{161}Lu
^{163}Lu
^{167}Lu
^{165}Lu
^{167}Ta
15
25
35
45
Spin ($\hbar$)

Wobbling frequency, defined by:

$E_{wobb} = E\,(I, n_\omega=1) - [E\,(I+1, n_\omega=0) + E\,(I-1, n_\omega=0)]/2$

$$\hbar\omega_w = \frac{j}{\mathcal{J}_3}\left[\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_1}-1\right)\right)\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_2}-1\right)\right)\right]^{1/2}$$

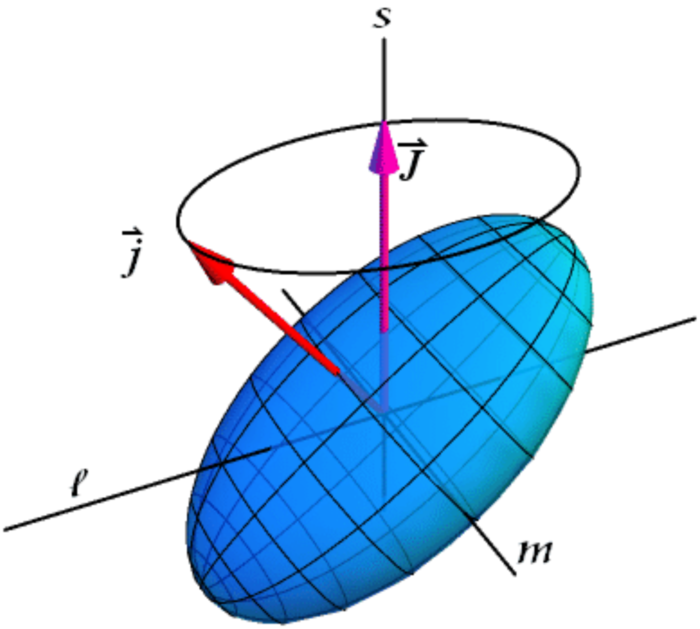
“Transverse“ wobbler:

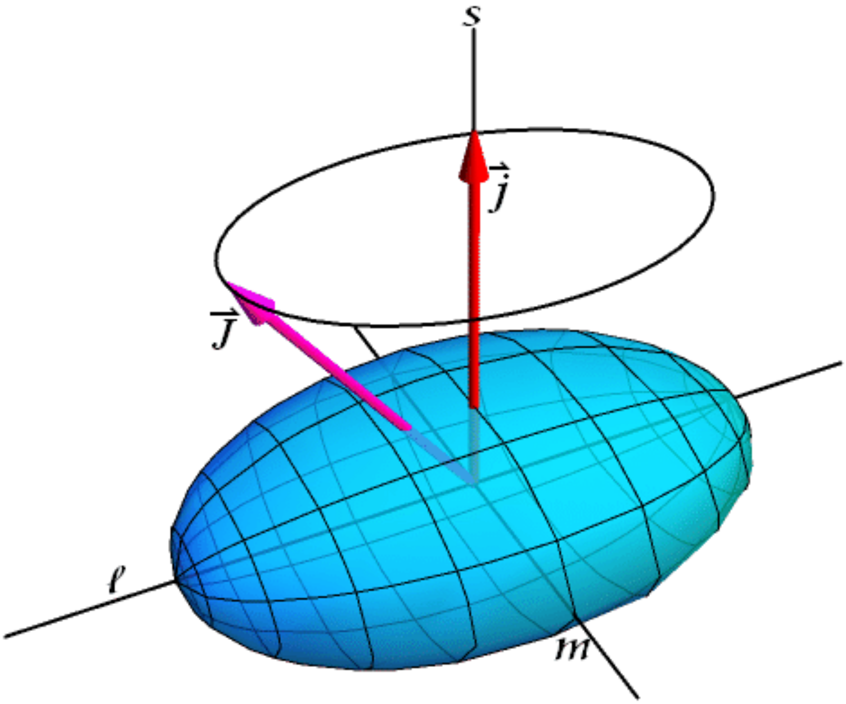
Odd-particle aligned with the “small” axis

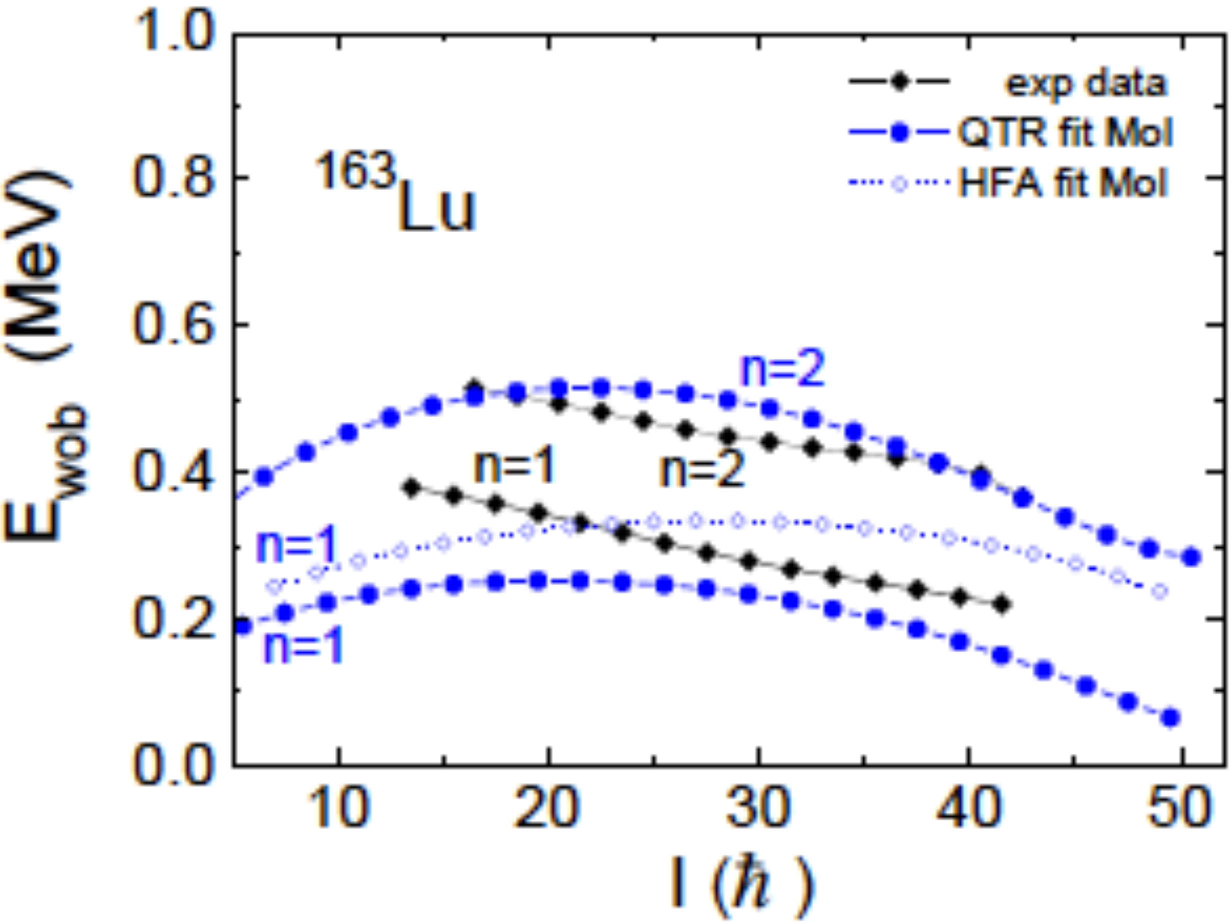
$\mathfrak{J}_3 < \mathfrak{J}_2\,;\ \mathfrak{J}_3 > \mathfrak{J}_1$

