E-b-E Fluctuations in Dense Baryonic Medium

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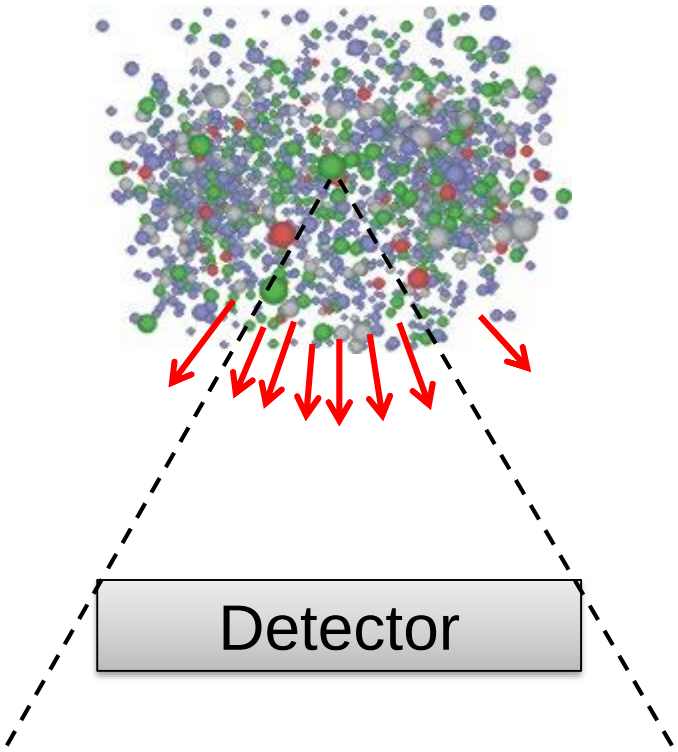
(Osaka U.)

Heavy Ion Cafe

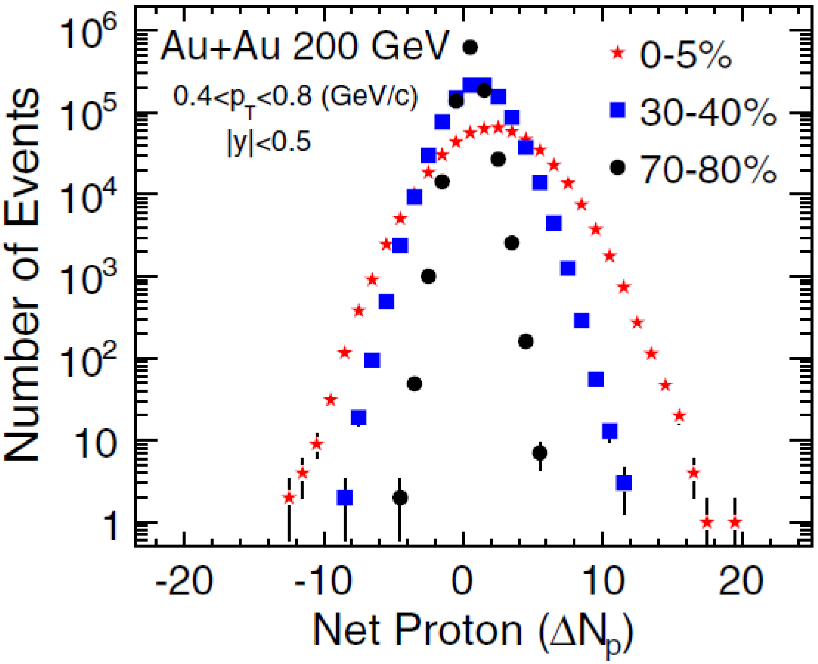
Sophia University, Tokyo, 23/June/2019

Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)



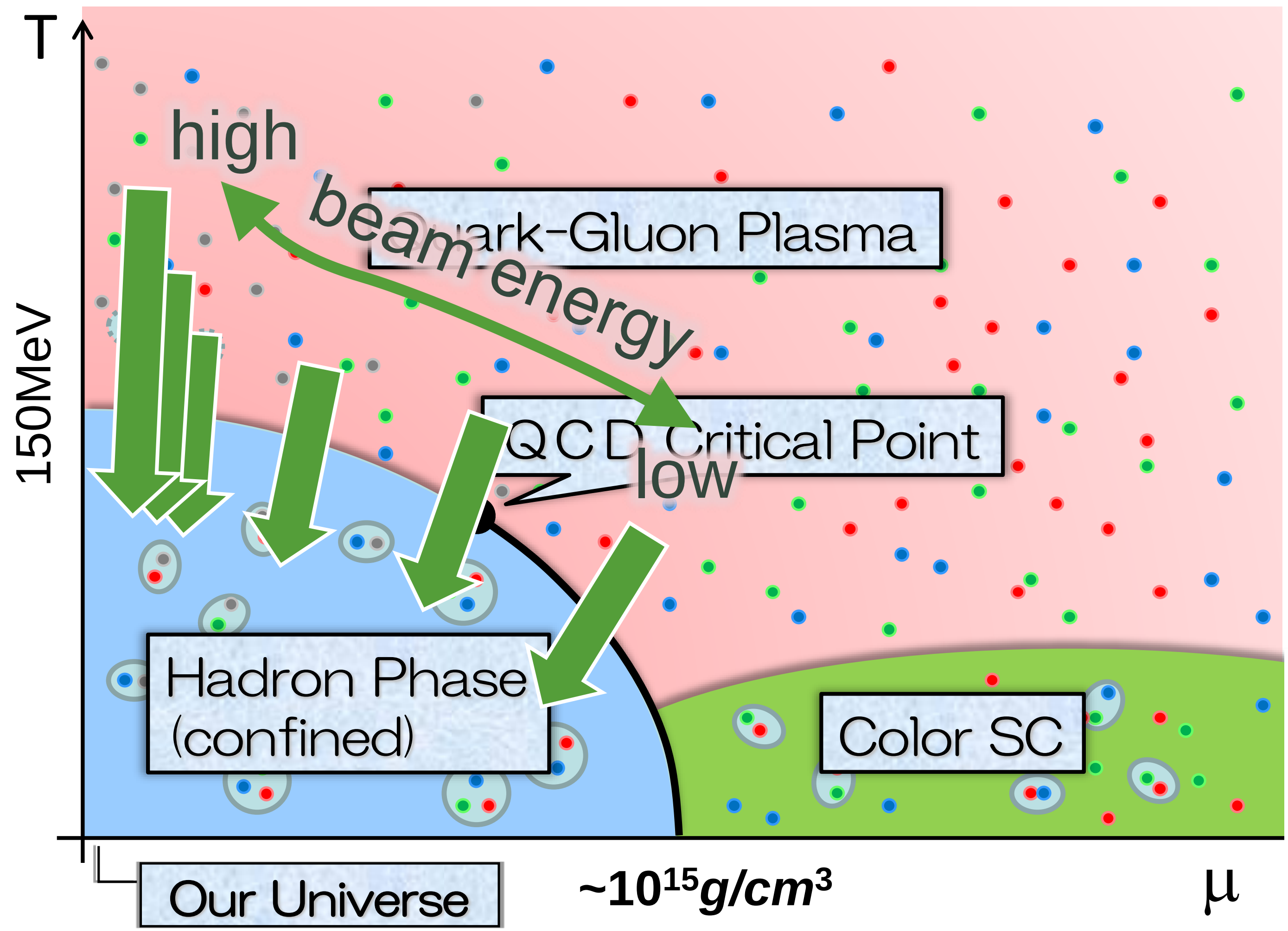
STAR, PRL**105** (2010)



Structure of distribution reflects microscopic properties

Cumulants: $\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$

Beam-Energy Scan Program in Heavy-Ion Collisions



A Coin Game

① Bet 25 Euro
② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

Same expectation value.

A Coin Game

① Bet 25 Euro

② You get head coins of

A. 50 x 1 Euro

B. 25 x 2 Euro

C. 1 x 50 Euro

Same expectation value.

But, different fluctuation.

Fluctuations in HIC: 2nd Order

Search for QCD CP

Fluctuation
increases

Stephanov, Rajagopal, Shuryak, 1998; 1999

Onset of QGP

Fluctuation
decreases

Asakawa, Heinz, Muller, 2000;
Jeon, Koch, 2000

Higher-order Cumulants

A. 50 x 1 Euro

B. 25 x 2 Euro

$2\langle \delta€^2 \rangle$  $=$ $\langle \delta€^2 \rangle$ 

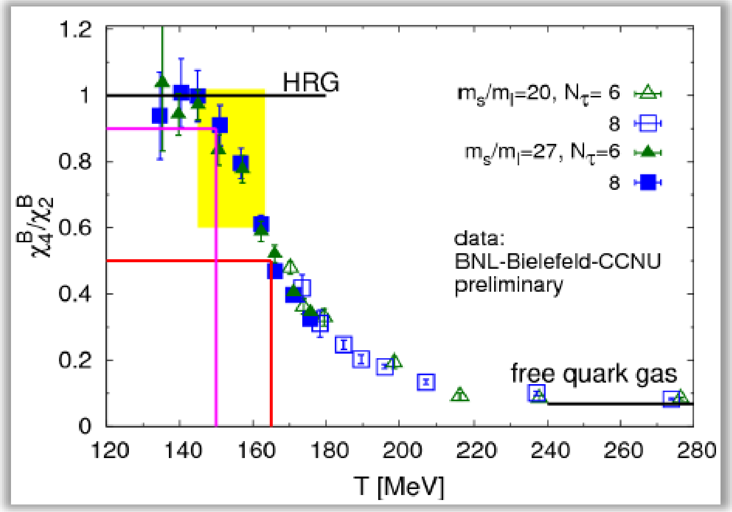
$4\langle \delta€^3 \rangle$  $=$ $\langle \delta€^3 \rangle$ 

$8\langle €^4 \rangle_c$  $=$ $\langle €^4 \rangle_c$ 

Asakawa, MK,
PPNP 90, 299
(2016)

Non-Gaussian Fluctuations

Onset of QGP



Fluctuation
decreases

Ejiri, Karsch, Redlich, 2006

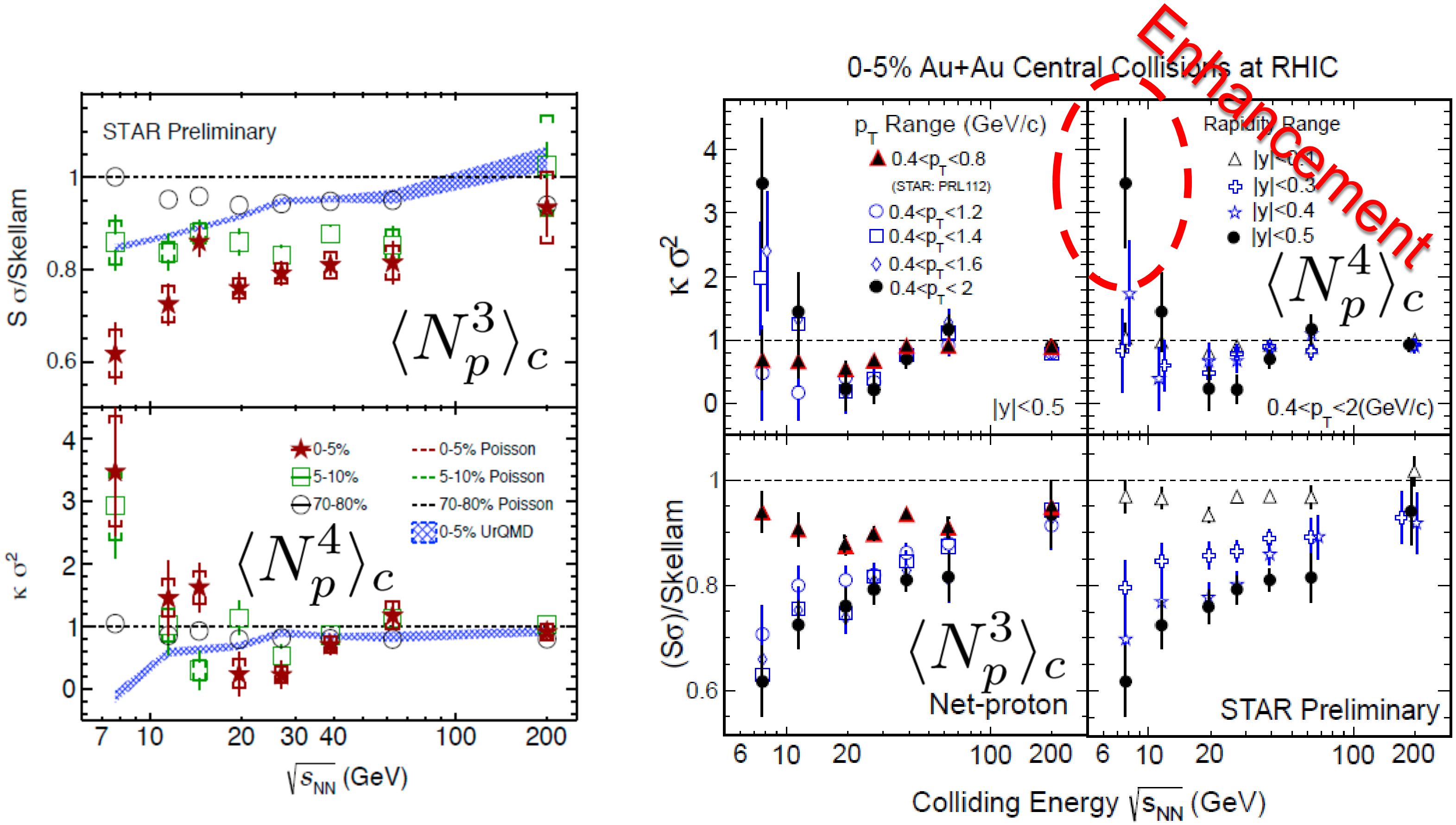
Search for QCD CP

Fluctuation
increases

Stephanov, 2009

Higher-Order Cumulants

STAR
2010~

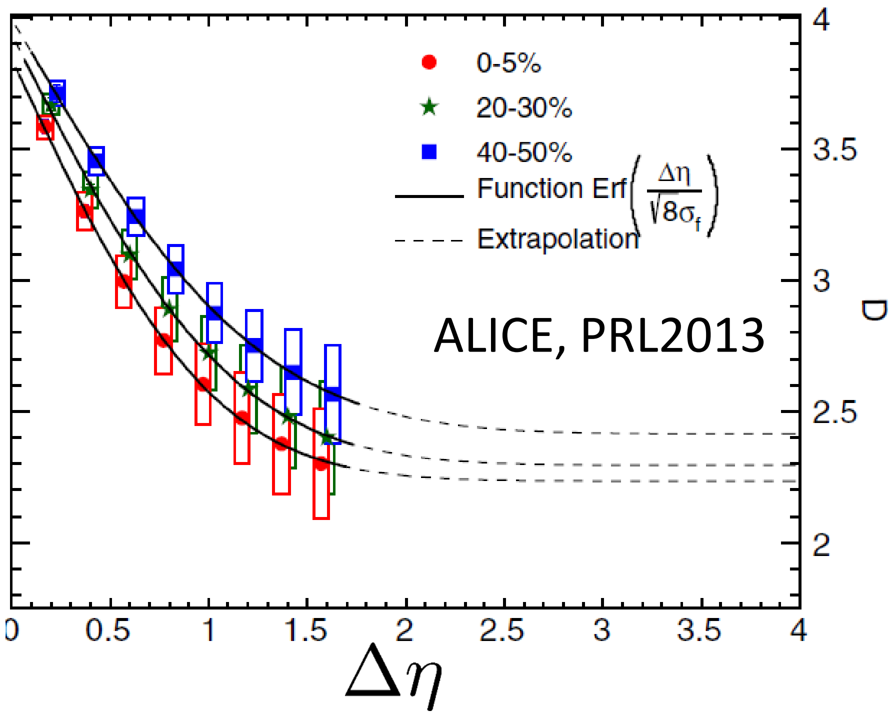


Non-zero **non**-Gaussian cumulants have been established!

General Review:
Asakawa, MK, PPNP (2016)

2nd Order @ ALICE

Net charge fluctuation

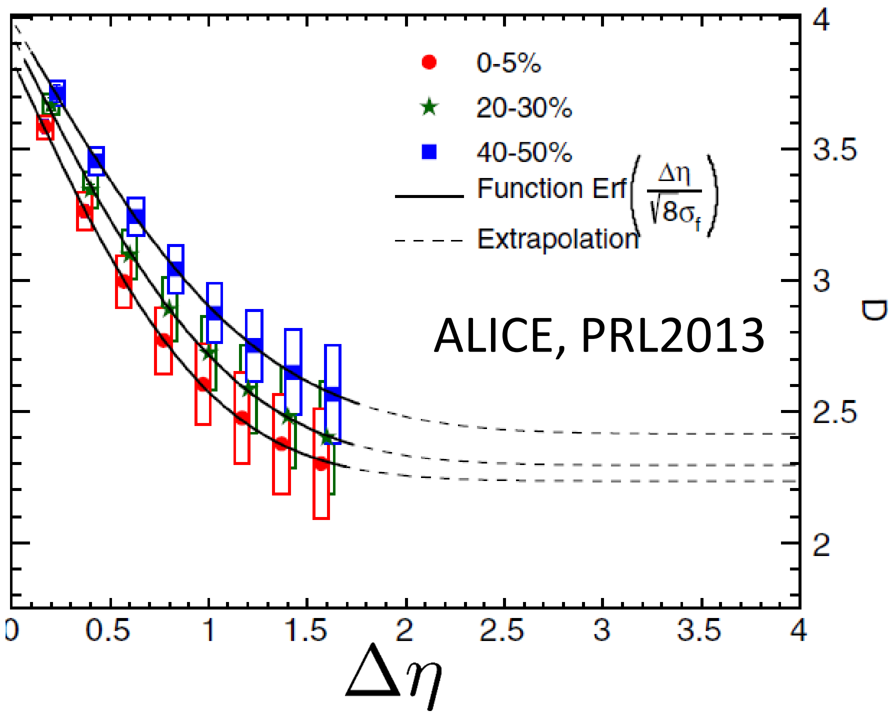


D-measure

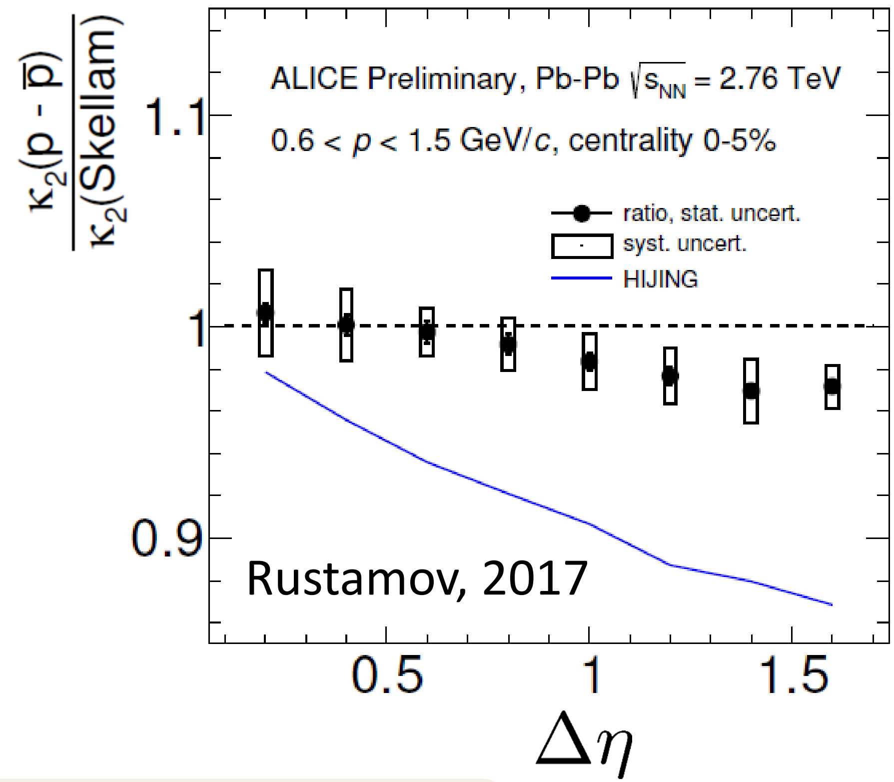
$$D \simeq 4 \frac{\langle \delta N_Q^2 \rangle}{\langle \delta N_Q^2 \rangle_{\rm HRG}}$$

2nd Order @ ALICE

Net charge fluctuation



Net proton fluctuation

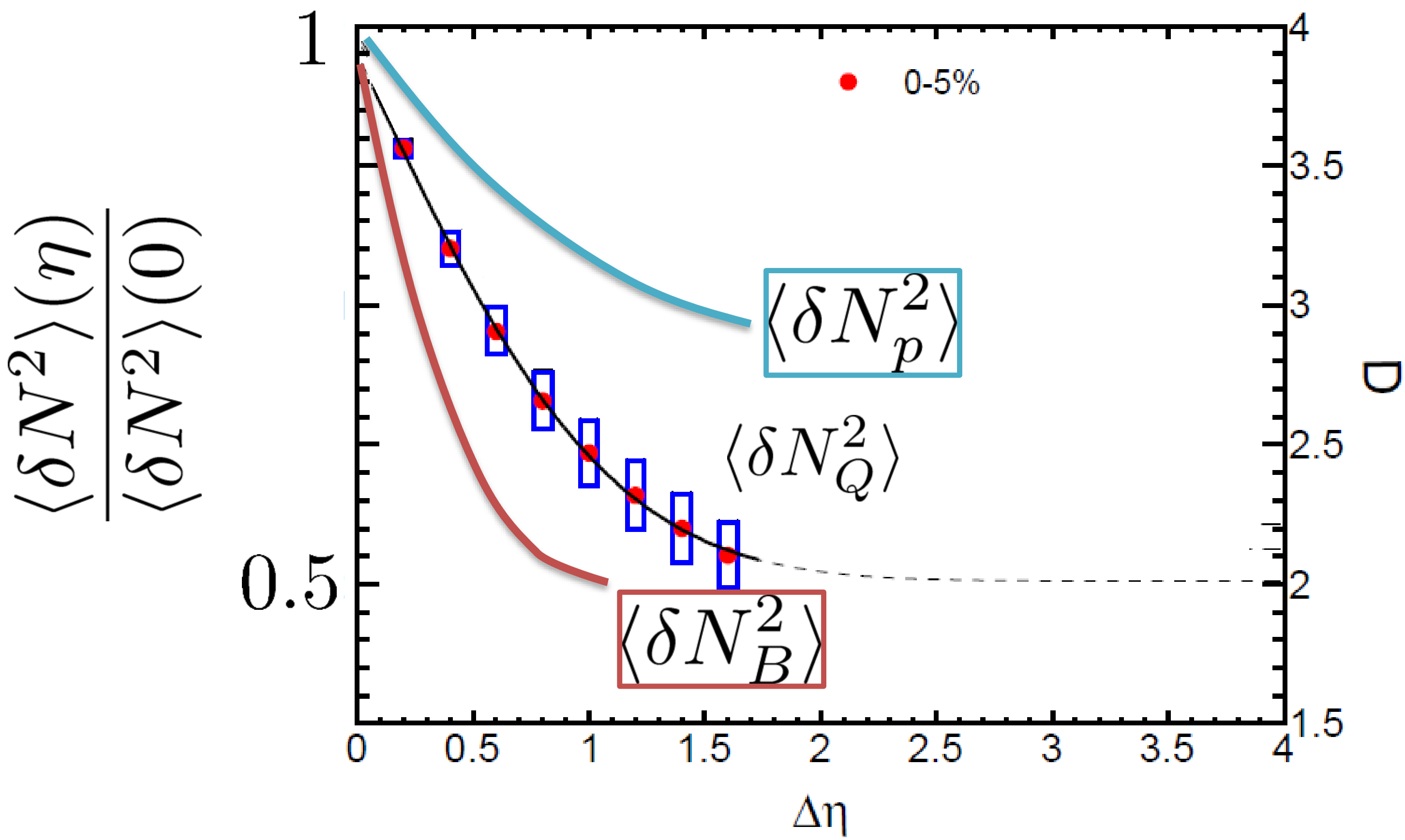


- Net-charge fluctuation has a suppression,
- but net-proton fluctuation does not. Why??

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

MK, presentations
GSI, Jan. 2013
Berkeley, Sep. 2014
FIAS, Jul. 2015
GSI, Jan. 2016
…

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.



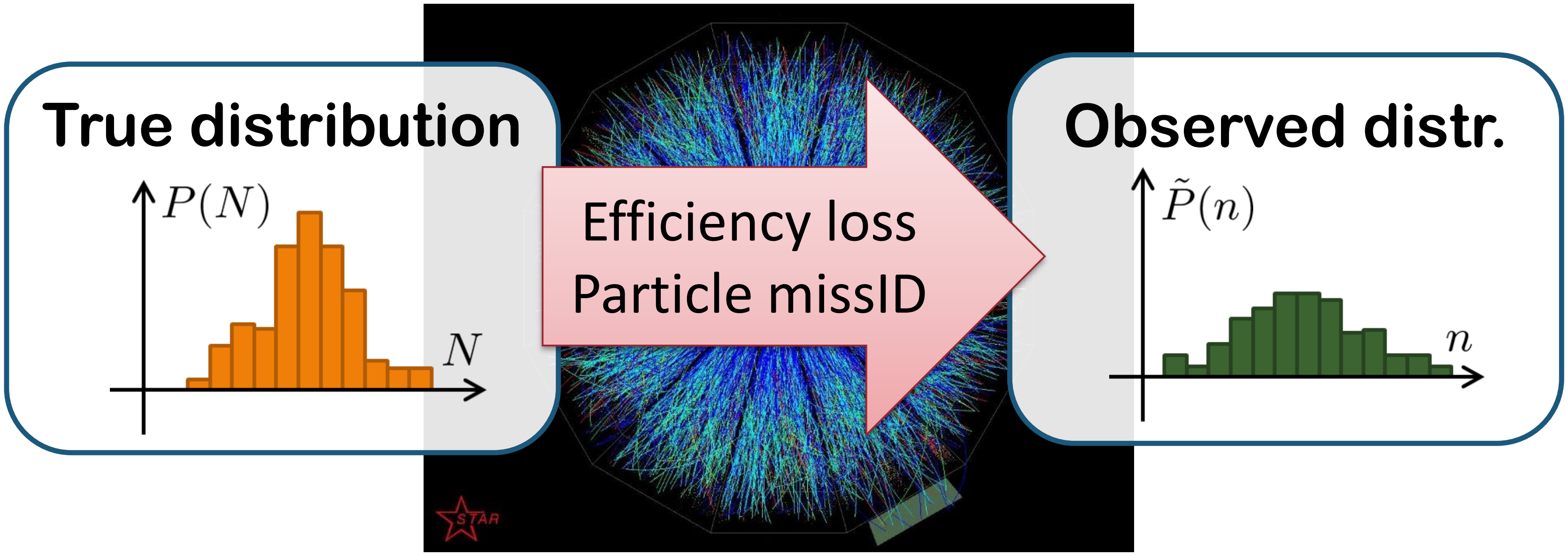
Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

Message

Understand 2nd-order fluctuations @ LHC & top-RHIC

1. Problems in experimental analysis
   - proper correction of detector's property
2. Conserved-charge fluctuations
3. Dynamics of non-Gaussian fluctuations
4. A suggestion: chiB/chiQ

Detector-Response Correction

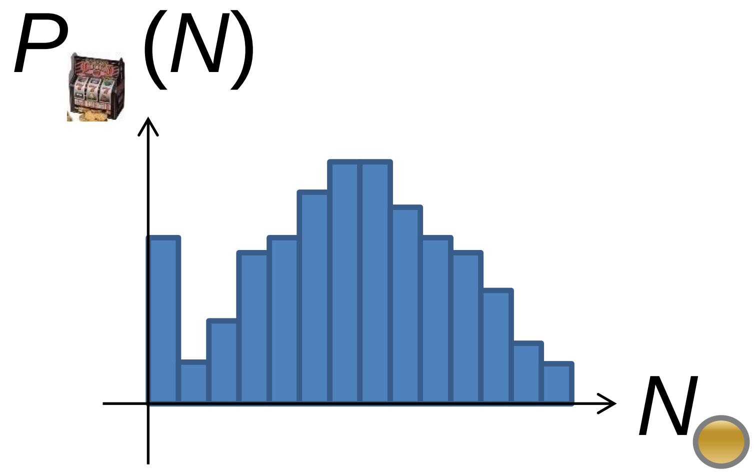


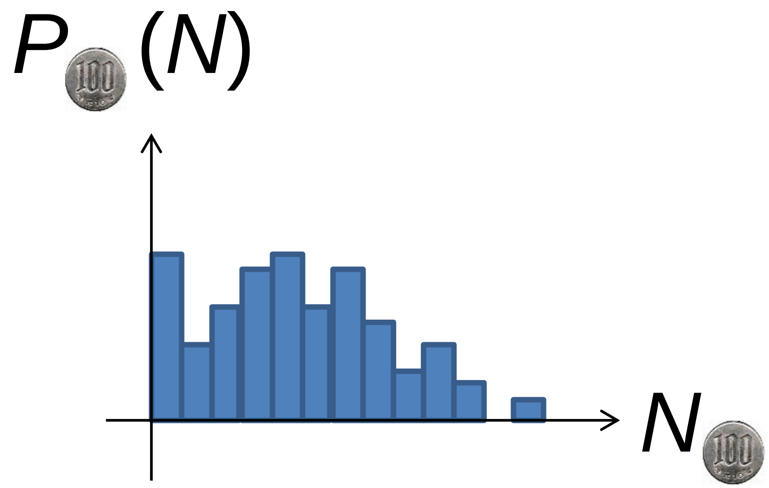
- **Correction assuming a binomial response**

  Bialas, Peschanski (1986);
  MK, Asakawa (2012); Bzdak, Koch (2012); ….