→ E_ω decreases with J

reaching 0 at $J_c = j\,\mathfrak{J}_2/(\mathfrak{J}_2 - \mathfrak{J}_3)$


s
$\vec{J}$
$\vec{j}$
ℓ
m


s
$\vec{j}$
$\vec{J}$
ℓ
m



S. Frauendorf and F. Doenau, Phys. Rev. C **89**, 014322 (2014)

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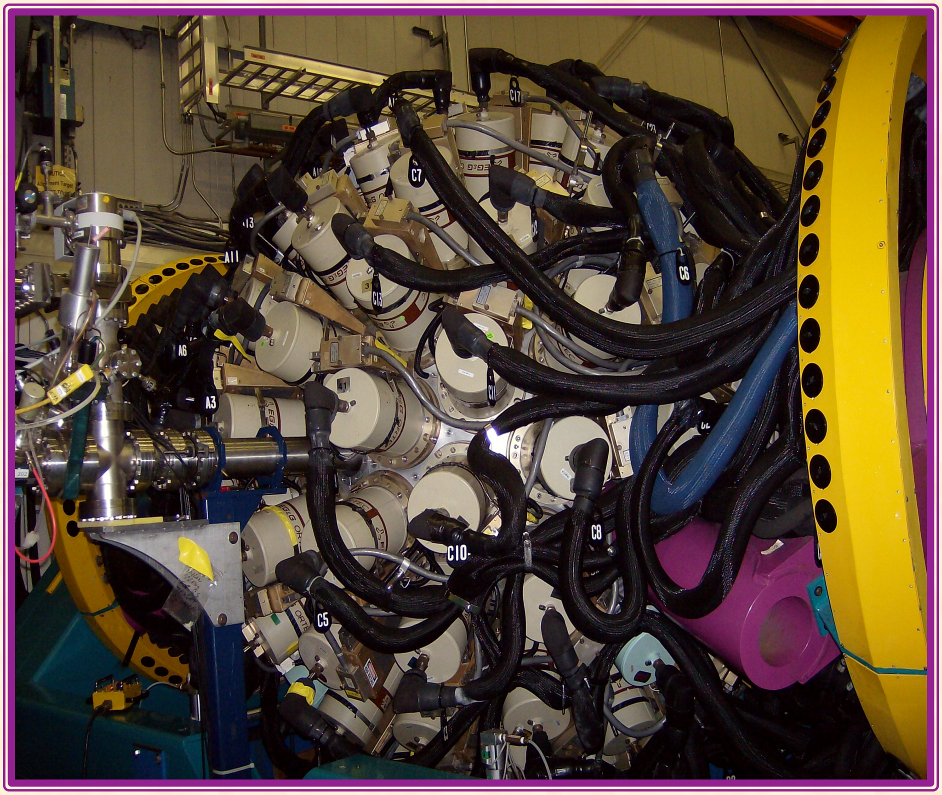
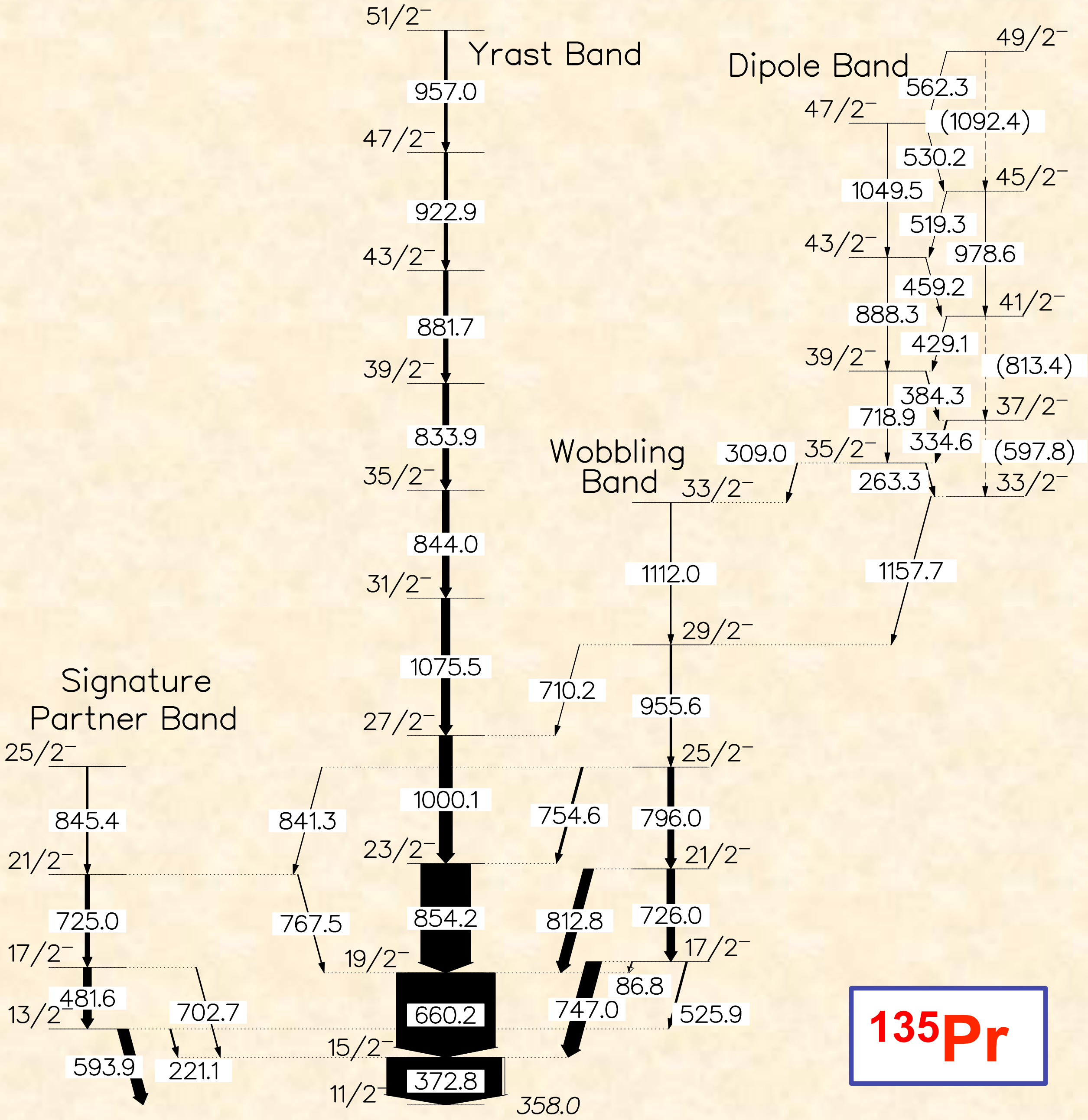
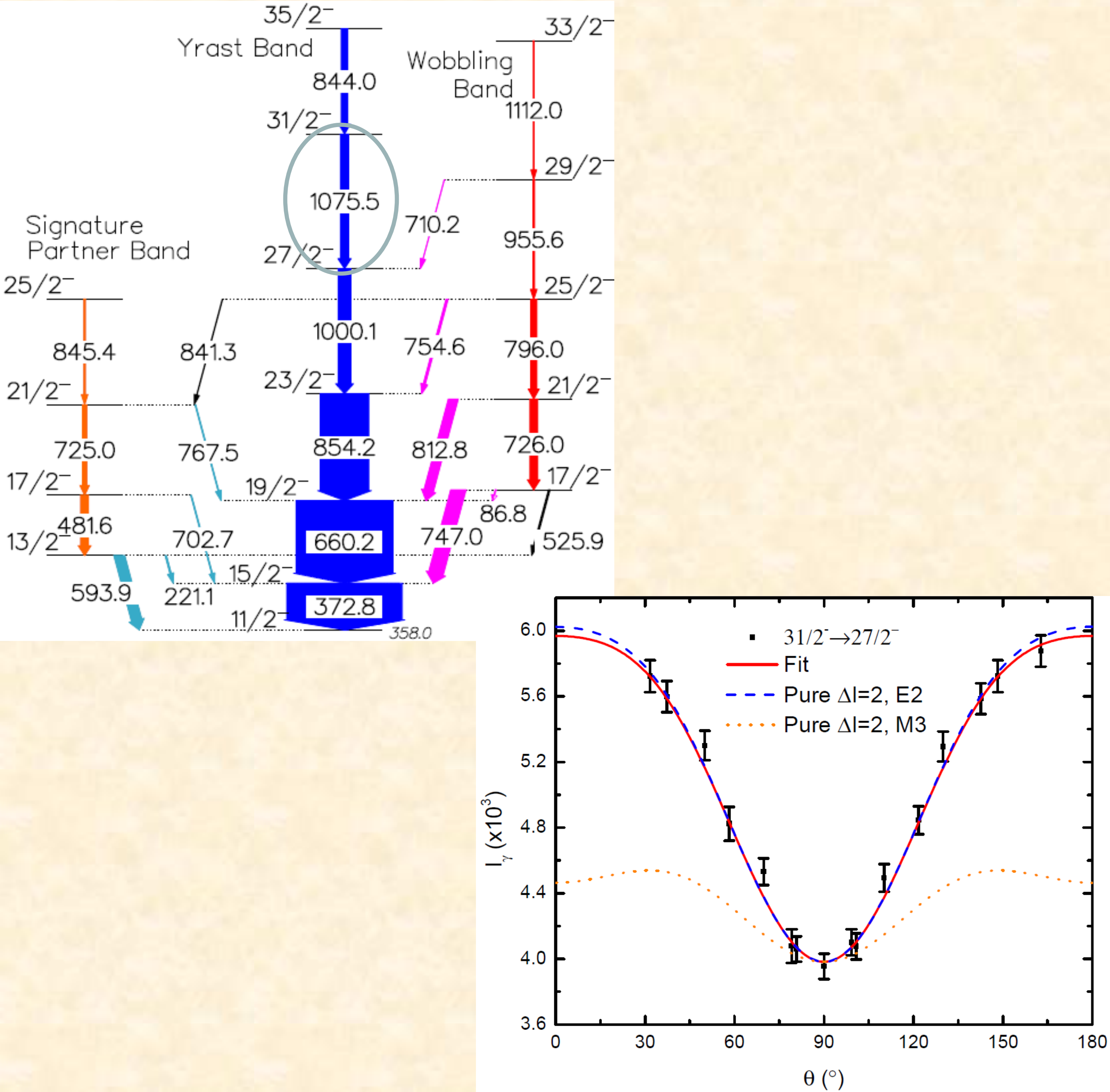


^{123}Sb (^{16}O, 4n)^{135}Pr @ 80 MeV
Gammasphere at ATLAS
(100 CSGe detectors)
γ–γ–γ coincidences
angular correlations



^{135}Pr

J. T. Matta *et al.*, Phys. Rev. Lett. **114**, 082501 (2015)


35/2⁻
Yrast Band
844.0
31/2⁻
1075.5
27/2⁻
1000.1
23/2⁻
854.2
19/2⁻
660.2
15/2⁻
372.8
11/2⁻
358.0
33/2⁻
Wobbling Band
1112.0
29/2⁻
710.2
955.6
25/2⁻
754.6
796.0
21/2⁻
812.8
726.0
17/2⁻
86.8
747.0
525.9
Signature Partner Band
25/2⁻
845.4
841.3
21/2⁻
725.0
767.5
17/2⁻
481.6
13/2⁻
702.7
593.9
221.1
31/2⁻→27/2⁻
Fit
Pure ΔI=2, E2
Pure ΔI=2, M3
I_γ (x10³)
6.0
5.6
5.2
4.8
4.4
4.0
3.6
0
30
60
90
120
150
180
θ (°)

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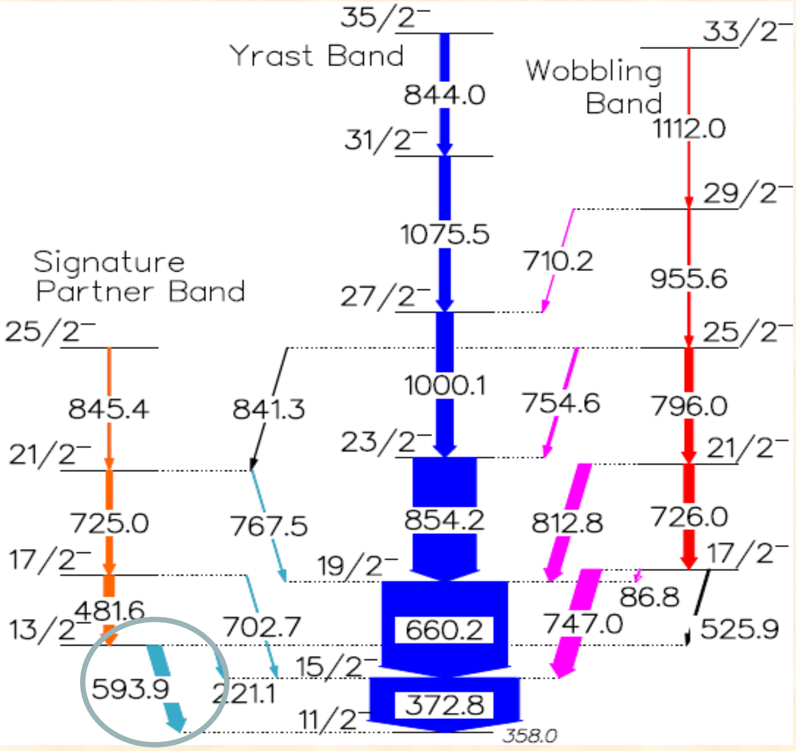
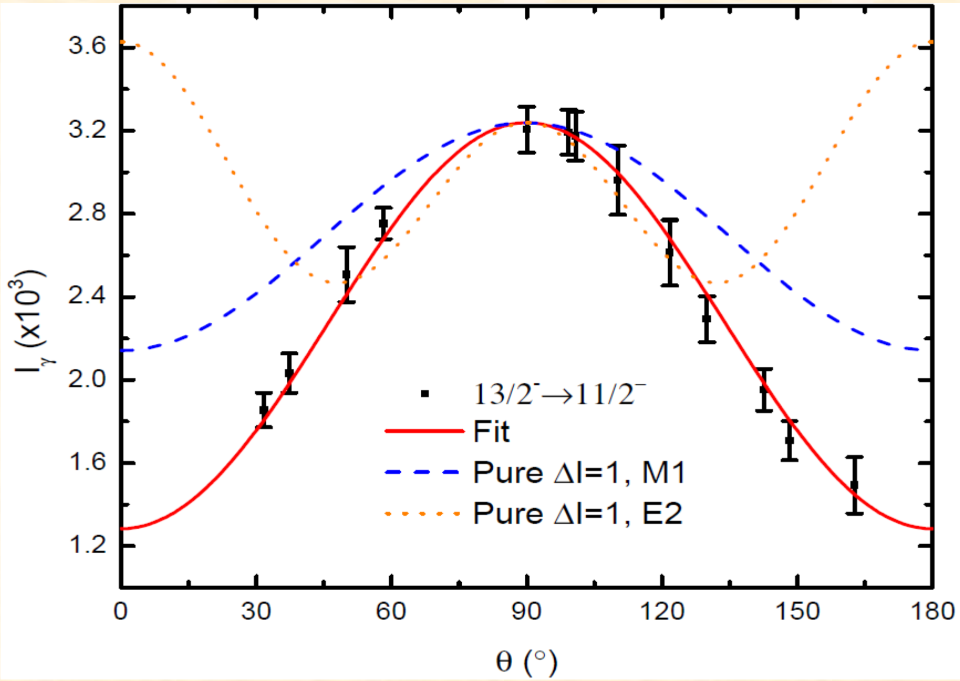
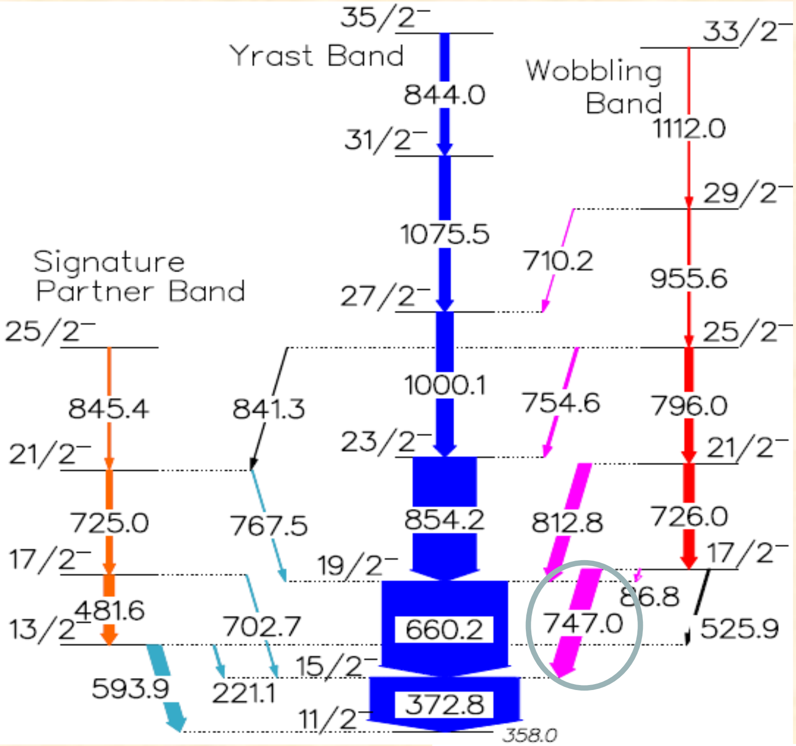