  But, the response of the detector is not binomial…

Slot Machine Analogy

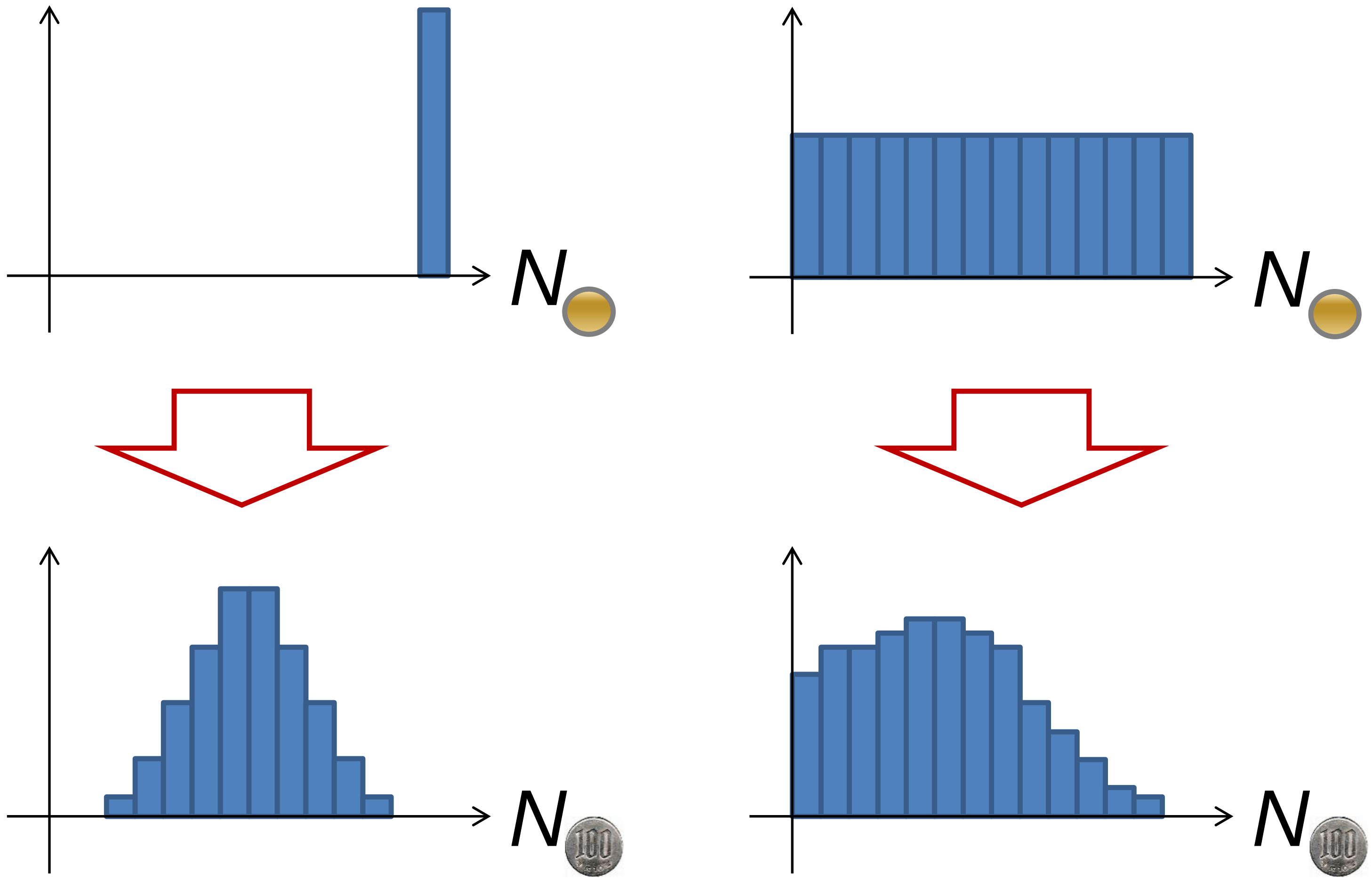




Extreme Examples

Fixed # of coins

Constant probabilities



Reconstructing Total Coin Number

$$P_{\text{100}}(N_{\text{100}}) = \sum_{\bullet} P_{\text{slot}}(N_{\bullet}) B_{1/2}(N_{\text{100}}; N_{\bullet})$$

$$B_p(k;N) = p^k (1-p)^{N-k} \ {}_kC_N$$:binomial distr. func.

Non-Binomial Correction

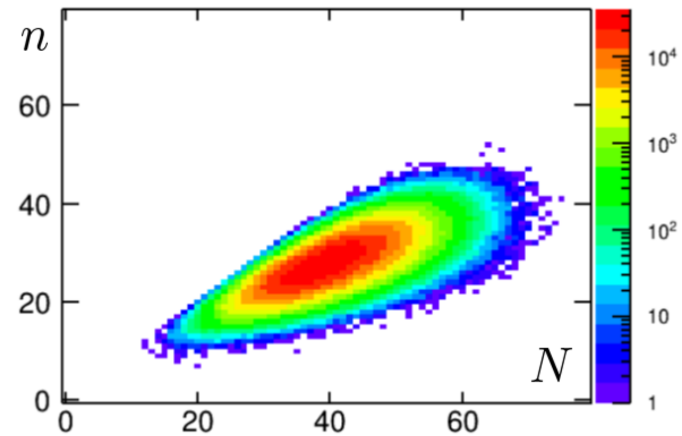
Nonaka, MK, Esumi (2018)

- Response matrix

$$\tilde{P}(n) = \sum_N \mathcal{R}(n;N)P(N)$$

Reconstruction for any R(n;N) with moments of R(n;N)

$$\langle n^m \rangle_R = \sum_n n^m \mathcal{R}(n;N)$$



- Caveats:
  - R(n;N) describes the property of the detector.
  - Detailed properties of the detector have to be known.
  - Multi-distribution function can be handled.
  - Huge numerical cost would be required.
  - Truncation is required in general: another systematics?

Result in a Toy-Model

Binomial w/ multiplicity-dependent efficiency

$$\epsilon(N) = \epsilon_0 + (N - N_{\rm ave})\epsilon'$$

Holtzman, Bzdak, Koch (16)

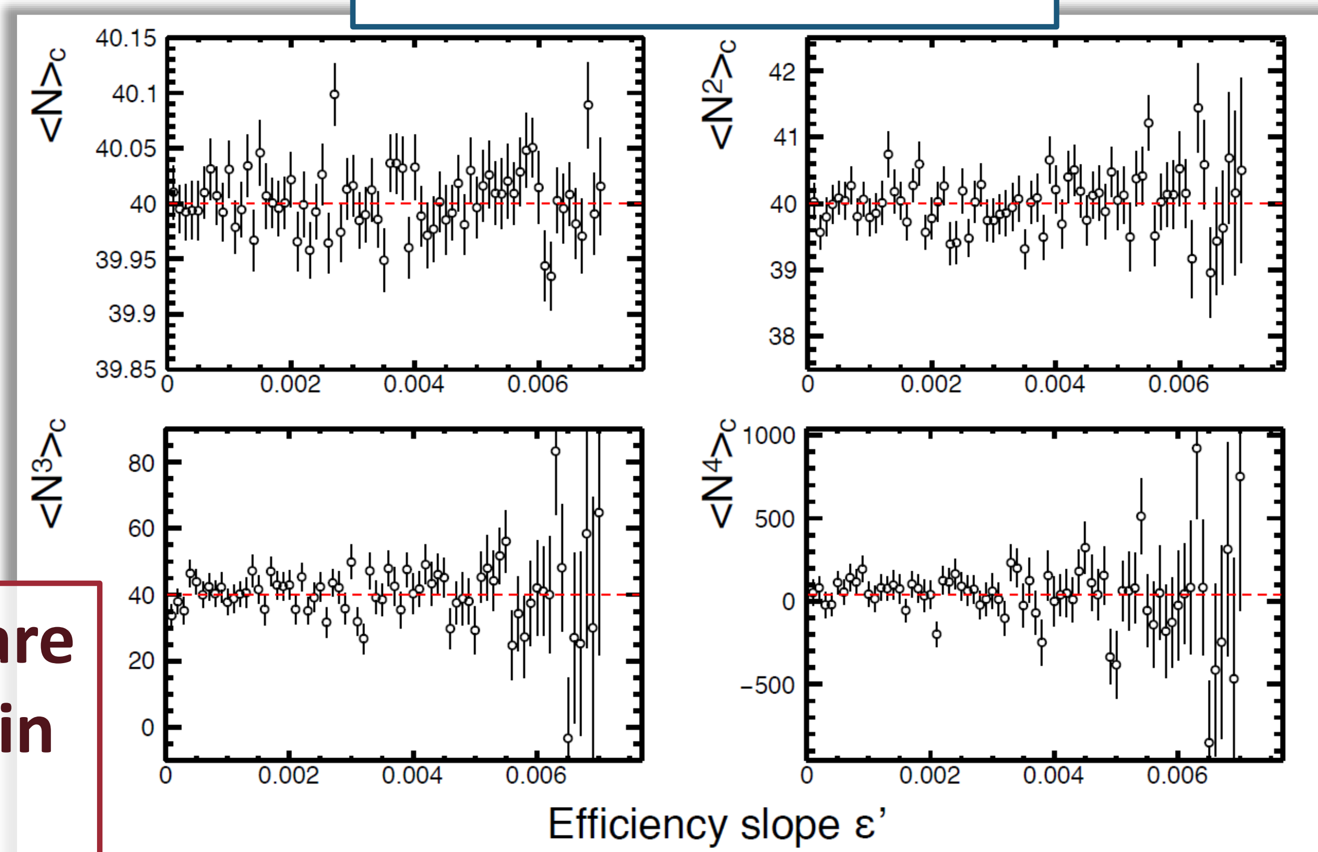
Nonaka, MK, Esumi (2018)

Input P(N): Poisson(λ=40)

$$\epsilon_0 = 0.7$$

Red: true cumulant

Reconstructed cumulants



True cumulants are reproduced within statistics!

Message

Understand 2nd-order fluctuations @ LHC & top-RHIC

1. Problems in experimental analysis
   - proper correction of detector's property

2. Conserved-charge fluctuations

3. Dynamics of non-Gaussian fluctuations

4. A suggestion: chiB/chiQ

Why Conserved Charges?

- Direct comparison with theory / lattice
  - Strong constraint from lattice
  - Ignorance on spatial volume of medium
- Slow time evolution

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- Slow time evolution

AHM-JK (2000)

D-measure

$$D \sim \frac{\langle \delta N_{\mathrm{Q}}^2 \rangle}{S}$$

S is model dependent

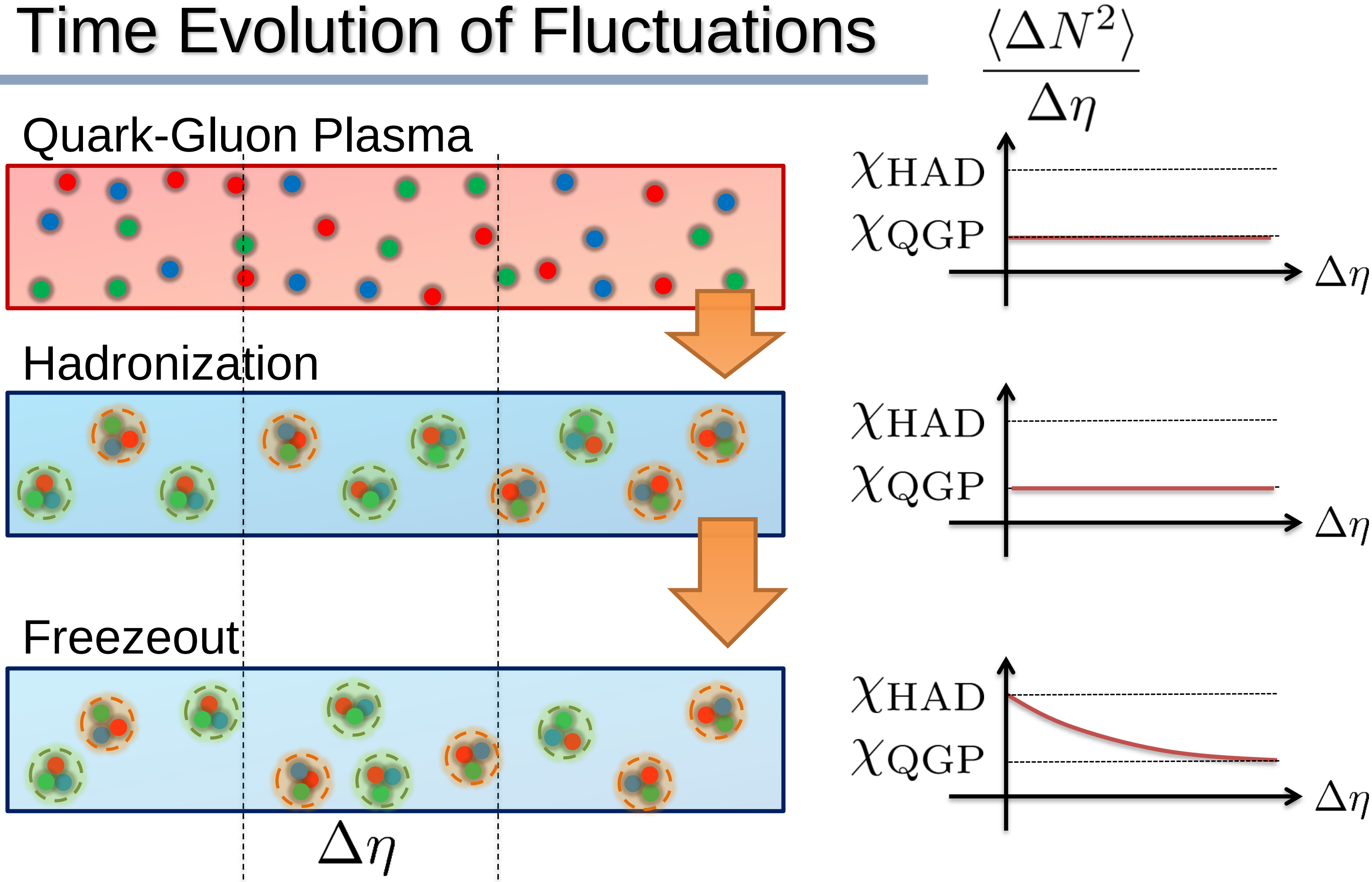
Ejiri-Karsch-Redlich

Ratio of cumulants

$$\frac{\langle N_{\mathrm{Q}}^4 \rangle_c}{\langle N_{\mathrm{Q}}^2 \rangle_c}, \quad \frac{\langle N_{\mathrm{B}}^4 \rangle_c}{\langle N_{\mathrm{B}}^2 \rangle_c}$$