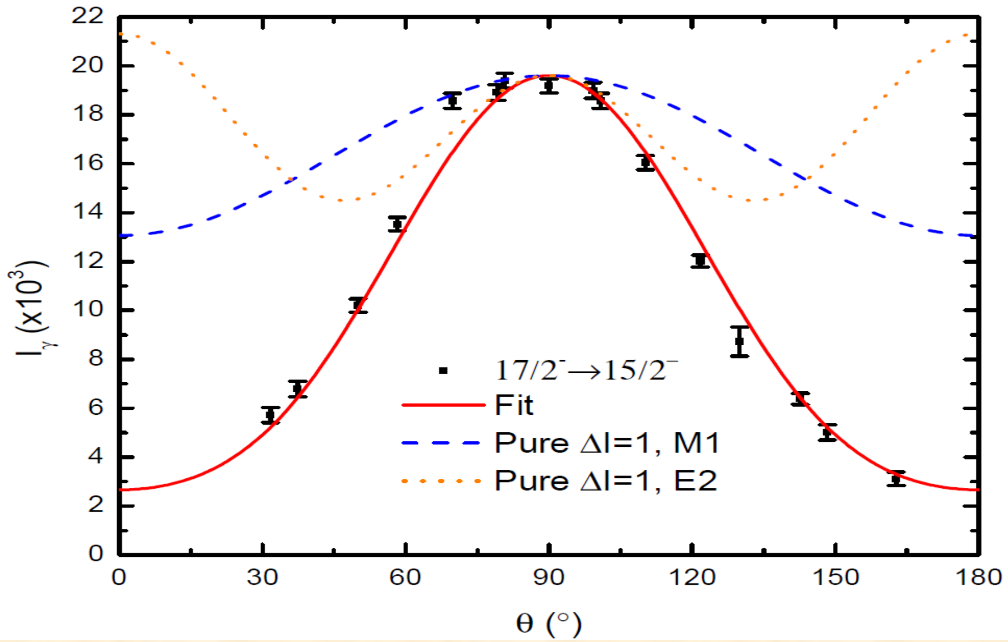
δ = -0.16 ± 0.04
E2% = 2.4 ± 1.2

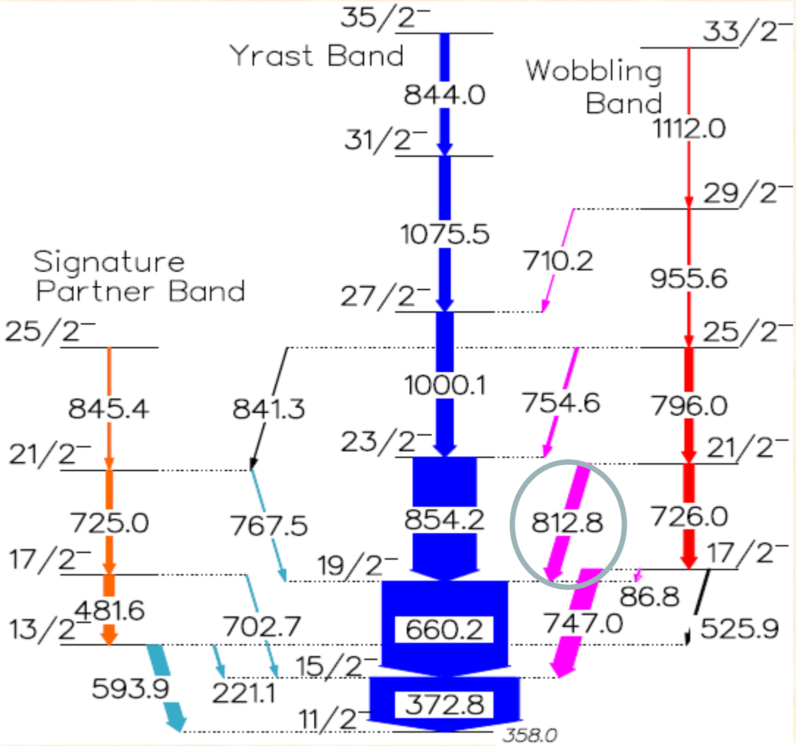




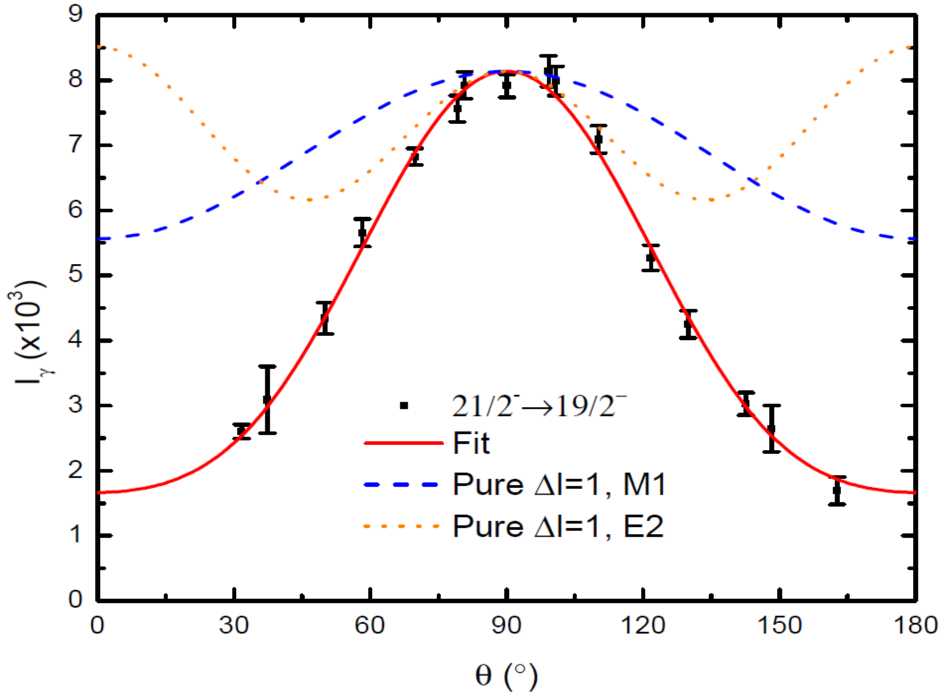
$\delta = -1.24 \pm 0.13$

$E2\% = 60.6 \pm 5.1$





$\delta = -1.54 \pm 0.09$
E2% = 70.3 ± 2.4



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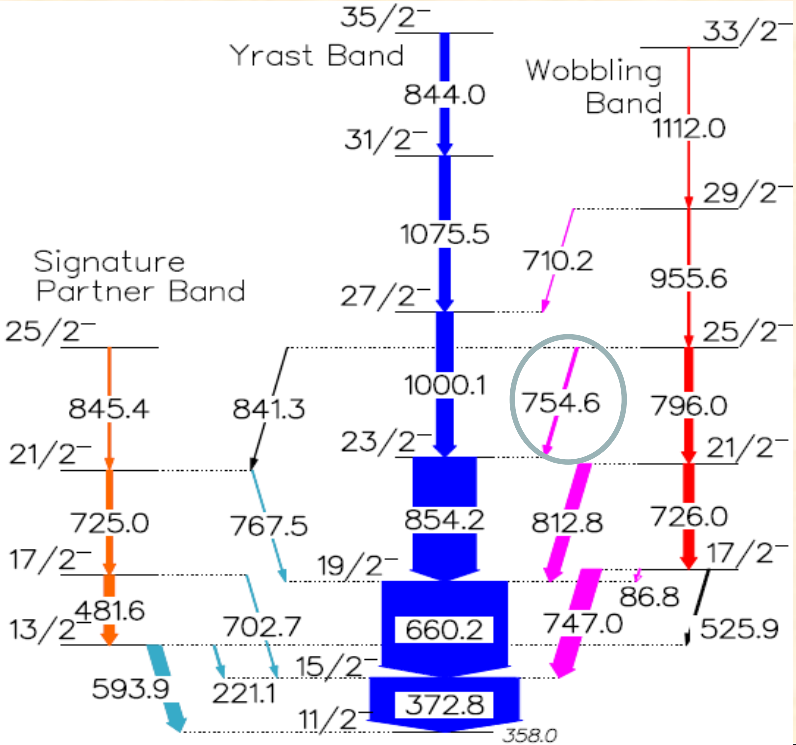
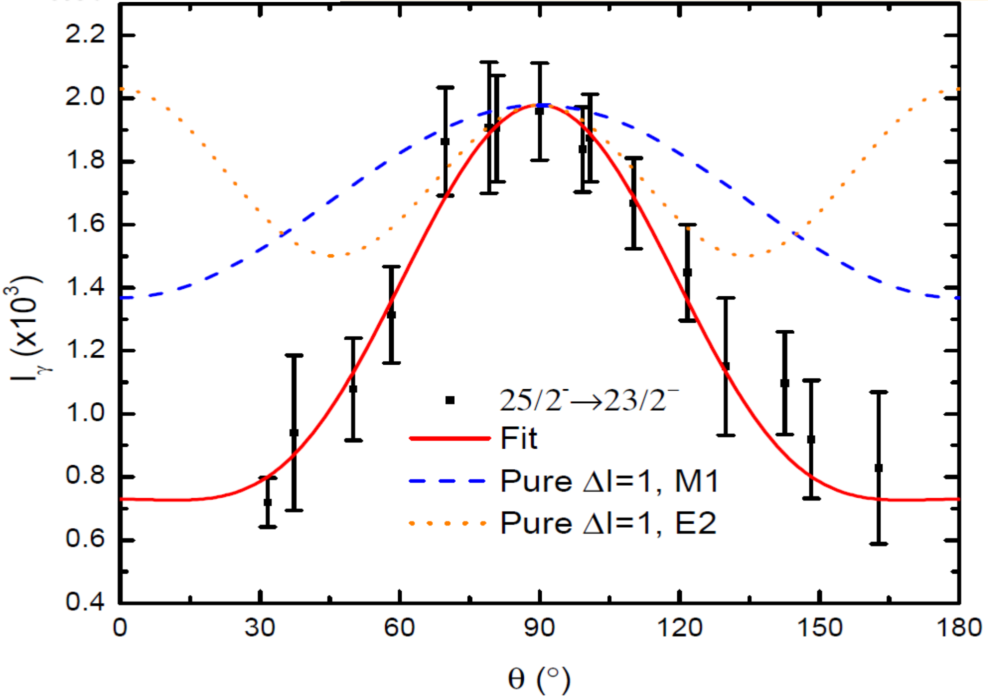
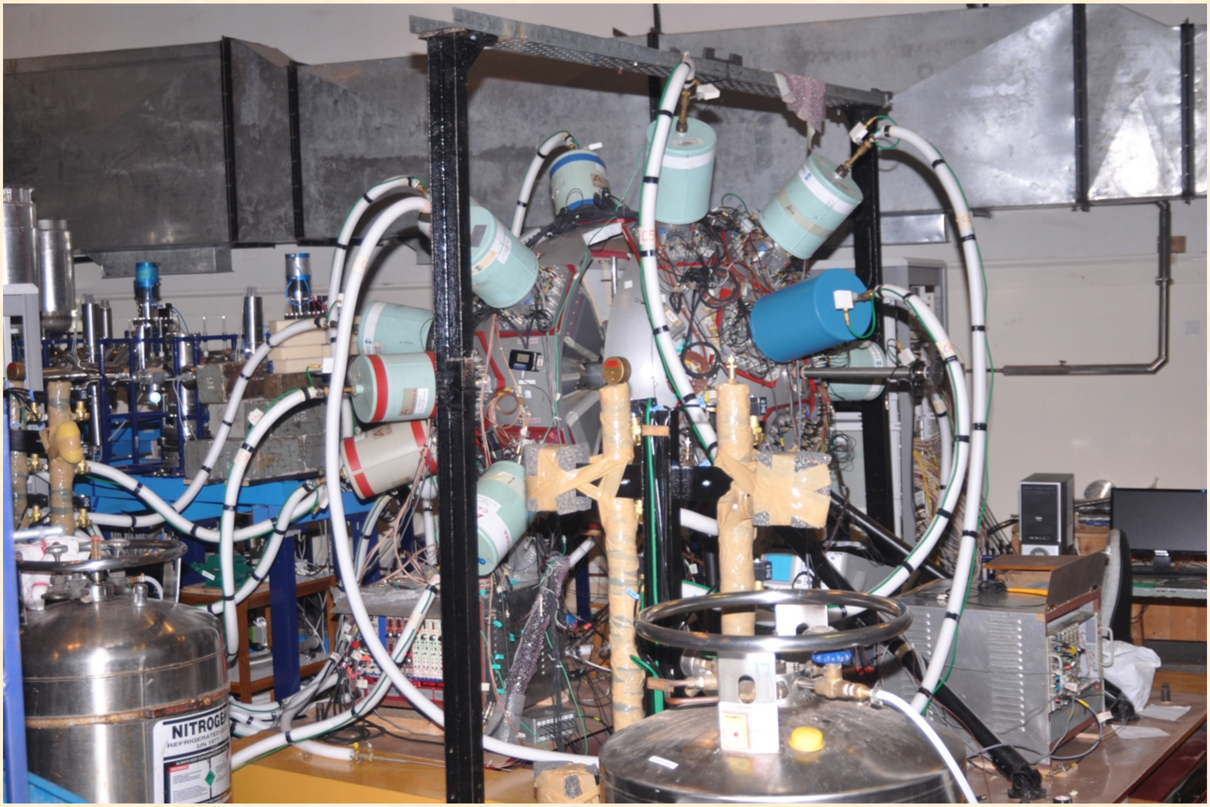


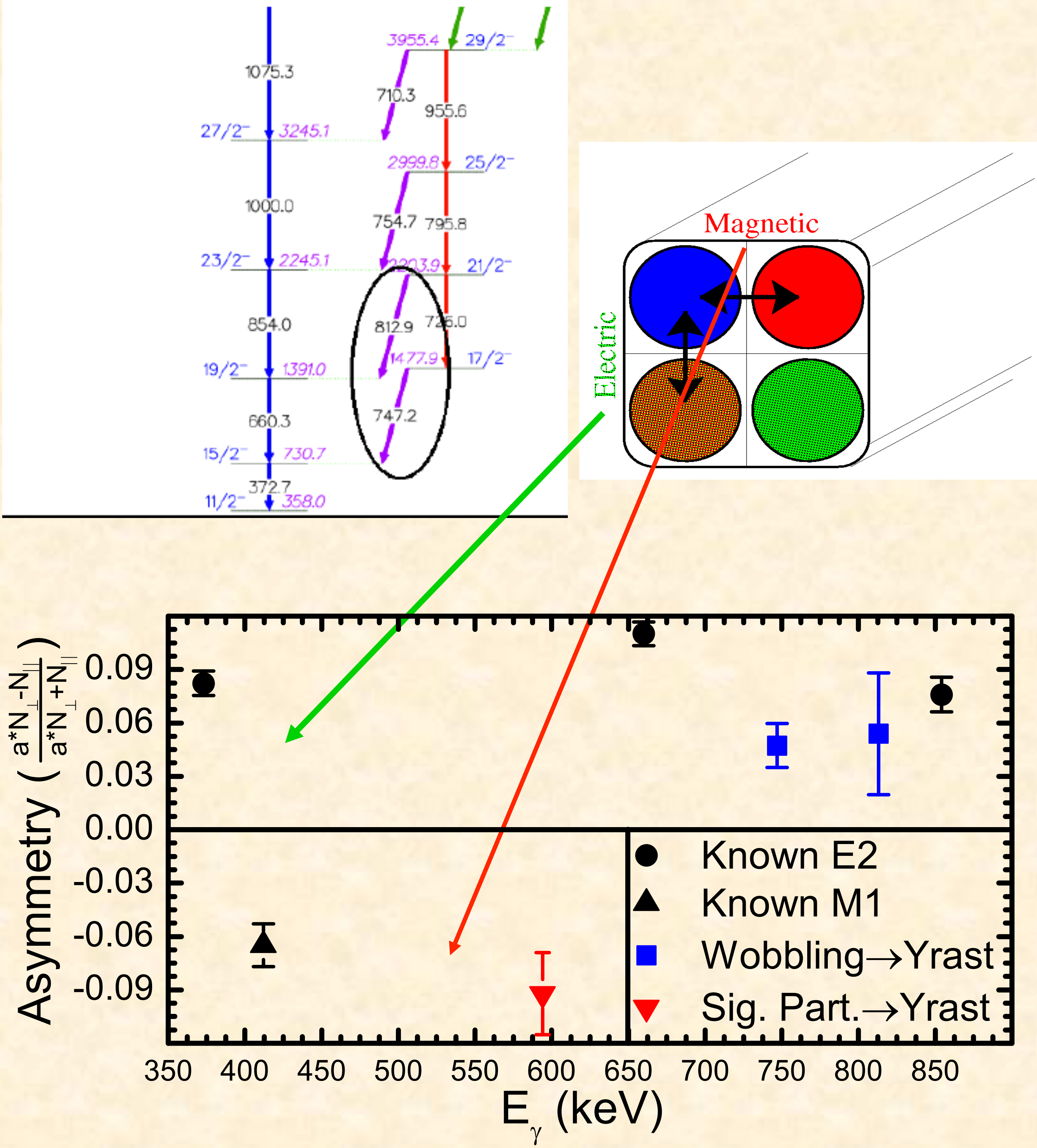
δ = -2.38 ± 0.37
E2% = 85.0 ± 4.0

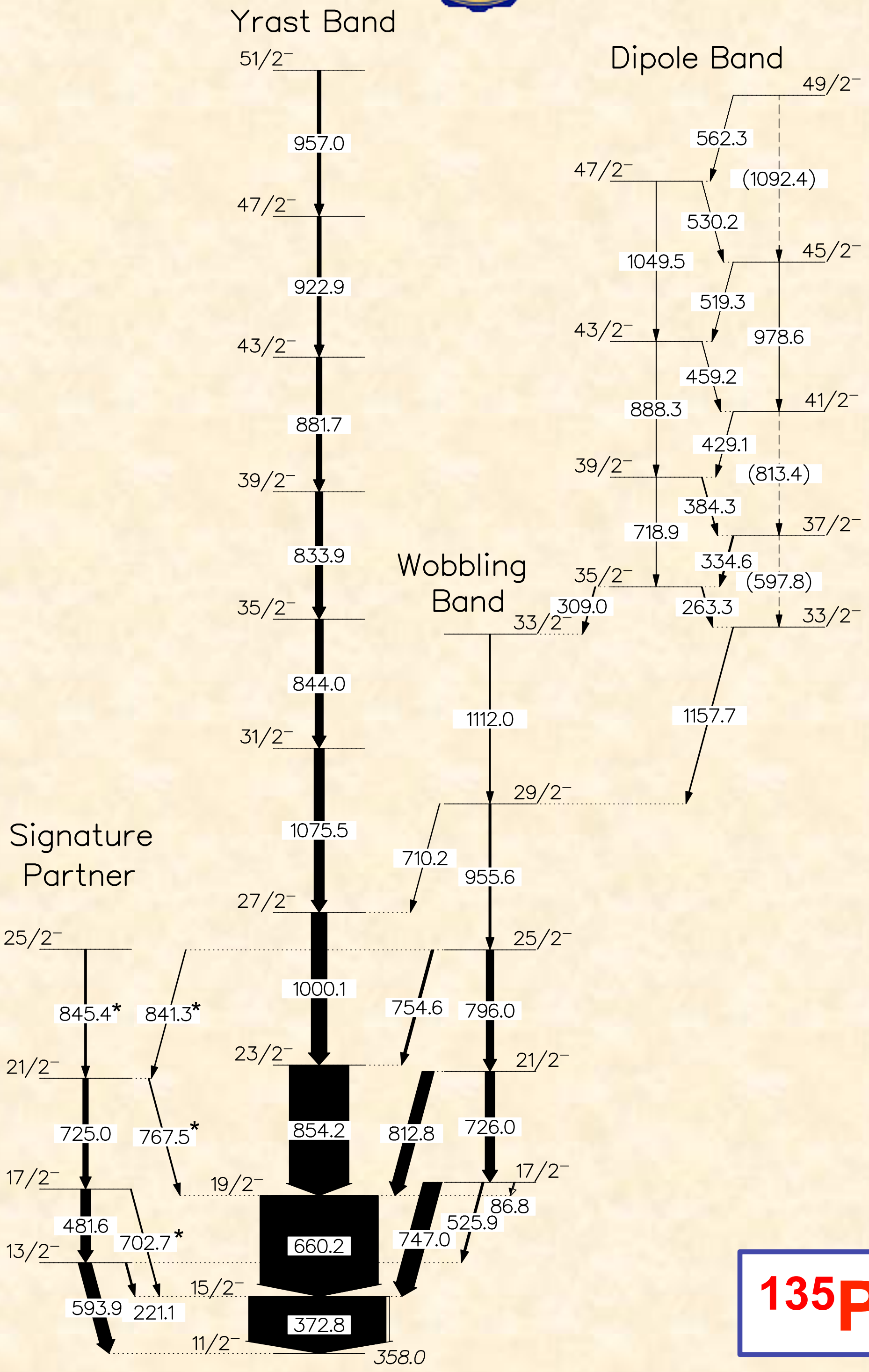




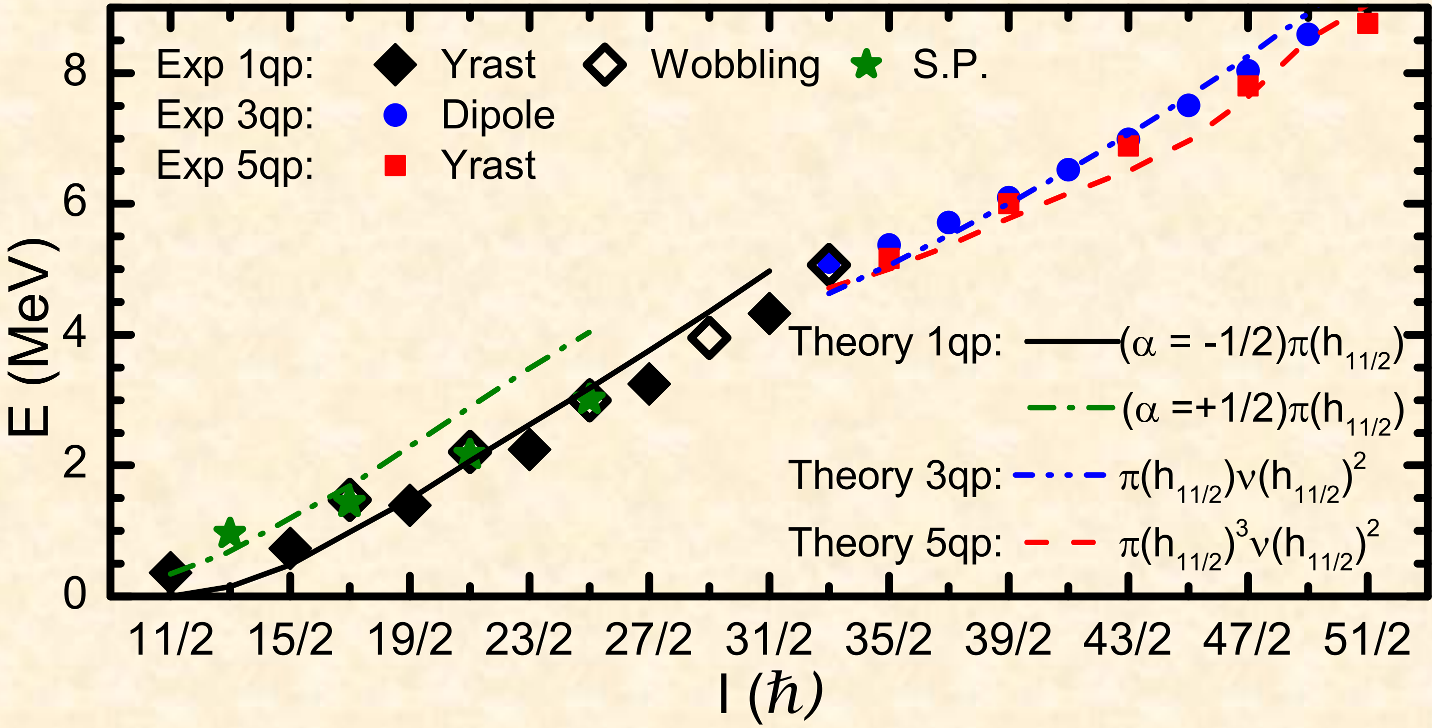
INGA @ TIFR
20 CS “clover” detectors
polarization measurements