Experimentally difficult

Time Evolution of Fluctuations



Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

Proton vs Baryon Cumulants

MK, Asakawa, 2012; 2012

Experiments
proton number cumulants

$$\langle N_p^n \rangle_c$$

Many theories
baryon number cumulants

$$\langle N_{\rm B}^n \rangle_c$$

measurement with
50% efficiency loss

- Clear difference b/w these cumulants.
- **Isospin randomization** justifies the reconstruction of $\langle N_B^n \rangle_c$ via the binomial model.
- Similar problem on the **momentum cut**…

Message

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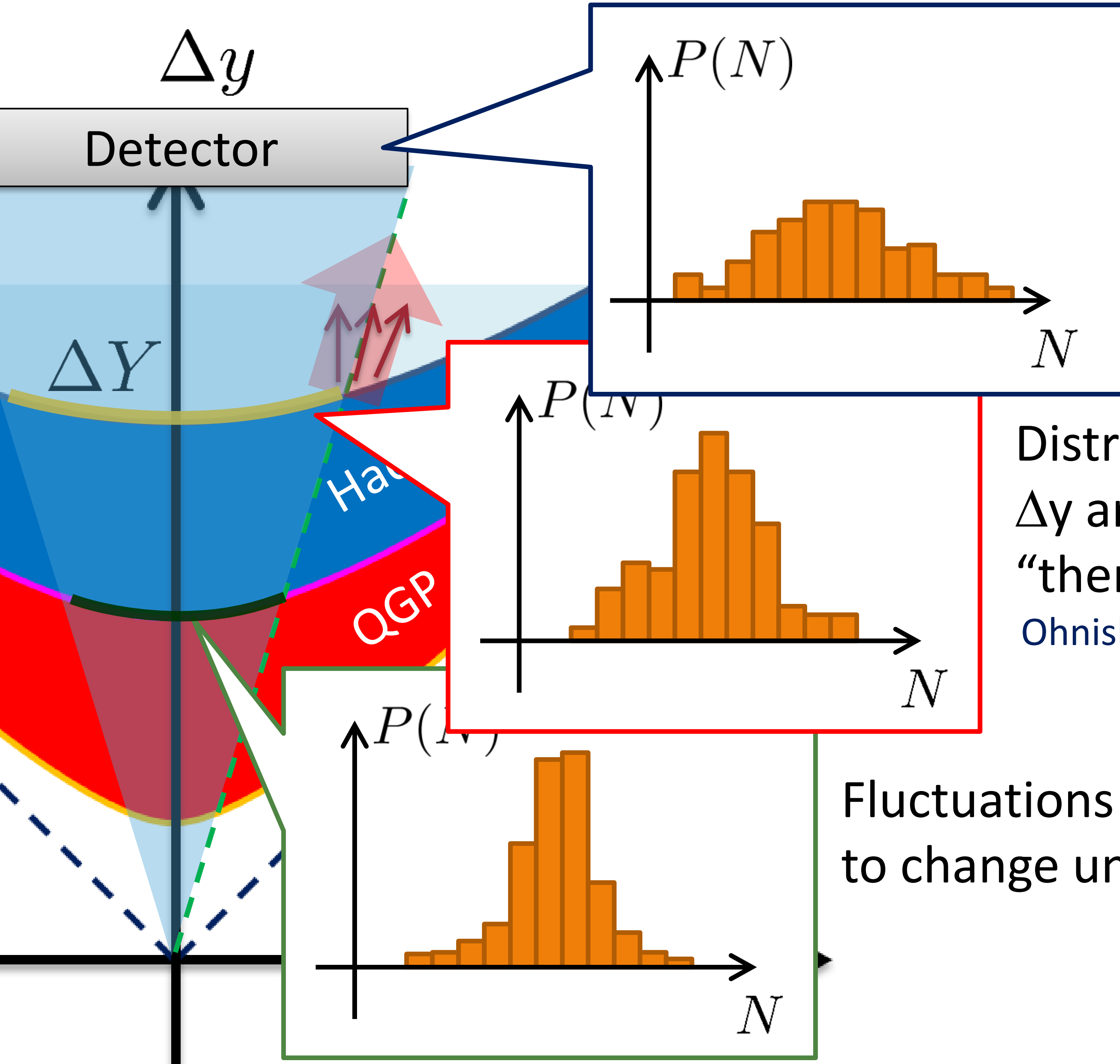
Time Evolution of Fluctuations

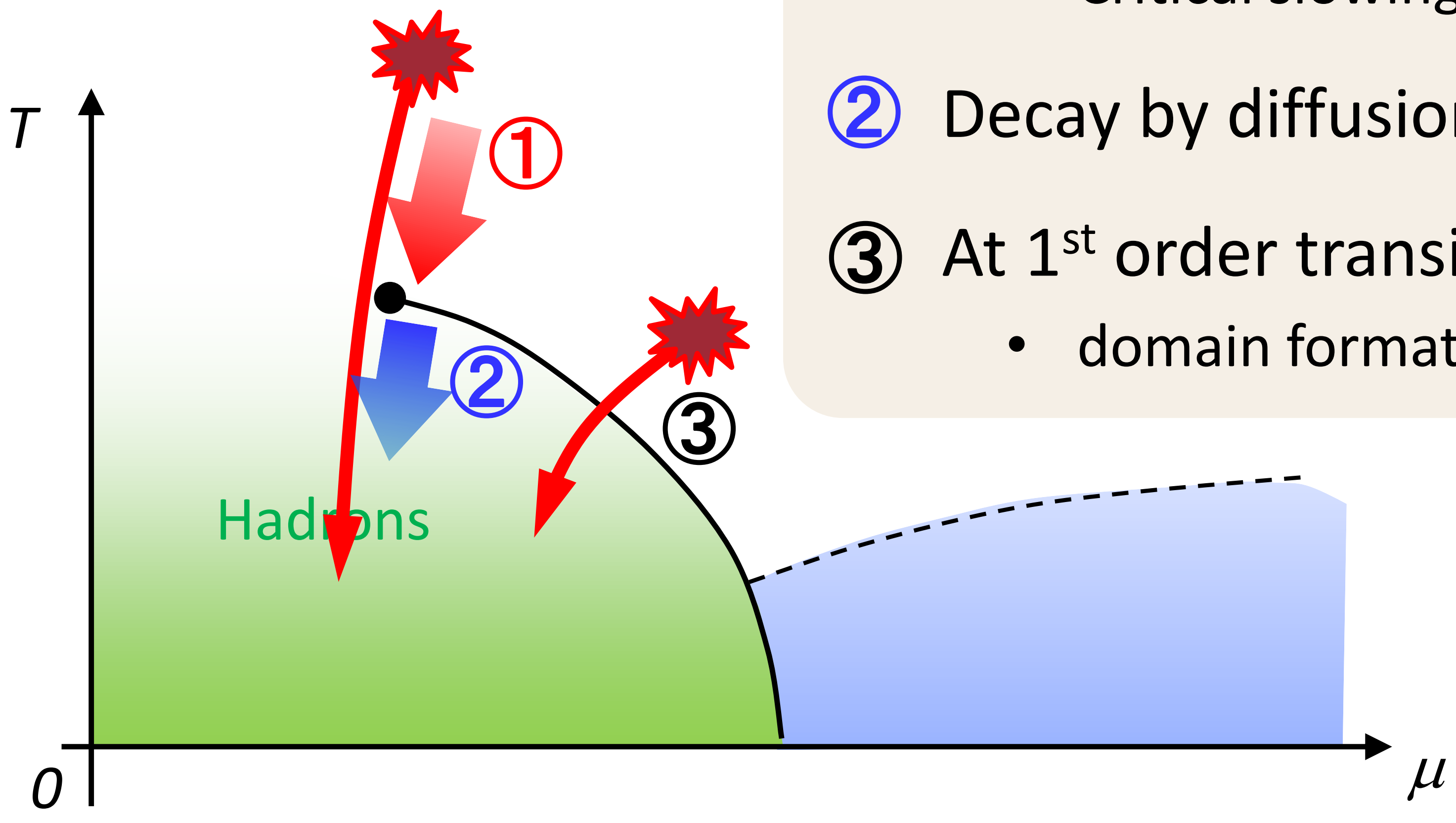


Distributions in ΔY and Δy are different due to "thermal blurring".

Ohnishi, MK, Asakawa, PRC(2016)

Fluctuations in ΔY continue to change until kinetic f.o.

Critical Fluctuation



① Growth of critical fluctuation
- Critical slowing down

② Decay by diffusion

③ At 1st order transition
- domain formation

Analysis of 2nd-order Cumulant

Sakaida+ (2017)

Evolution of baryon number density

Stochastic Diffusion Equation

$$\partial_t n = D(t)\partial_x^2 n + \partial_x \xi$$

$$\langle \xi(x_1,t_1)\xi(x_2,t_2)\rangle = 2D\chi_2\delta^{(2)}(1-2)$$

$D(t),\ \chi_2(t)$:parameters characterizing criticality

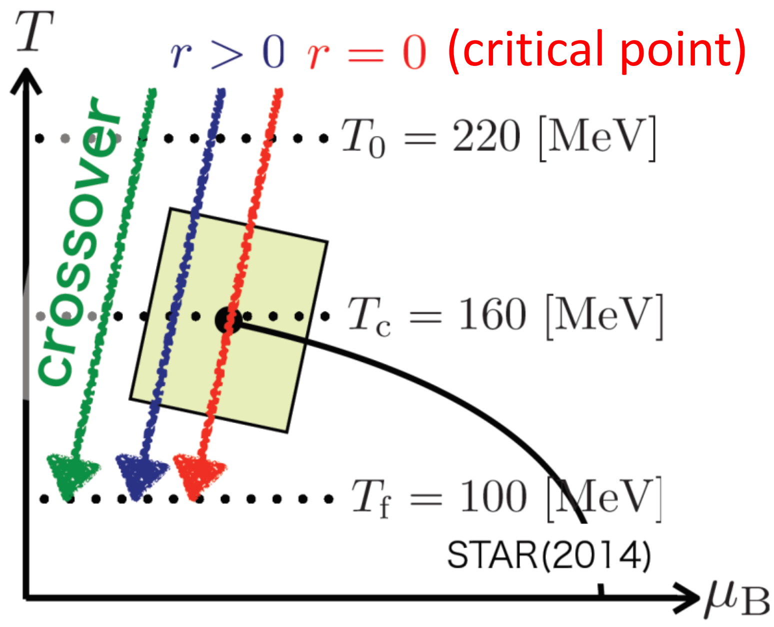
- ☐ Analytic solution is obtained.
- ☐ Study 2nd order cumulant & correlation function.

Parametrizing D(τ) and χ(τ)

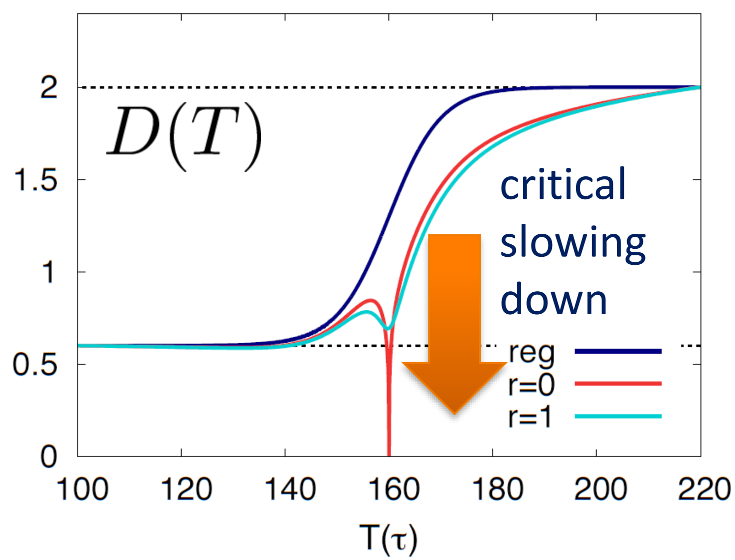
☐ Critical behavior

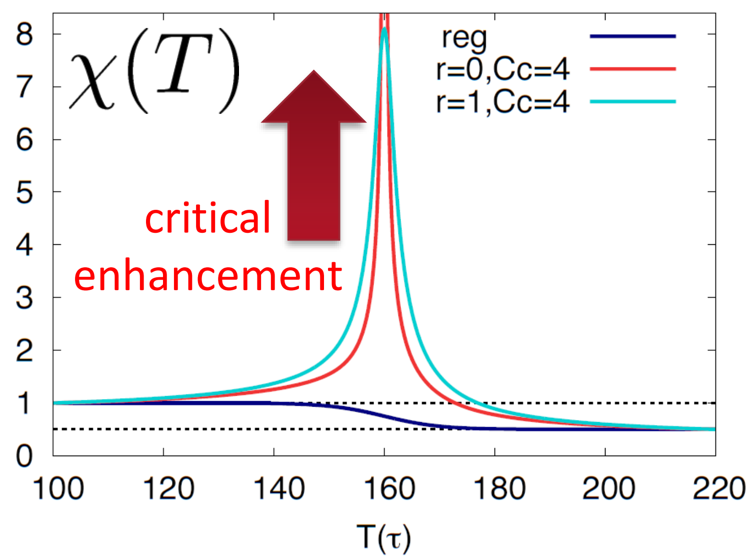
- 3D Ising (r,H)
- model H

Berdnikov, Rajagopal (2000)
Stephanov (2011); Mukherjee+(2015)



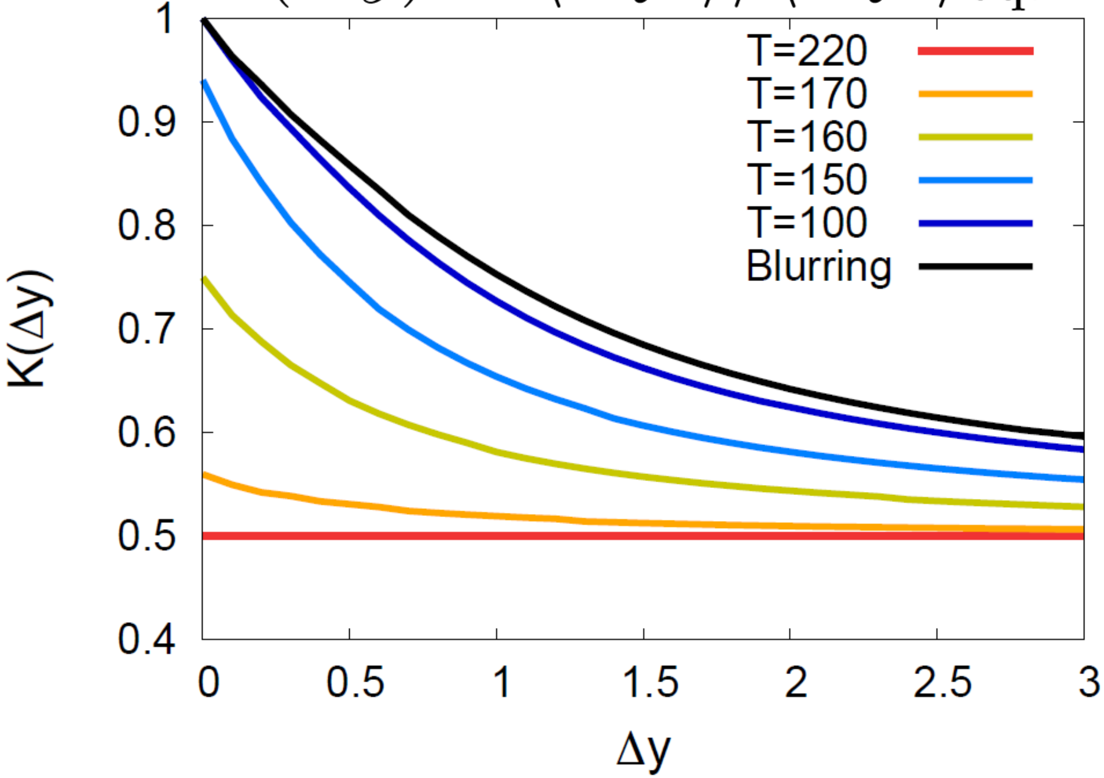
☐ Temperature dep.

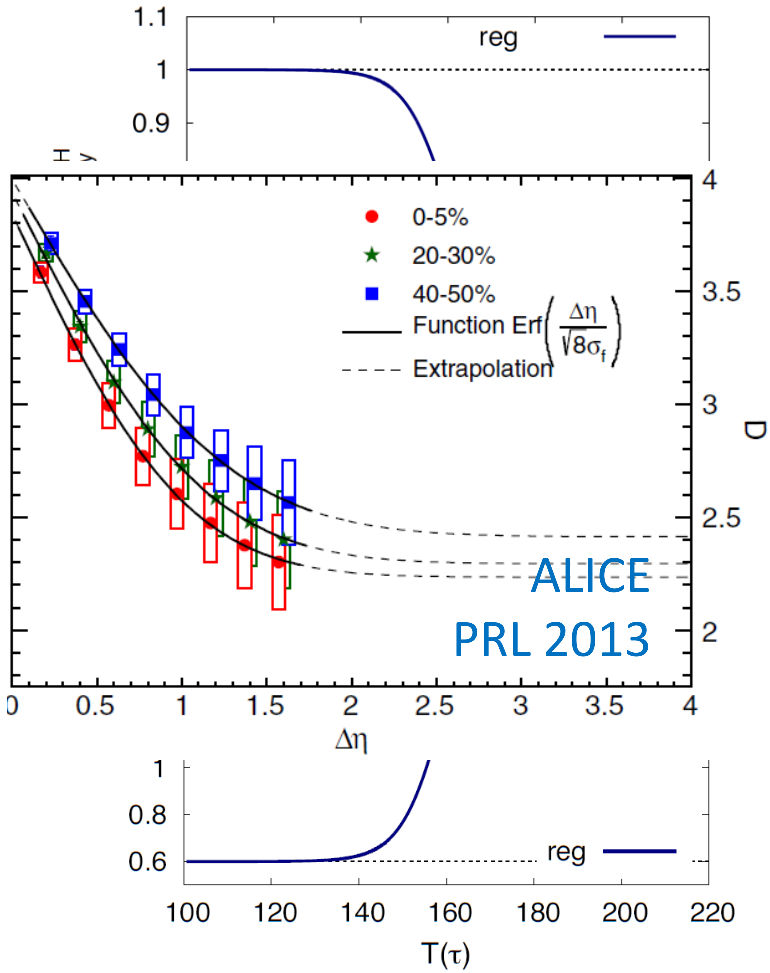




Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$



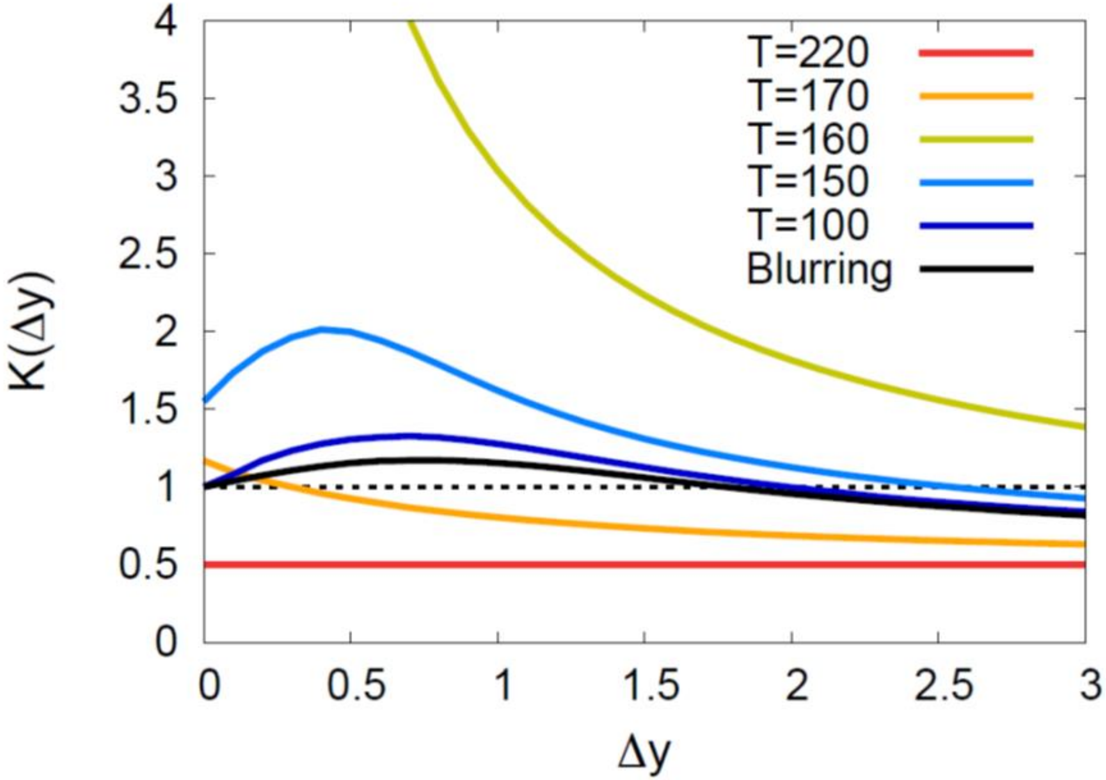


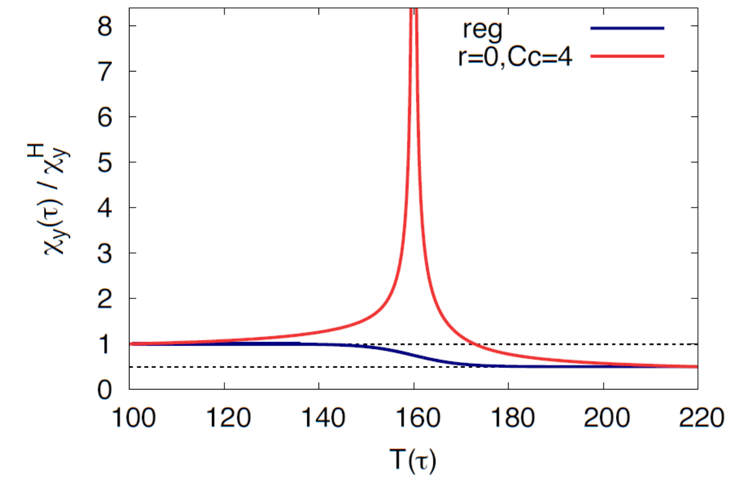
- monotonically decresing

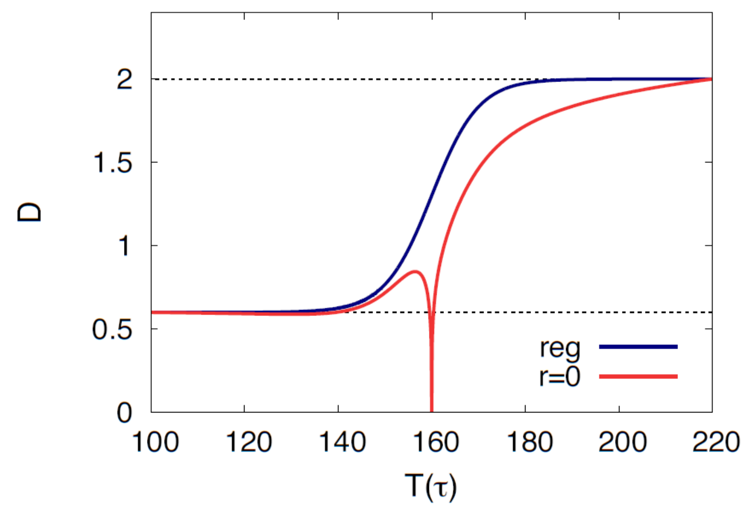
Analytic result: $\chi(\tau)$ monotonically increasing ⇒ $K(\Delta y)$ monotonically decreasing

Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.}$$







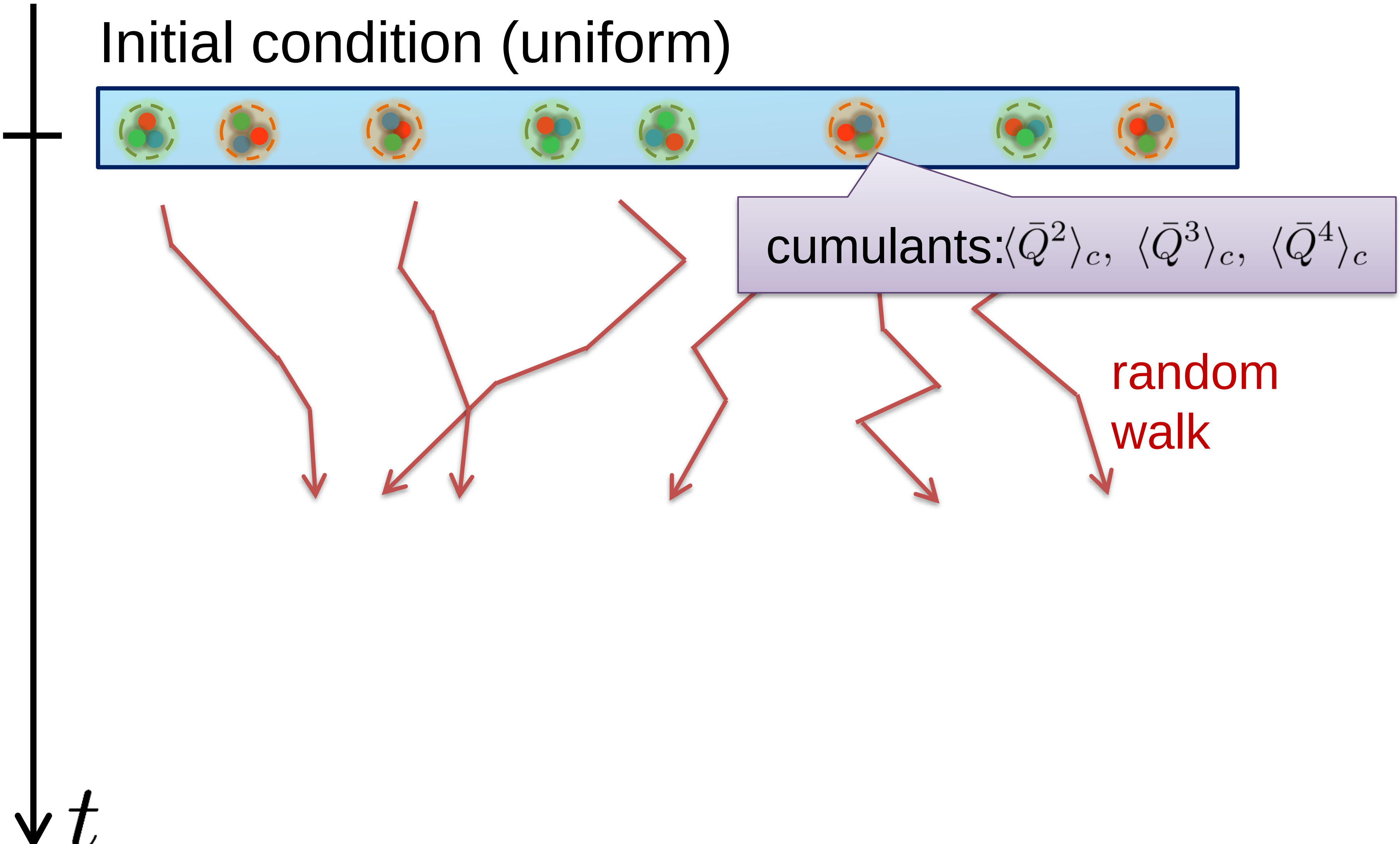
- non-monotonic Δy dep.