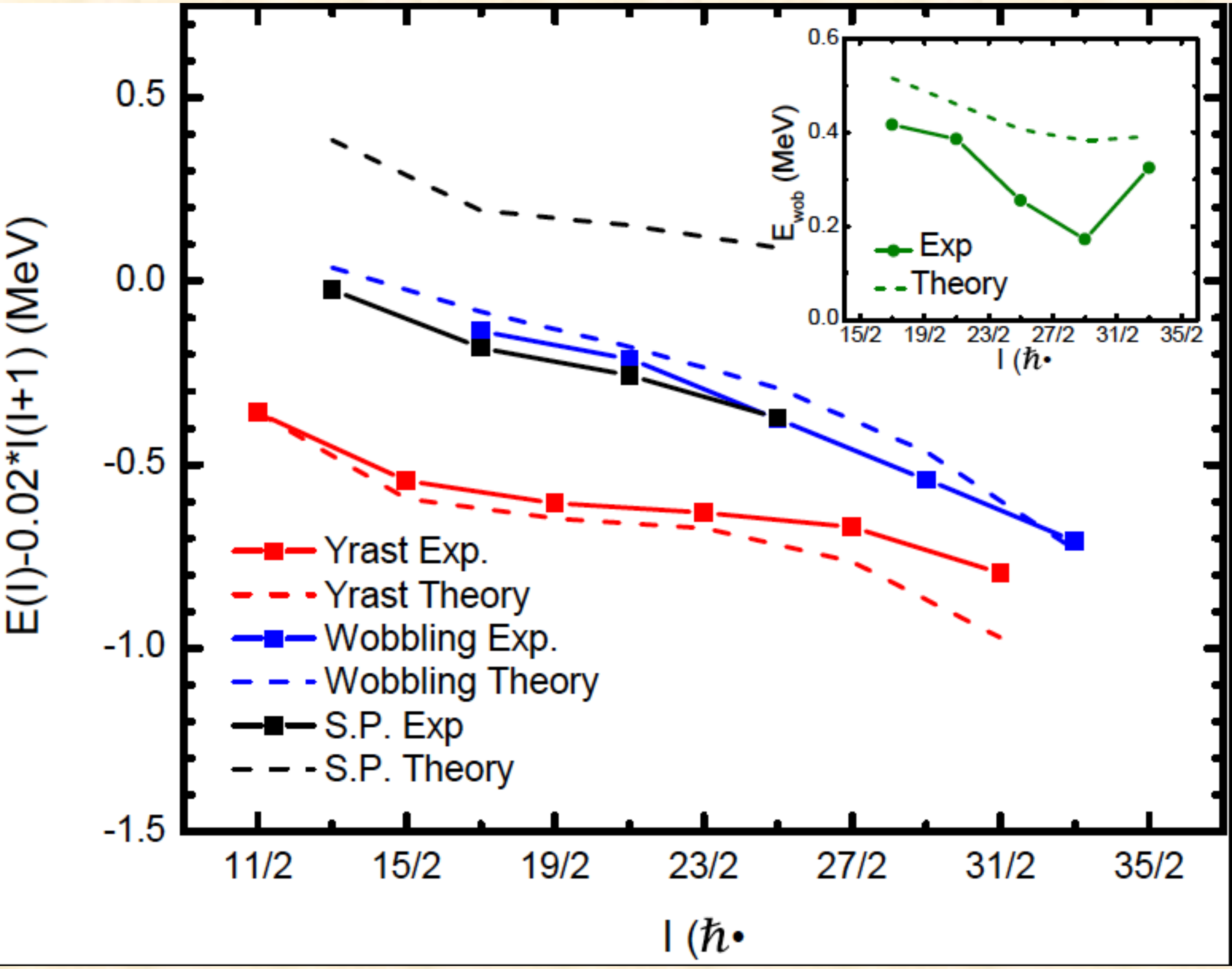
Polarization Asymmetries




Yrast Band
Dipole Band
Wobbling Band
Signature Partner
51/2⁻
957.0
47/2⁻
922.9
43/2⁻
881.7
39/2⁻
833.9
35/2⁻
844.0
31/2⁻
1075.5
27/2⁻
1000.1
23/2⁻
854.2
19/2⁻
660.2
15/2⁻
372.8
11/2⁻
358.0
49/2⁻
562.3
(1092.4)
47/2⁻
530.2
1049.5
45/2⁻
519.3
978.6
43/2⁻
459.2
888.3
41/2⁻
429.1
(813.4)
39/2⁻
384.3
718.9
37/2⁻
334.6
(597.8)
35/2⁻
309.0
263.3
33/2⁻
33/2⁻
1112.0
1157.7
29/2⁻
710.2
955.6
25/2⁻
754.6
796.0
21/2⁻
812.8
726.0
17/2⁻
86.8
525.9
747.0
25/2⁻
845.4*
841.3*
21/2⁻
725.0
767.5*
17/2⁻
481.6
702.7*
13/2⁻
593.9
221.1

^{135}Pr


Exp 1qp: ◆ Yrast ◇ Wobbling ★ S.P.
Exp 3qp: ● Dipole
Exp 5qp: ■ Yrast
Theory 1qp: $(\alpha = -1/2)\pi(h_{11/2})$
$(\alpha = +1/2)\pi(h_{11/2})$
Theory 3qp: $\pi(h_{11/2})\nu(h_{11/2})^2$
Theory 5qp: $\pi(h_{11/2})^3\nu(h_{11/2})^2$
E (MeV)
0
2
4
6
8
I (ħ)
11/2
15/2
19/2
23/2
27/2
31/2
35/2
39/2
43/2
47/2
51/2


E(I)-0.02*I(I+1) (MeV)
0.5
0.0
-0.5
-1.0
-1.5
11/2
15/2
19/2
23/2
27/2
31/2
35/2
I (ħ•
Yrast Exp.
Yrast Theory
Wobbling Exp.
Wobbling Theory
S.P. Exp
S.P. Theory
E_{wob} (MeV)
0.6
0.4
0.2
0.0
15/2
19/2
23/2
27/2
31/2
35/2
I (ħ•
Exp
Theory

✓ Measurements of level energies, angular distributions, and polarizations of the associated γ rays, have established a "wobbler" sequence in ^{135}Pr.

First observation of wobbling in any nuclei away from A~160 region.

✓ Comparison with calculations in QTR model establishes the observed structure as corresponding to a "transverse wobbler"

✓ The transmutation of the transverse wobbler into a longitudinal wobbler and then to a magnetic rotation structure is observed in line with theoretical predictions.

Clear indications of gradual change of the rotational axis from "short" into a planar geometry akin to magnetic rotation.

J. T. Matta *et al*., Phys. Rev. Lett. **114**, 082501 (2015)

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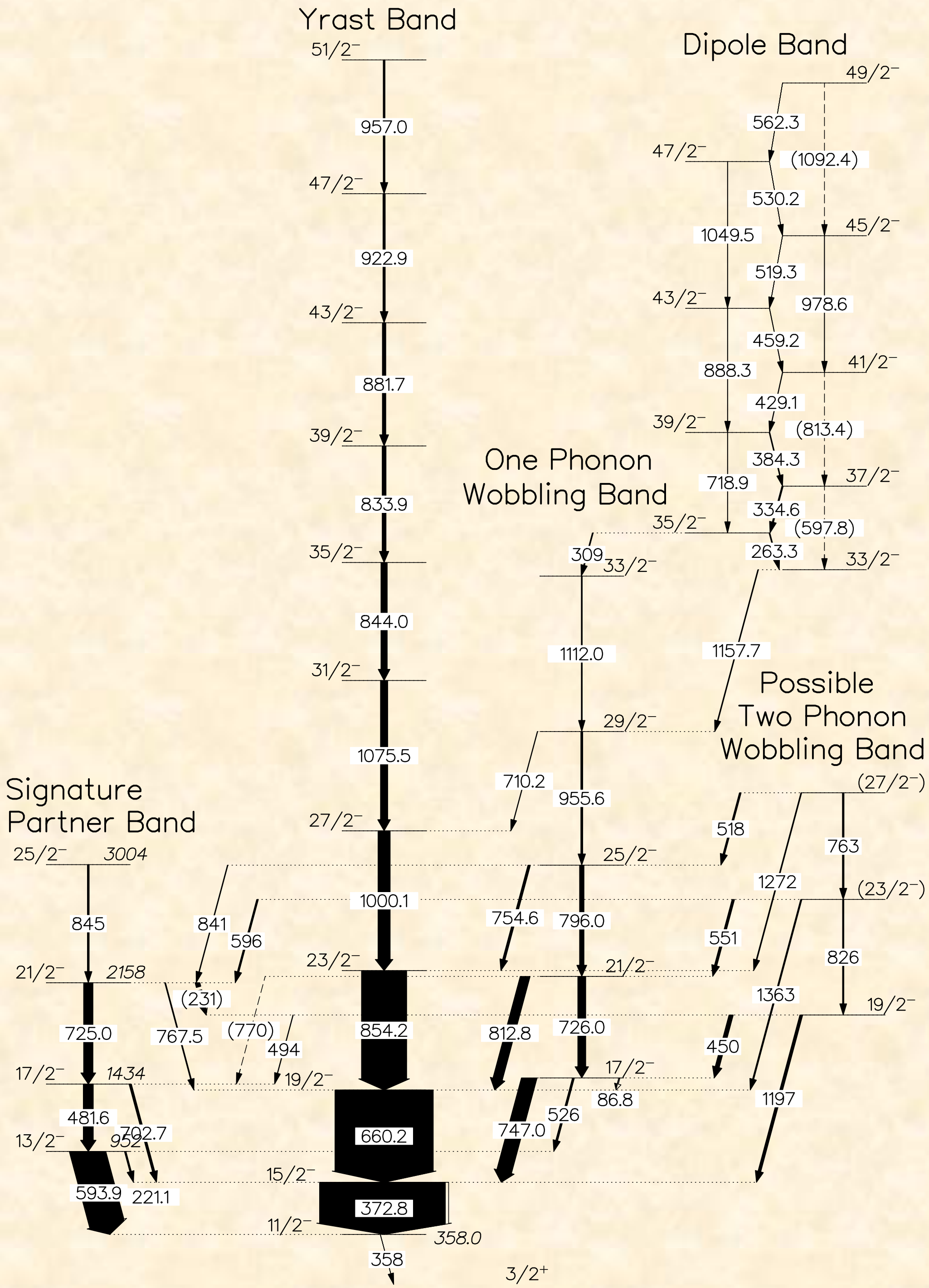

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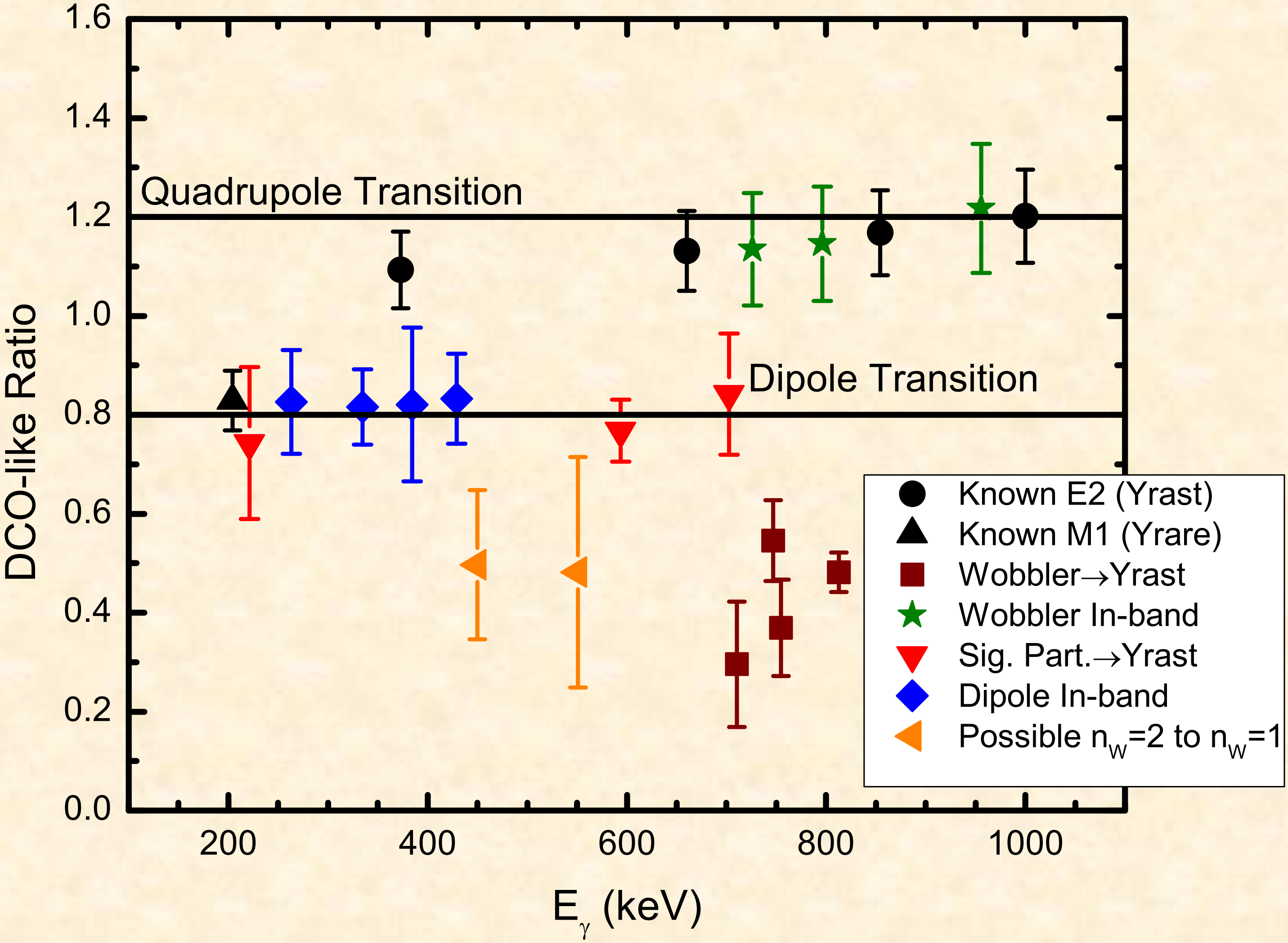
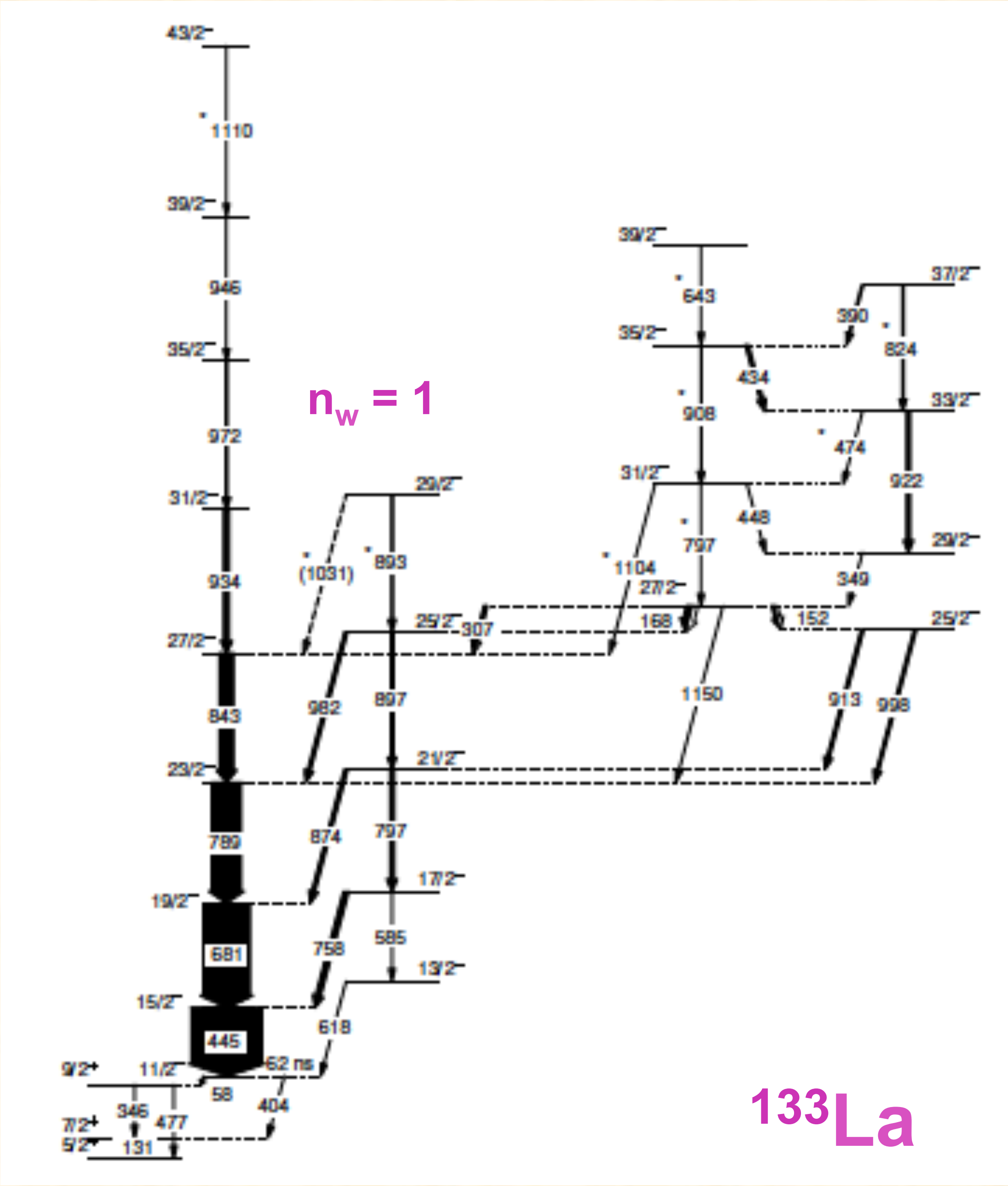
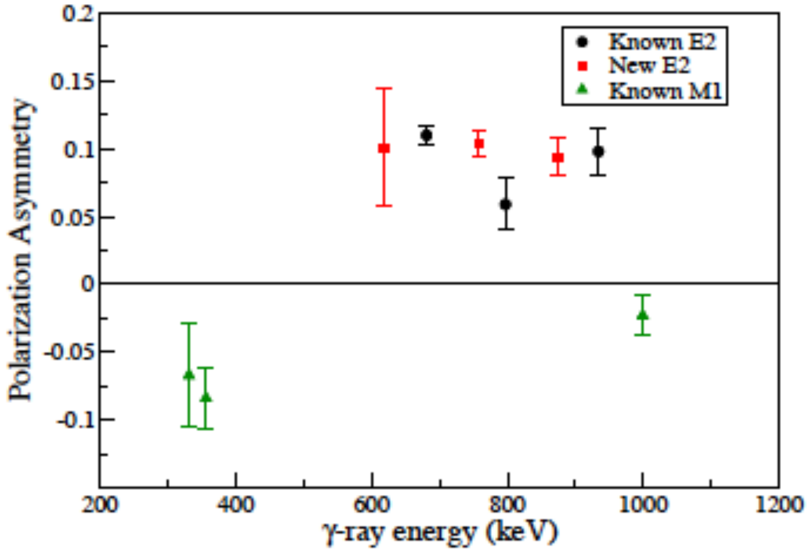




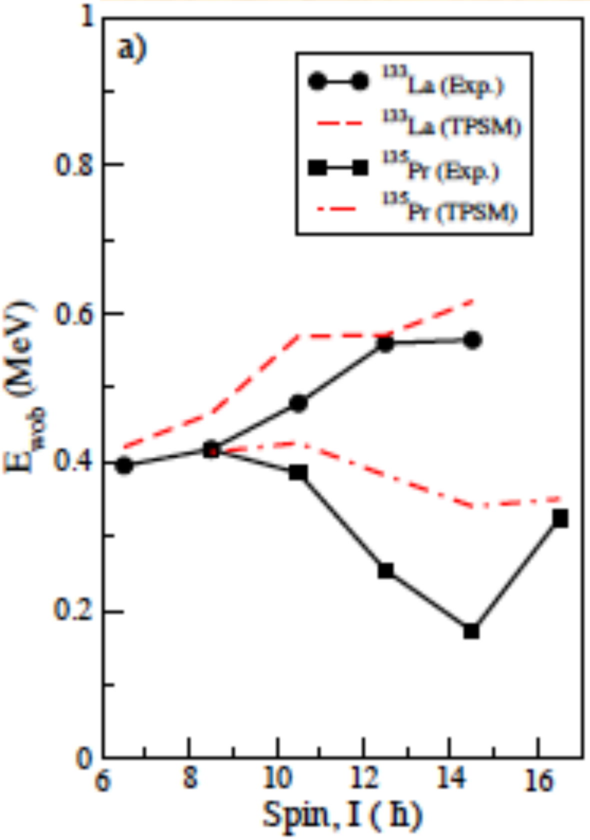
$n_w = 1$

^{133}La

Courtesy of R. Palit, TIFR, Mumbai



^{133}La n_{ω} = 1 band



Courtesy of R. Palit, TIFR, Mumbai

Octupole Condensation

- Strong octupole correlations have been observed in nuclei in the A~230 region.

  Such octupole correlations come from long-range interactions between valence nucleons occupying states with $\Delta j = \Delta l = 3$

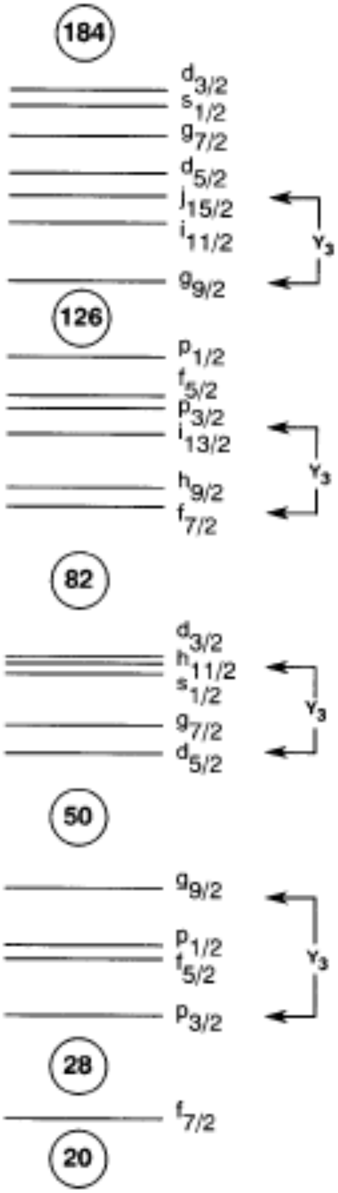


FIG. 4. Nuclear spherical single-particle levels. The most important octupole couplings are indicated.

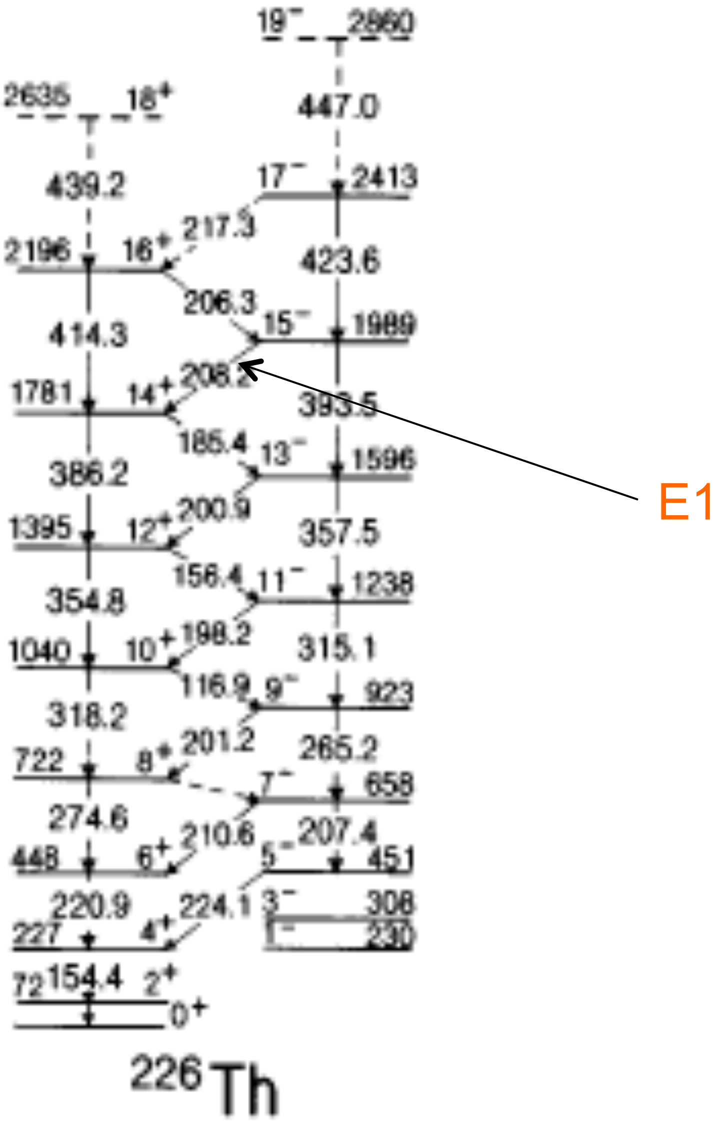
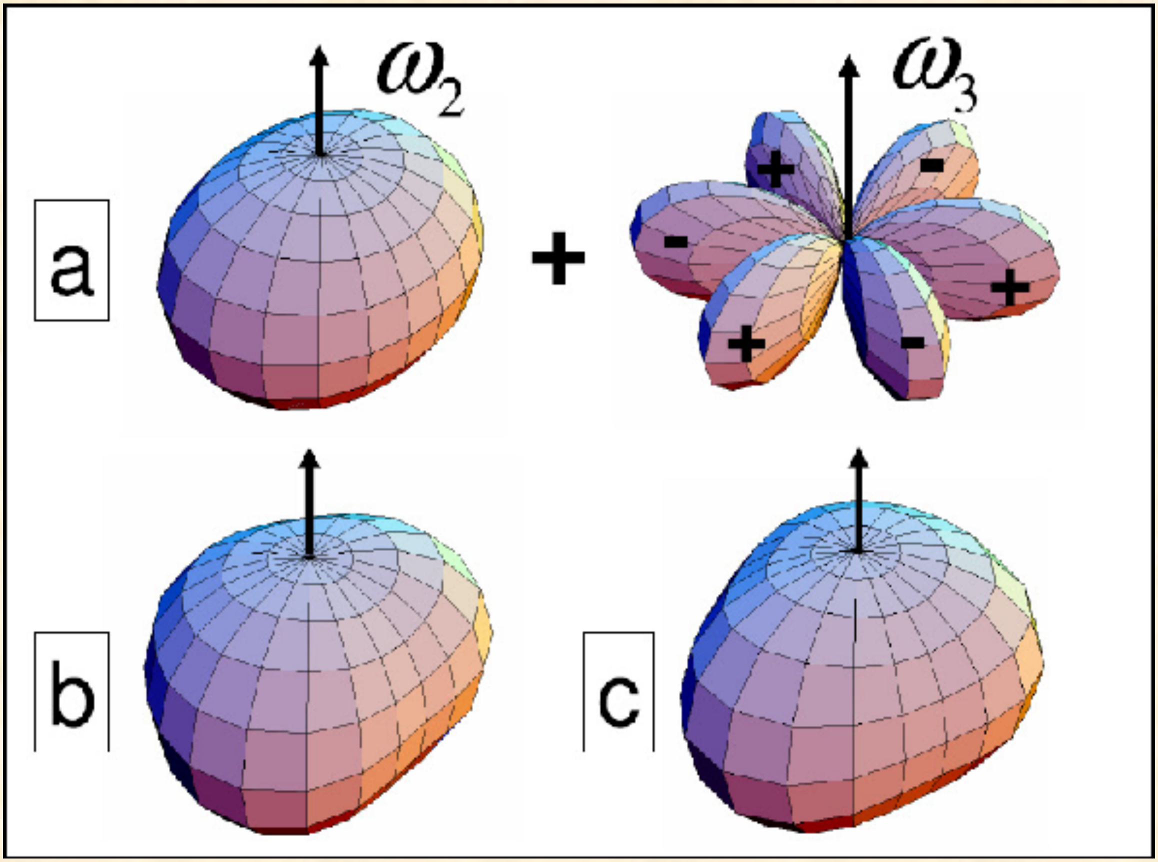


FIG. 17. Level scheme of ^{226}Th, taken from Schüler *et al.* (1986; see also Ackermann *et al.*, 1993). The level and transition energies are in keV.