Analytic result: $K(\Delta y)$ non-monotonic ⇒ $\chi(\tau)$ non-monotonic

See also,
Wu, Song
arXiv: 1903.06075

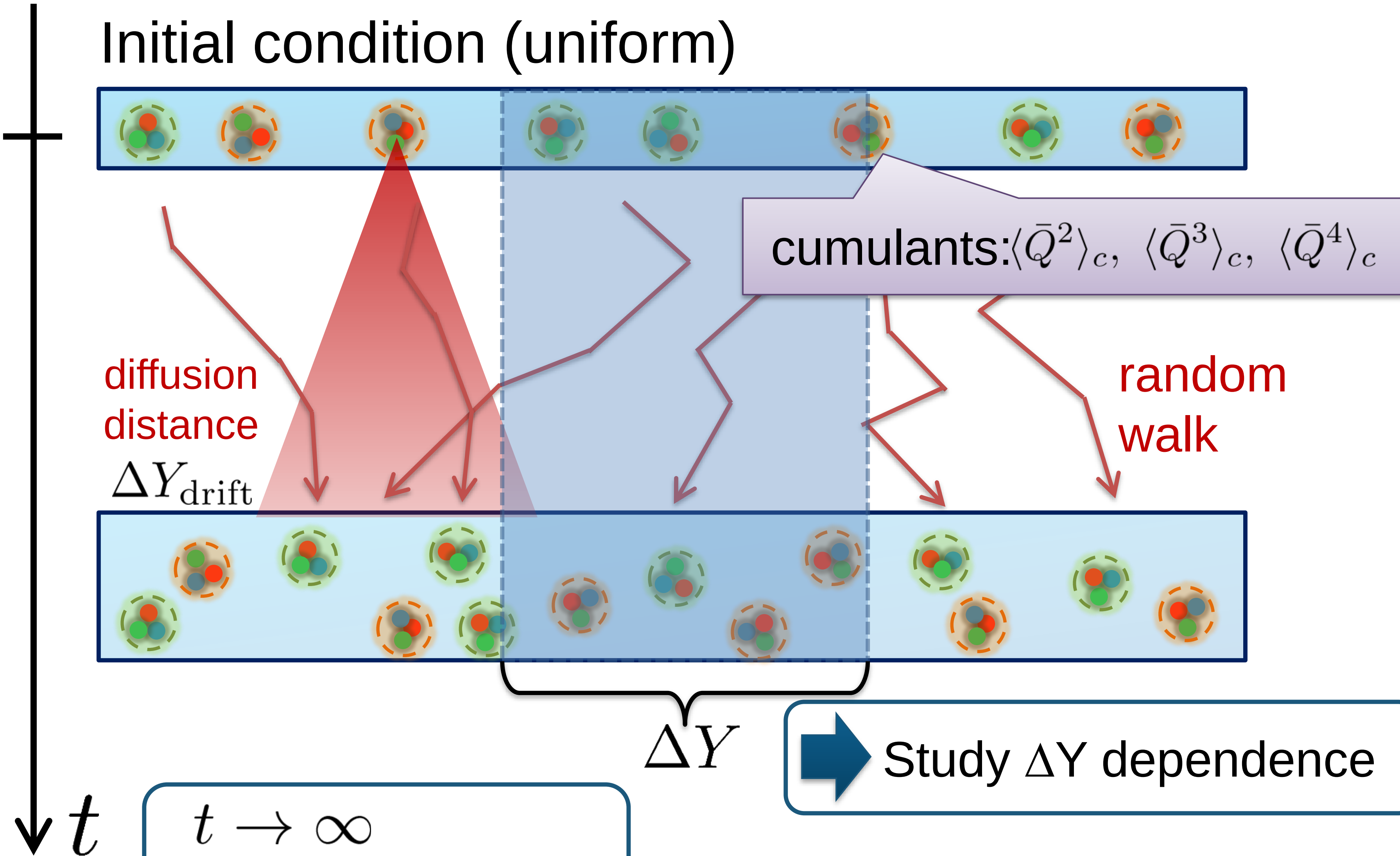
Extension to
Higher-order Cumulants

(Non-Interacting) Brownian Particle Model



diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model



$t \to \infty$
Poisson distribution

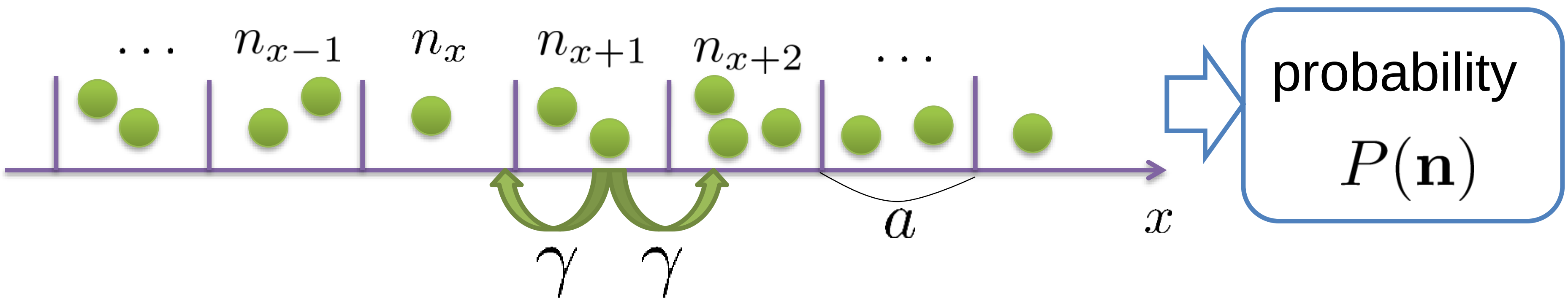
Study ΔY dependence

diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

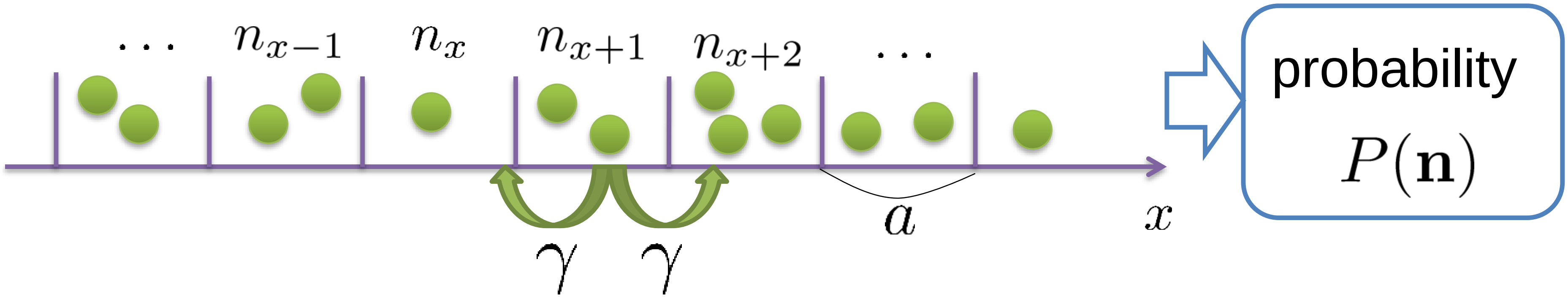
Divide spatial coordinate into discrete cells



Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



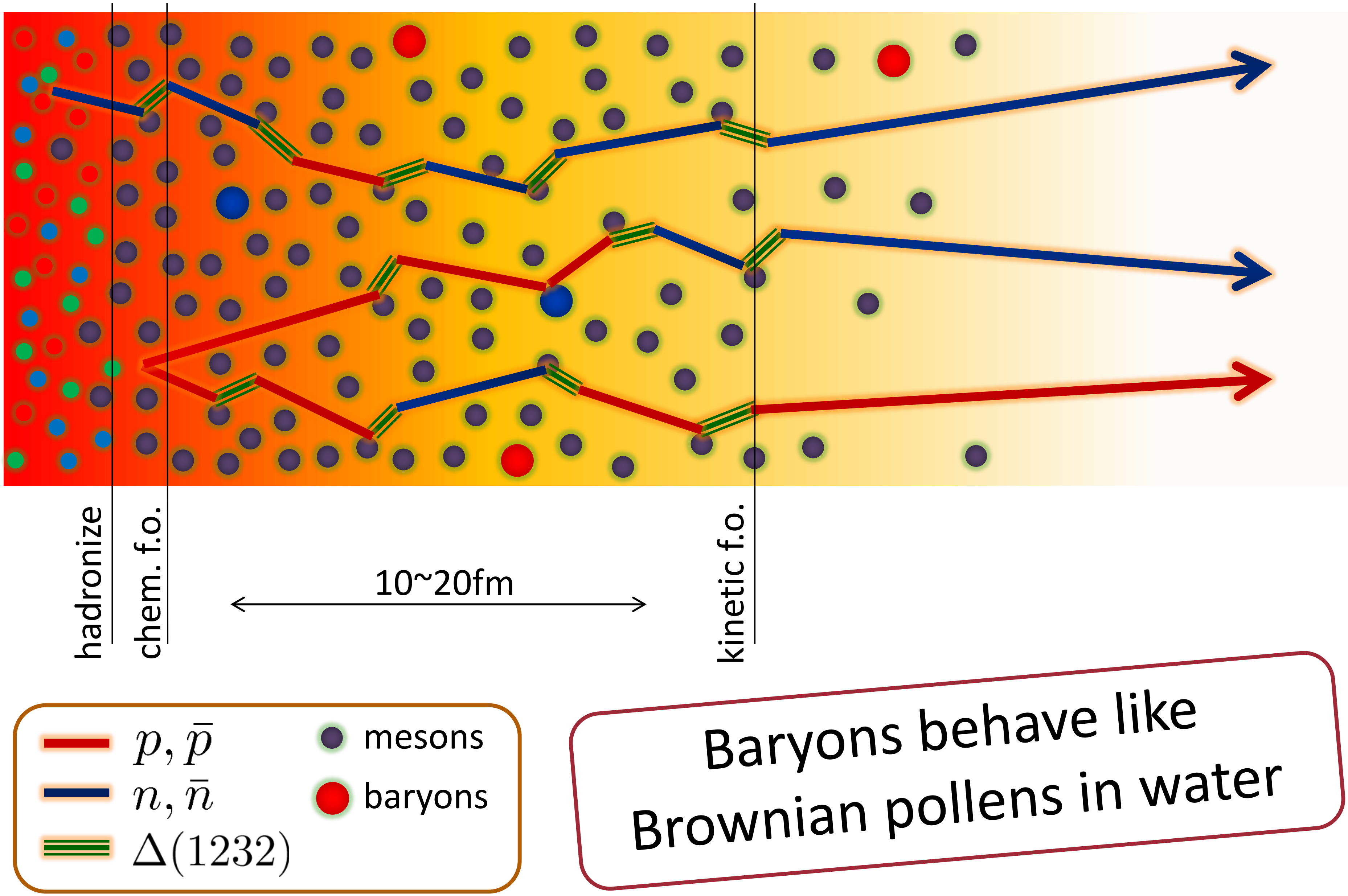
Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1}) + P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

Baryons in Hadronic Phase

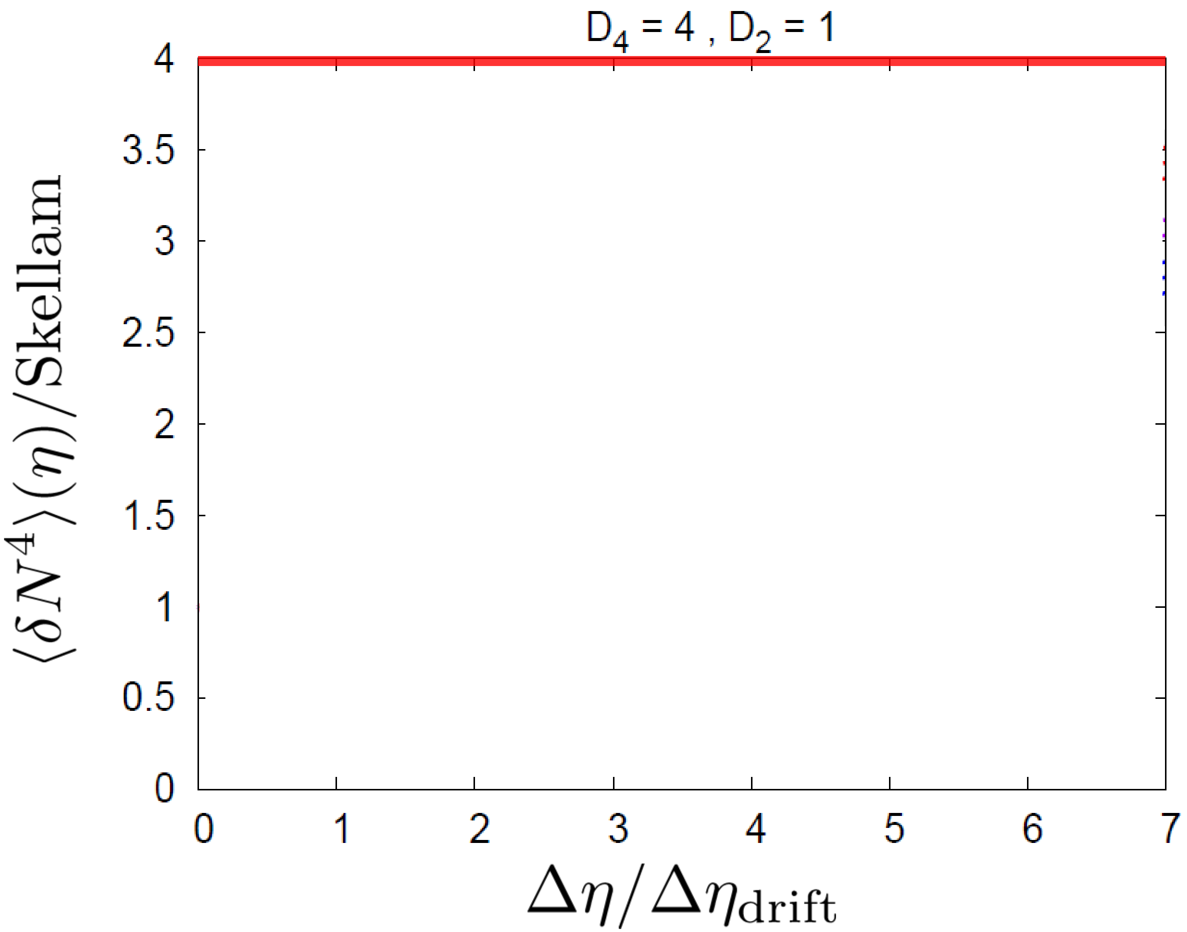


Baryons behave like Brownian pollens in water

4^{th} Order Cumulant

MK+ (2014)
MK (2015)