A possible explanation for these correlations has been proposed by Frauendorf in terms of "octupole condensation".

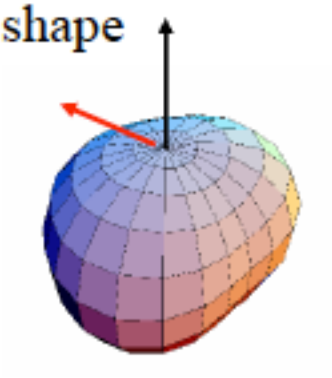
The quadrupole-deformed nucleus is assumed to be a rigid rotor and the octupole vibration is harmonic with a frequency Ω_3.
No interaction between the octupole phonons and the deformed potential of the nucleus.

The superposition of these two modes resembles an octupole deformed nucleus and the running of the wave gives the appearance of an octupole nucleus rotating or vibrating.

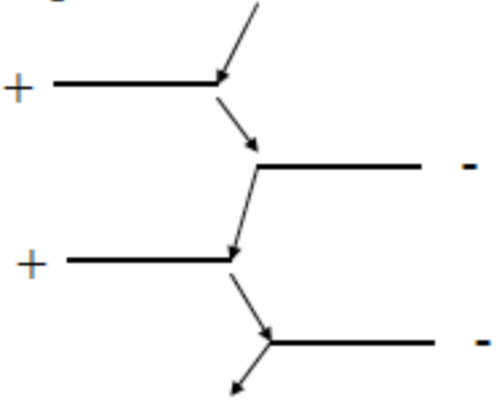


S. Frauendorf, Phys. Rev C **77**, 021304(R) (2008)

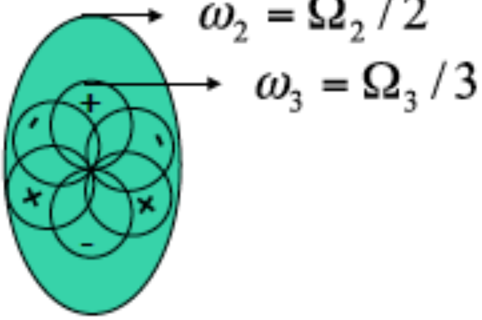
Strong coupling
rotating heart shape

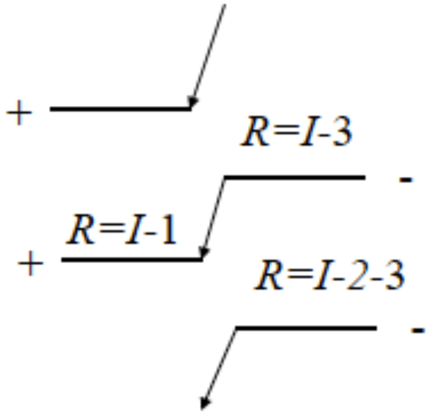


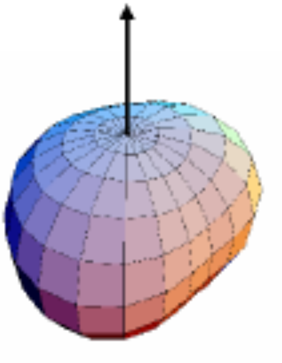
Dipole transitions



Weak coupling
phonon condensation







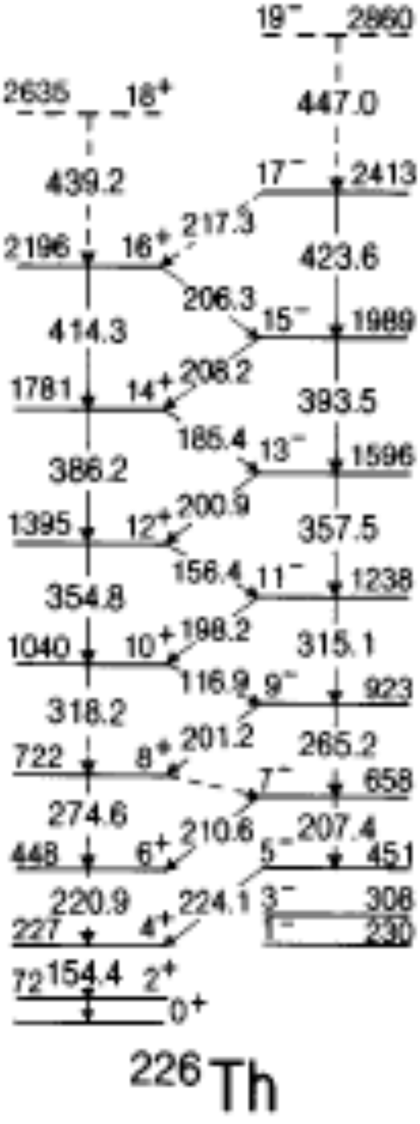
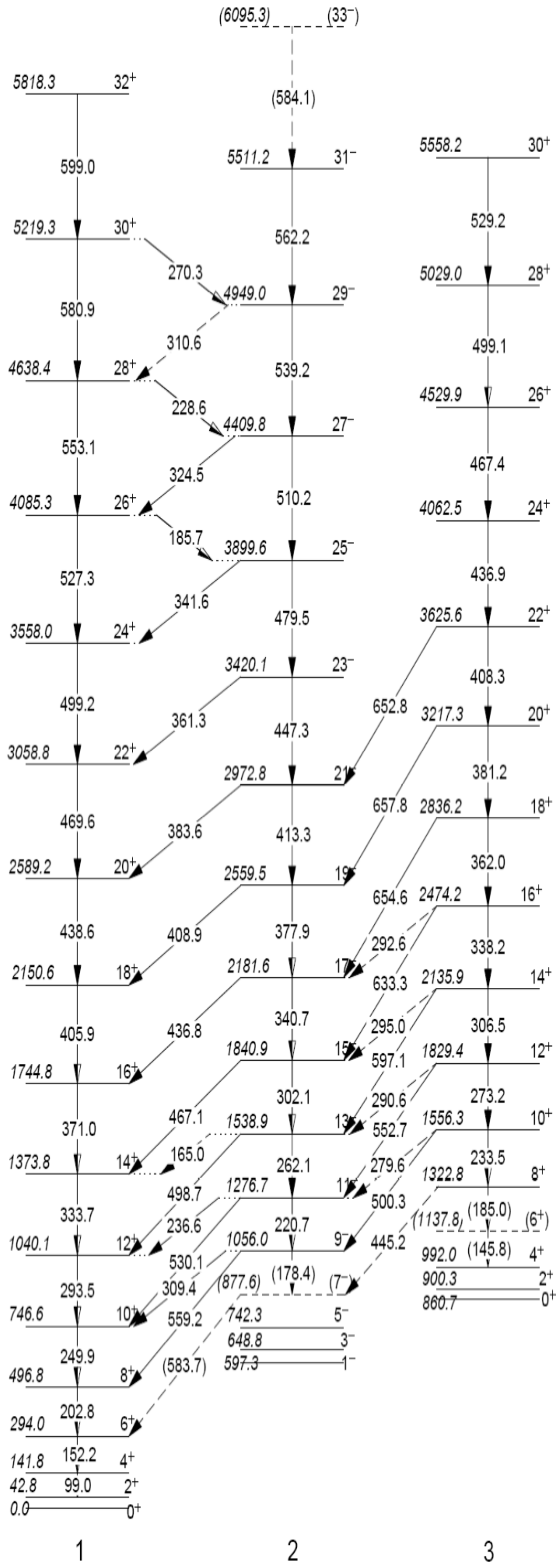


FIG. 17. Level scheme of ^{226}Th, taken from Schüler *et al.* (1986; see also Ackermann *et al.*, 1993). The level and transition energies are in keV.


(6095.3)
(33⁻)
(584.1)
5818.3
32⁺
599.0
5219.3
30⁺
580.9
4638.4
28⁺
553.1
4085.3
26⁺
527.3
3558.0
24⁺
499.2
3058.8
22⁺
469.6
2589.2
20⁺
438.6
2150.6
18⁺
405.9
1744.8
16⁺
371.0
1373.8
14⁺
333.7
1040.1
12⁺
293.5
746.6
10⁺
249.9
496.8
8⁺
202.8
294.0
6⁺
141.8
152.2
4⁺
42.8
99.0
2⁺
0.0
0⁺
270.3
310.6
228.6
324.5
185.7
341.6
361.3
383.6
408.9
436.8
467.1
165.0
498.7
236.6
530.1
309.4
559.2
(583.7)
5511.2
31⁻
562.2
4949.0
29⁻
539.2
4409.8
27⁻
510.2
3899.6
25⁻
479.5
3420.1
23⁻
447.3
2972.8
21
413.3
2559.5
19
377.9
2181.6
17
340.7
1840.9
15
302.1
1538.9
13
262.1
1276.7
11
220.7
1056.0
9⁻
(178.4)
(877.6)
(7⁻)
742.3
5⁻
648.8
3⁻
597.3
1⁻
5558.2
30⁺
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4529.9
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467.4
4062.5
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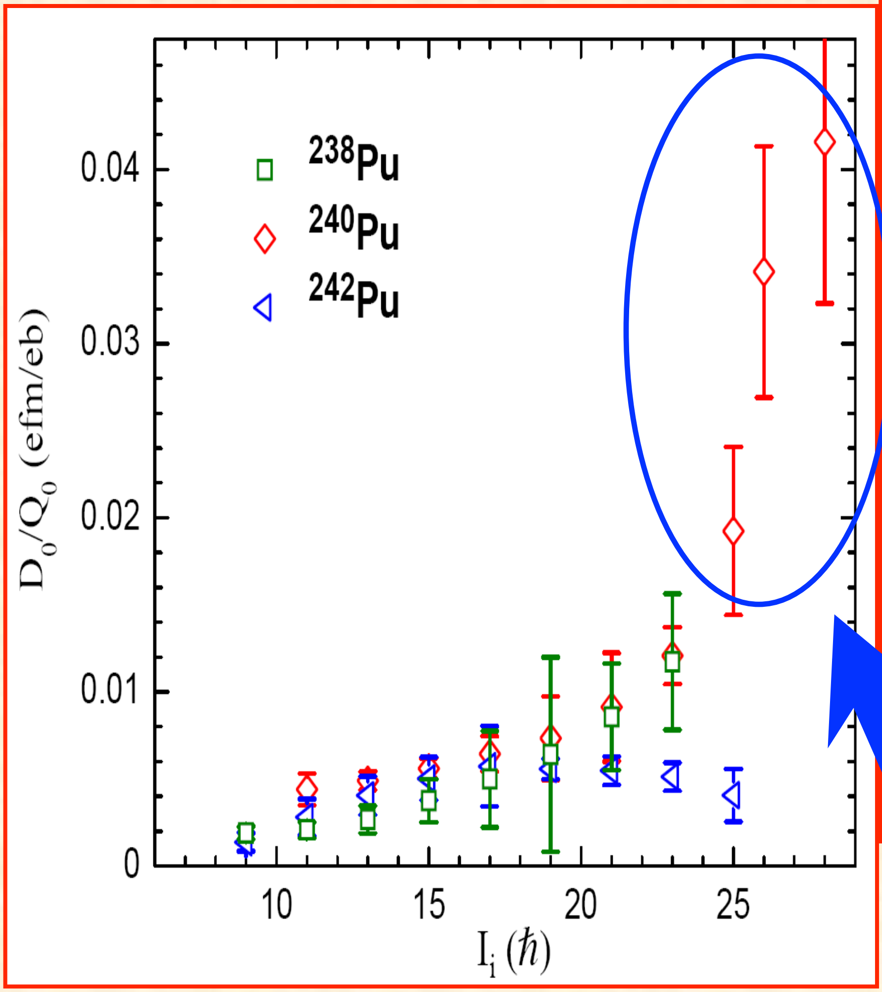
436.9
3625.6
22⁺
408.3
3217.3
20⁺
381.2
2836.2
18⁺
362.0
2474.2
16⁺
338.2
2135.9
14⁺
306.5
1829.4
12⁺
273.2
1556.3
10⁺
233.5
1322.8
8⁺
(185.0)
(1137.8)
(6⁺)
(145.8)
992.0
4⁺
900.3
2⁺
860.7
0⁺
652.8
657.8
654.6
292.6
633.3
295.0
597.1
290.6
552.7
279.6
500.3
445.2
1
2
3

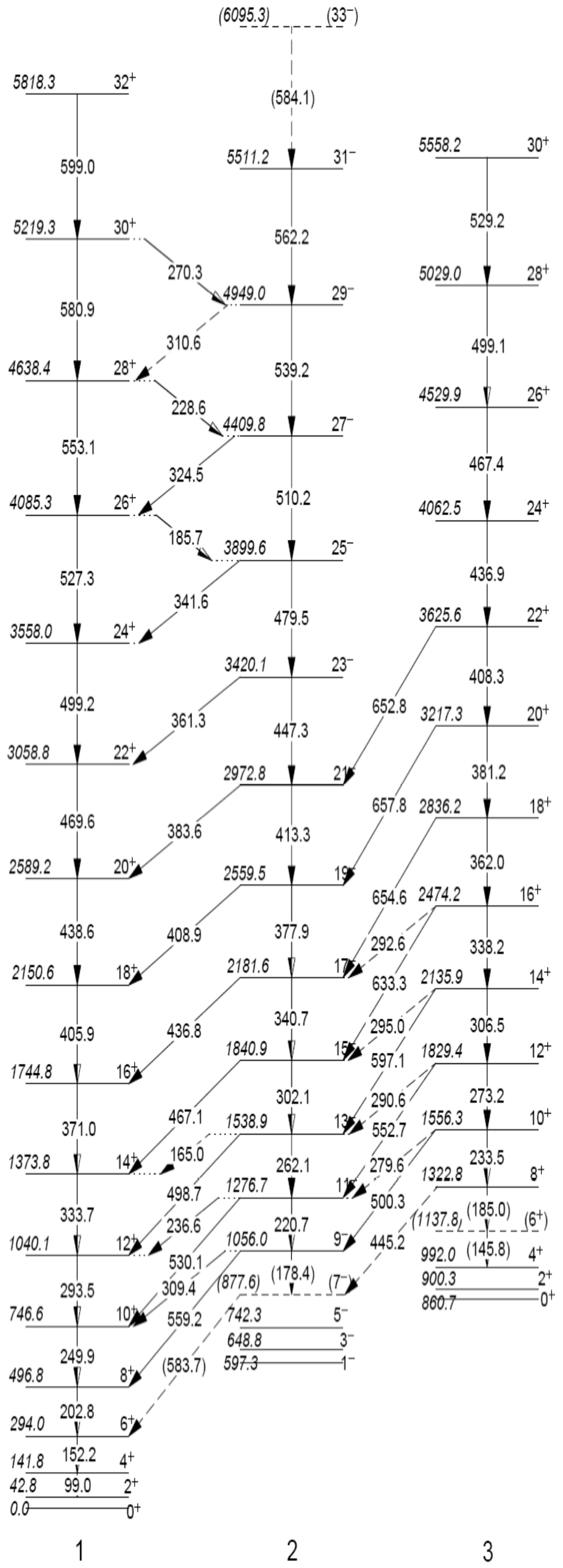
^{240}Pu



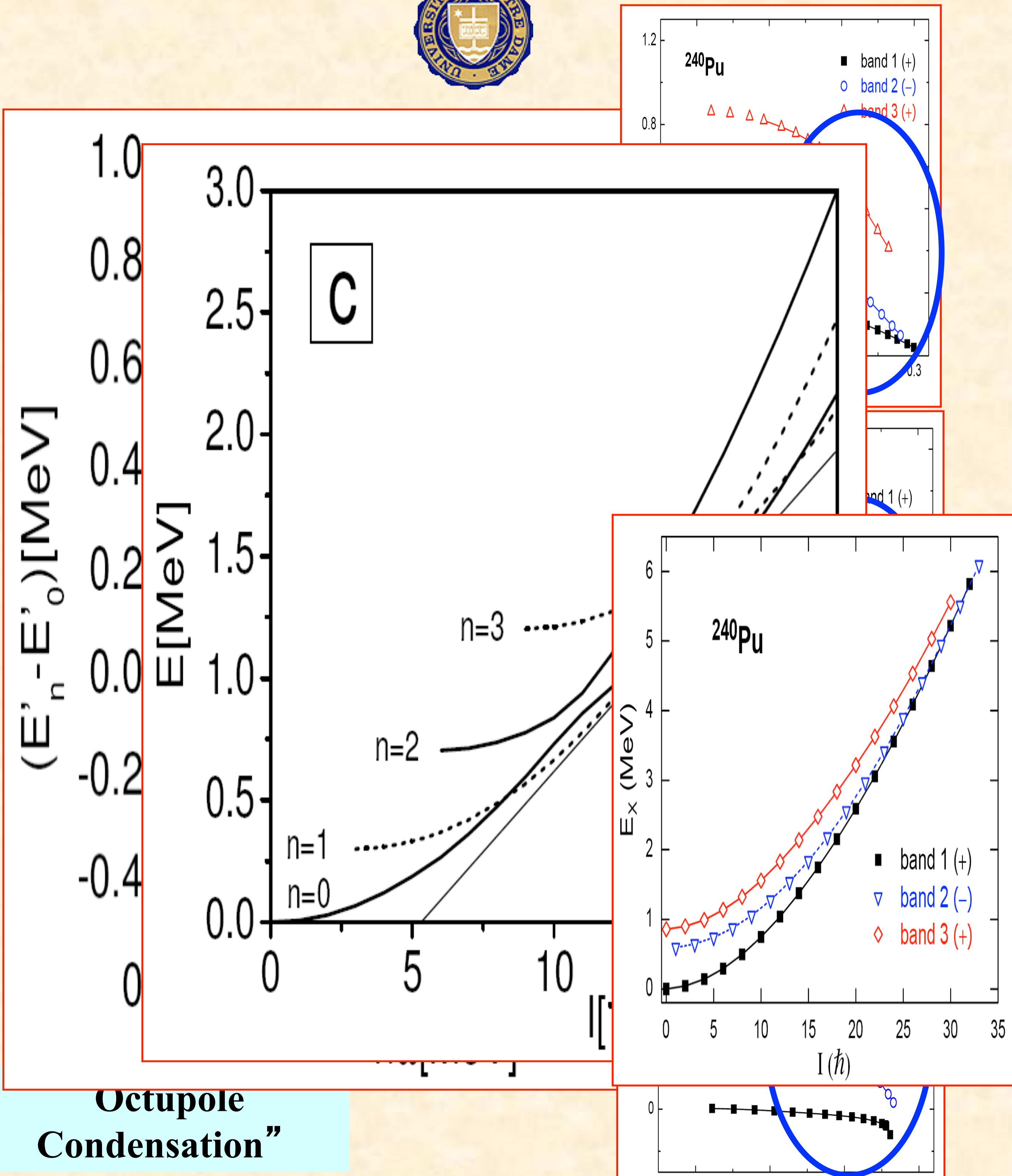
D_0/Q_0 (efm/eb)
0.04
0.03
0.02
0.01
0
10
15
20
25
I_i ($\hbar$)
^{238}Pu
^{240}Pu
^{242}Pu

^{240}Pu
(6095.3)
5818.3
32+
(584.1
599.0
5511.2
5219.3
30+
562.2
270.3
4949.0
580.9
310.6
4638.4
28+
539.2
228.6
4409.8
553.1
324.5
510.2
4085.3
26+
185.7
3899.6
527.3
341.6
479.5
3558.0
24+
3420.1
499.2
361.3
058.8
2972.8
1
2

$D_0 > 0.2$ e fm


(6095.3) (33⁻)
(584.1)
5818.3 32+
599.0
5511.2 31⁻
5558.2 30+
529.2
5219.3 30+
270.3
562.2
5029.0 28+
4949.0 29⁻
580.9
310.6
499.1
539.2
4638.4 28+
4529.9 26+
228.6
4409.8 27⁻
553.1
467.4
324.5
510.2
4085.3 26+
4062.5 24+
185.7
3899.6 25⁻
527.3
341.6
436.9
479.5
3625.6 22+
3558.0 24+
3420.1 23⁻
408.3
499.2
361.3
652.8
447.3
3217.3 20+
3058.8 22+
2972.8 21⁻
381.2
469.6
657.8
383.6
413.3
2836.2 18+
2589.2 20+
2559.5 19⁻
362.0
438.6
408.9
654.6
377.9
2474.2 16+
292.6
338.2
2150.6 18+
2181.6 17⁻
633.3
2135.9 14+
405.9
436.8
340.7
295.0
306.5
1744.8 16+
1840.9 15⁻
597.1
1829.4 12+
371.0
467.1
302.1
290.6
273.2
1538.9 13⁻
552.7
1556.3 10+
165.0
1373.8 14+
262.1
279.6
233.5
498.7
1276.7 11⁻
1322.8 8+
333.7
236.6
220.7
500.3
(1137.8) (185.0) (6+)
1040.1 12+
1056.0 9⁻
445.2
992.0 (145.8) 4+
530.1
(178.4)
900.3 2+
293.5
309.4
(877.6) (7⁻)
860.7 0+
746.6 10+
742.3 5⁻
559.2
648.8 3⁻
249.9
597.3 1⁻
(583.7)
496.8 8+
202.8
294.0 6+
152.2
141.8 4+
99.0
42.8 2+
0.0 0+
1
2
3

^{240}Pu



Octupole
Condensation”

- ✓ **Strong octupole correlations have been observed in the nucleus ^{240}Pu.**

- ✓ **The data appears to agree well with the predictions of the new “octupole condensation” picture.**

- ✓ **Another good test case would be ^{230}Th.**

X. Wang *et al.*, Phys. Rev. Lett. **102**, 122501 (2009)

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Excellent resources (if you are so inclined):

- J. A. Cerny, Editor, *Nuclear Reactions and Spectroscopy*, Academic Press, 1974.
- H. Morinaga and T. Yamazaki, *In-beam Gamma-ray Spectroscopy*, North Holland, 1976.
- H. Ejiri and M.A.J. de Voigt, *Gamma-ray and Electron Spectroscopy in Nuclear Physics*, Clarendon Press, 1989
- S. Frauendorf, Rev. Mod. Phys. **73**, 463 (2001).

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ありがとう

धन्यवाद

Thanks!

The Question Kitten

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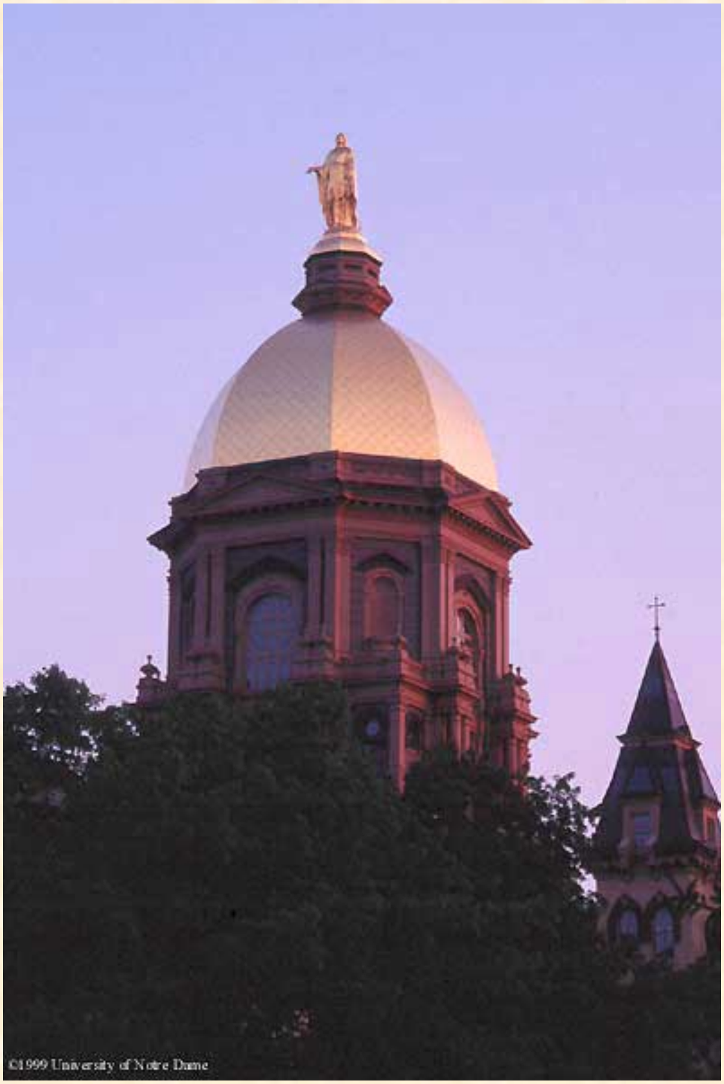

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