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

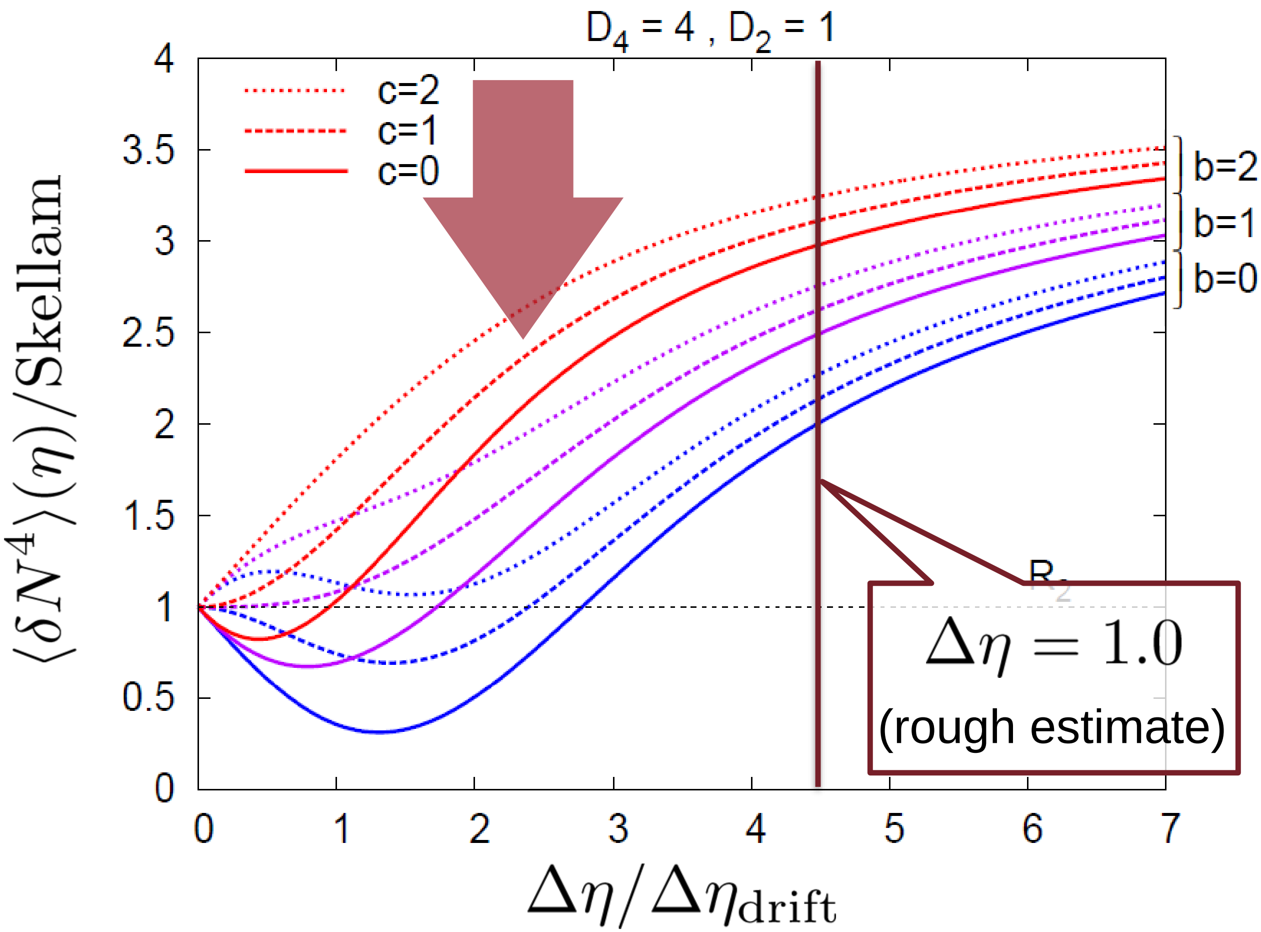
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

4th Order Cumulant

MK+ (2014)
MK (2015)

After the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

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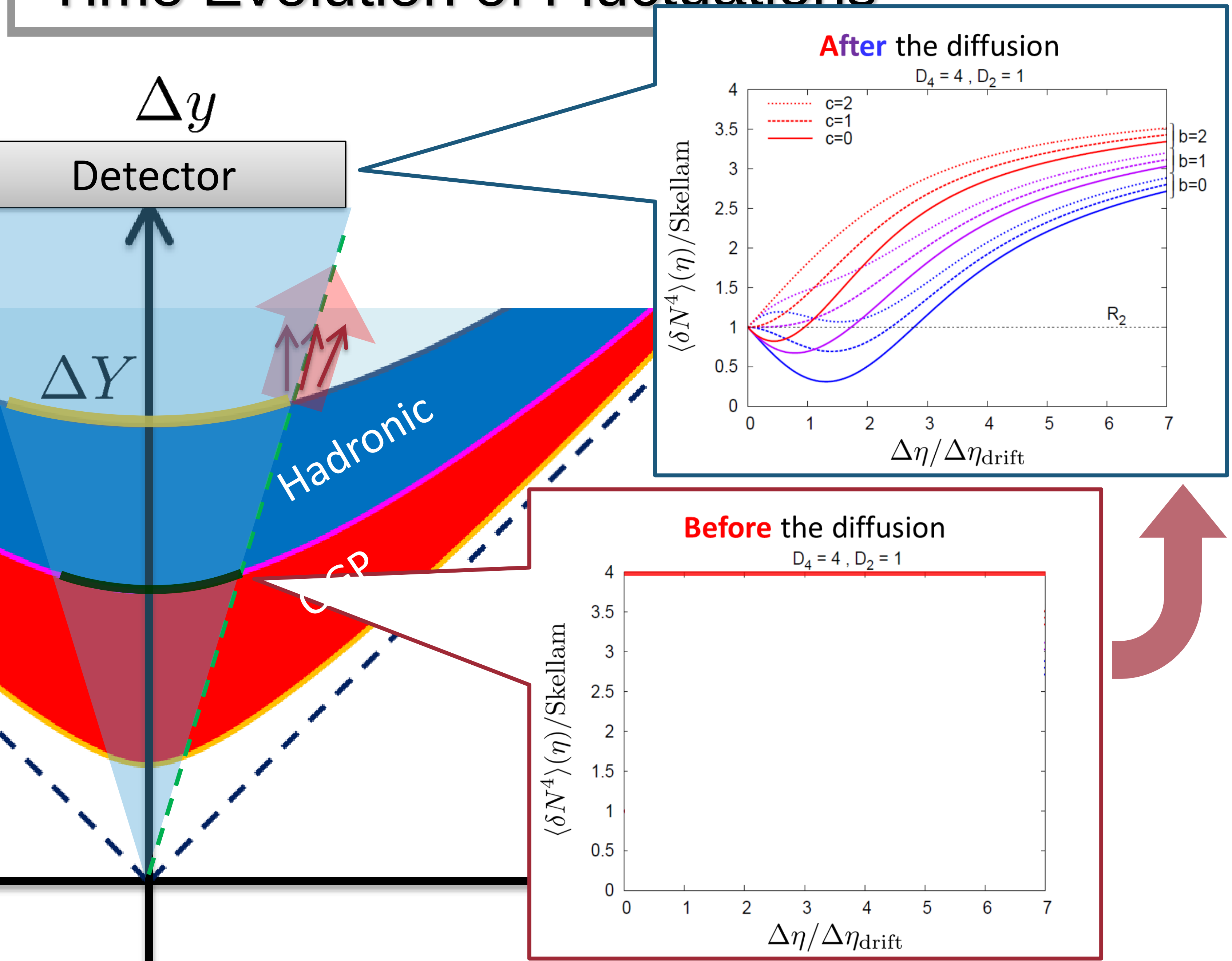
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

- Cumulant at small $\Delta\eta$ is modified toward a Poisson value.
- Non-monotonic behavior can appear.

Time Evolution of Fluctuations



As a result of a simple random walk...

Rapidity Window Dep.

4^{th}-order cumulant

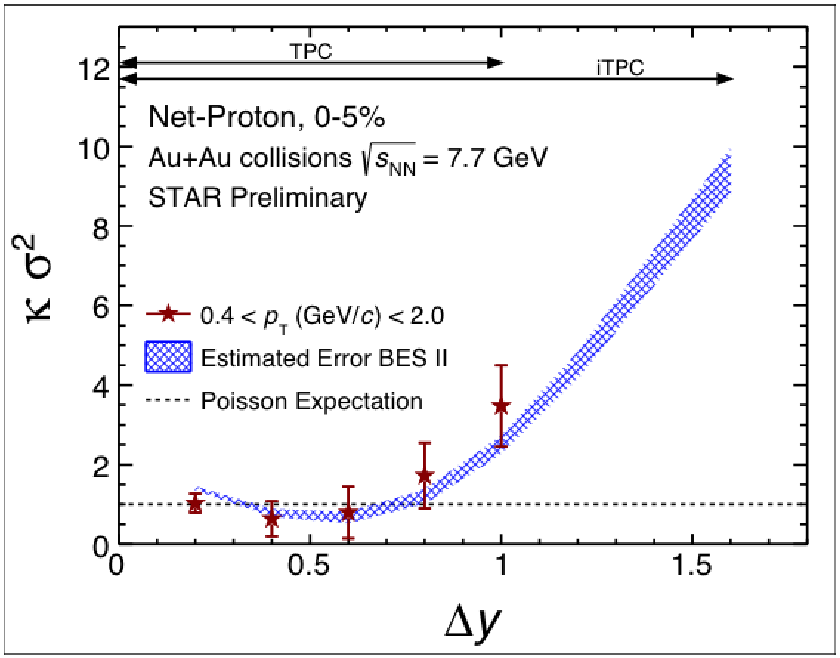
MK+, 2014
MK, 2015

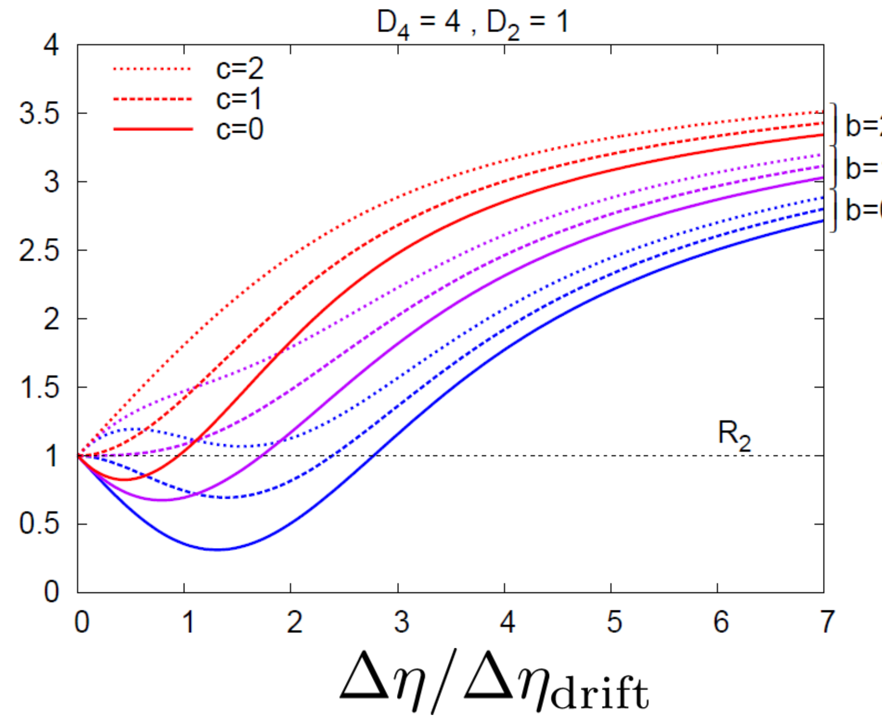
Initial Conditions

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} \qquad c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

STAR Collab. (X. Luo, CPOD2014)



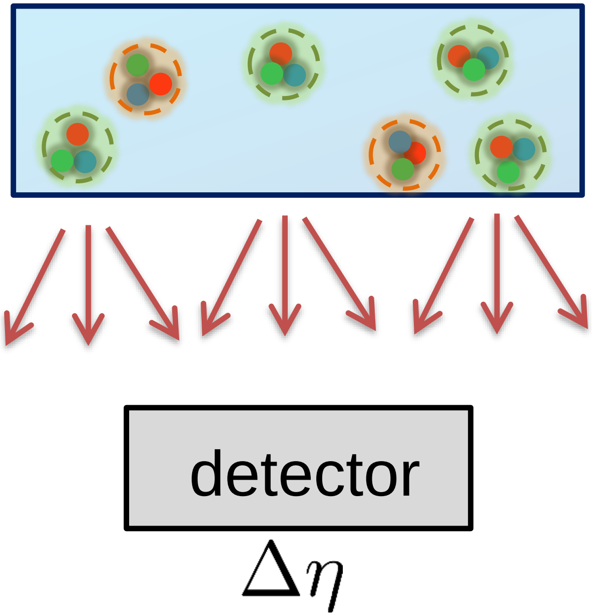


- ☐ Is non-monotonic $\Delta\eta$ dependence already observed?
- ☐ Different initial conditions give rise to different characteristic $\Delta\eta$ dependence. → Study initial condition

Finite volume effects: Sakaida+, PRC90 (2015)

Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling

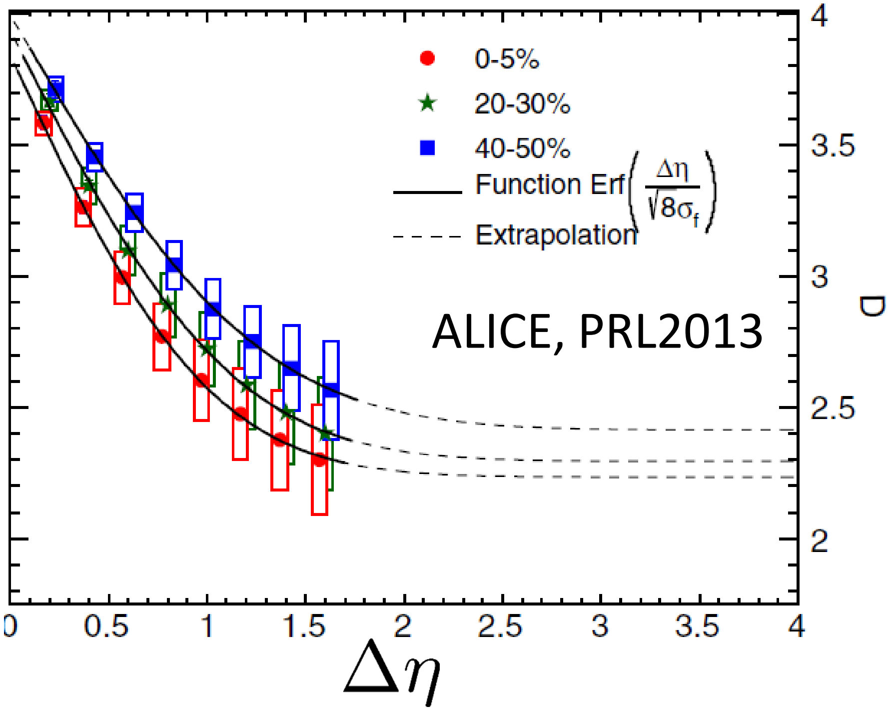


Careful treatment is required to interpret fluctuations at low beam energies!

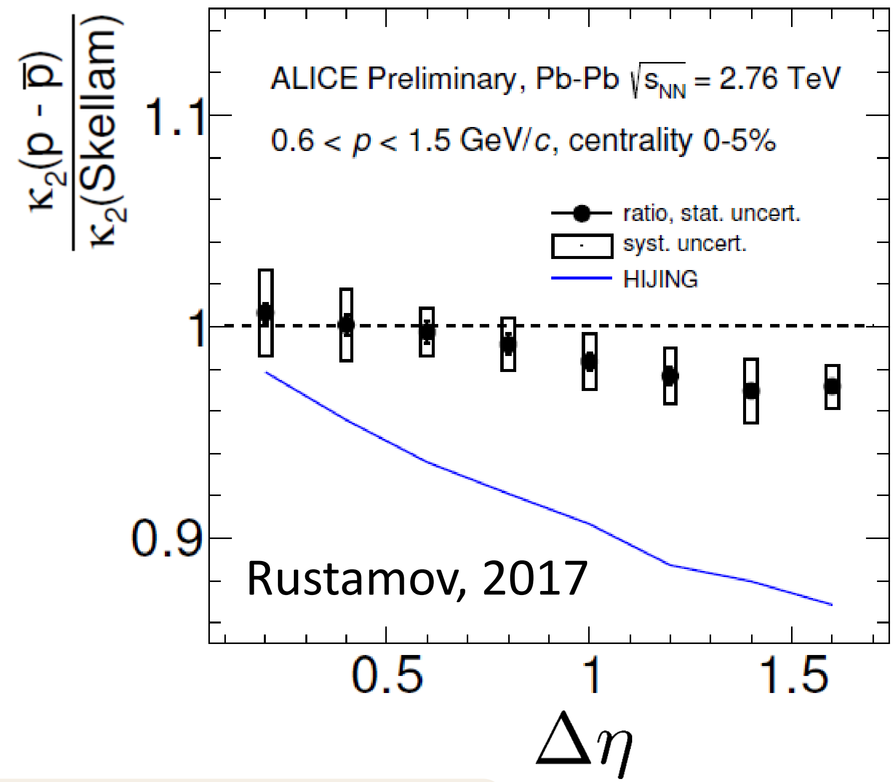
Many information should be encoded in $\Delta\eta$ dep.

2^{nd} Order @ ALICE

Net charge fluctuation



Net proton fluctuation



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$<\delta N_B^2>$ and $<\delta N_p^2>$ @ LHC ?

MK, presentations
GSI, Jan. 2013
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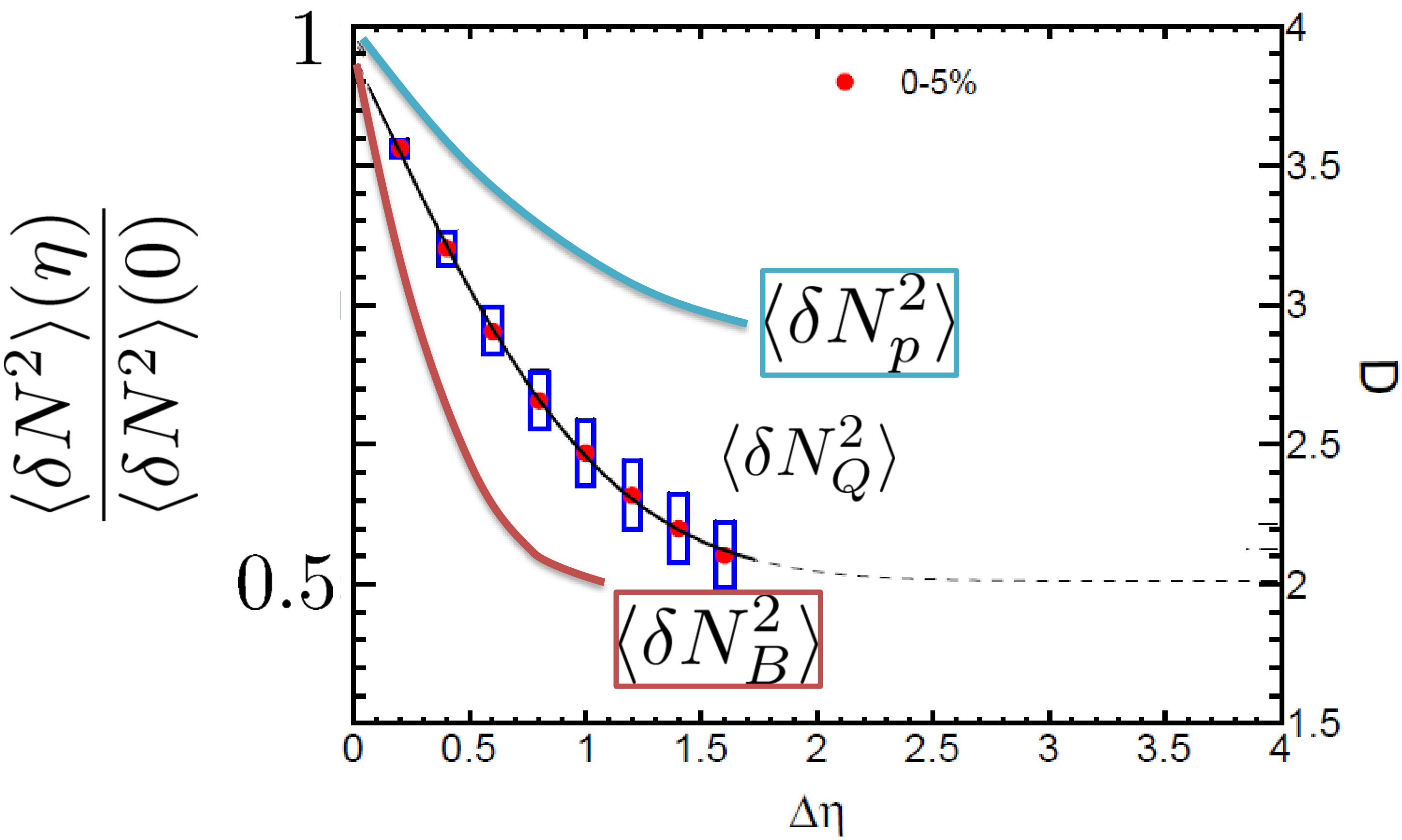
…

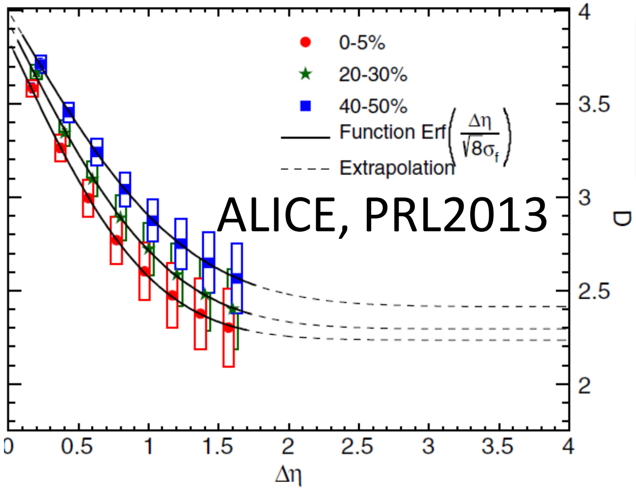
$\langle\delta N_Q^2\rangle, \langle\delta N_B^2\rangle, \langle\delta N_p^2\rangle$ should have different $\Delta\eta$ dependence.



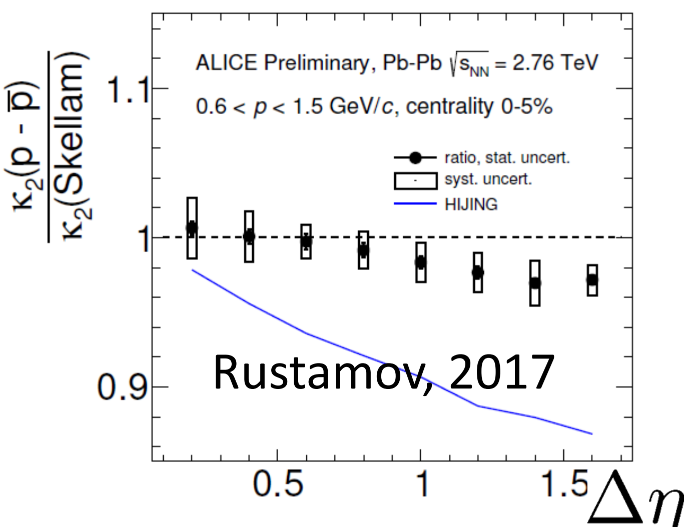
Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

Suggestion

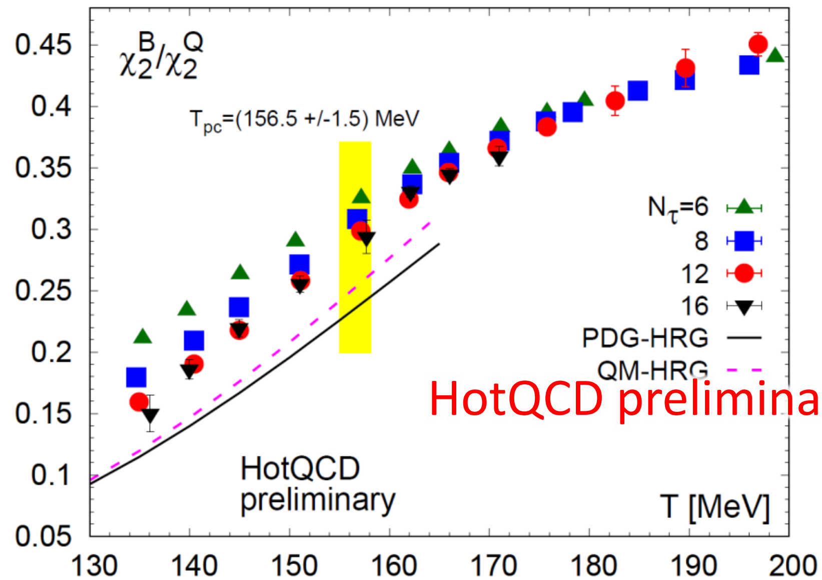
Net charge fluctuation



Net proton fluctuation



- ☐ Construct $\langle \delta N_B^2 \rangle$ $(\langle \delta N_N^2 \rangle)$, $\langle \delta N_Q^2 \rangle$
- ☐ Then, take ratio $\langle \delta N_B^2 \rangle / \langle \delta N_Q^2 \rangle$
- ☐ Compare it with lattice



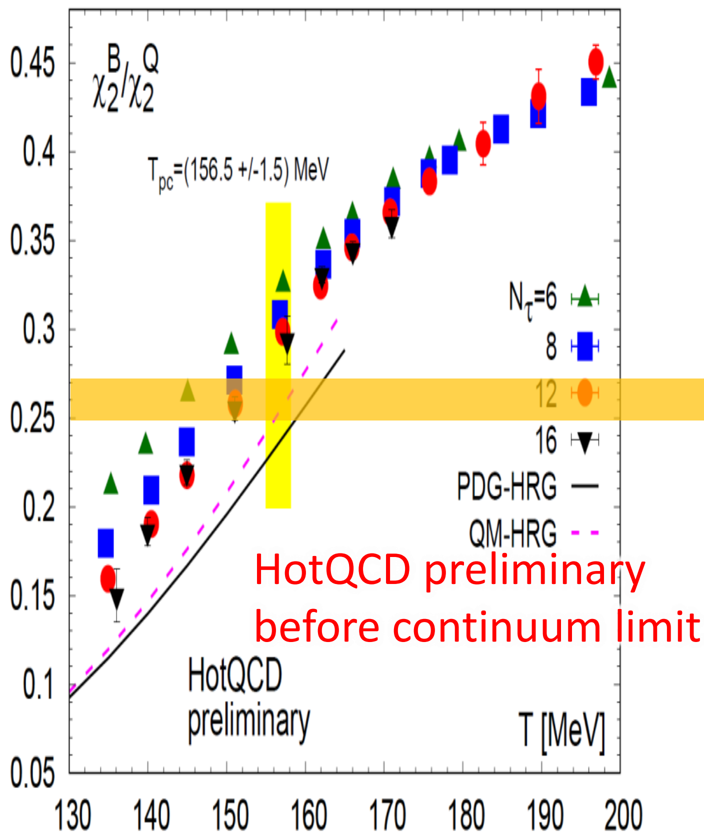
✓ **linear T dependence near Tc !!**

✓ **only 2[nd] order: reliable !!**

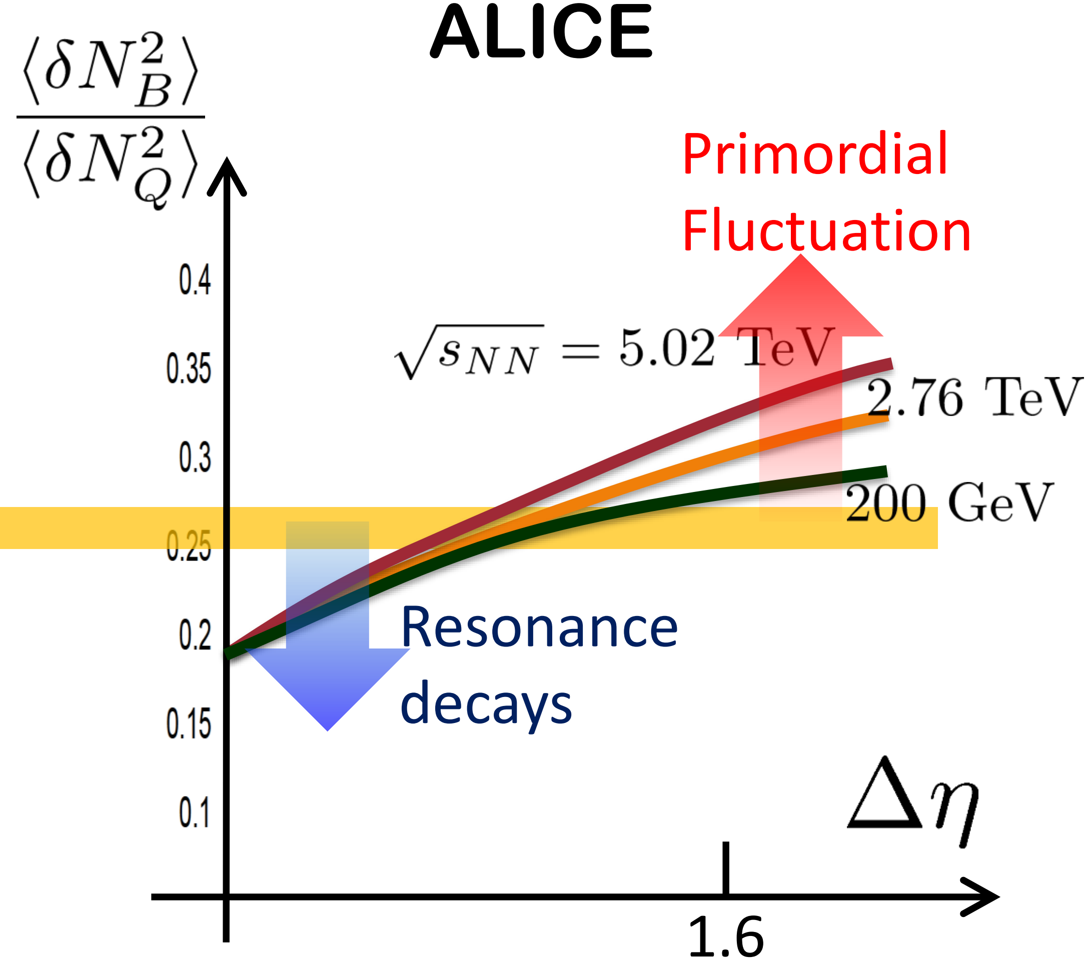
First reliable comparison of LAT/HIC

Prediction

LATTICE



ALICE



Δη dependence for tracing back the history!

Summary

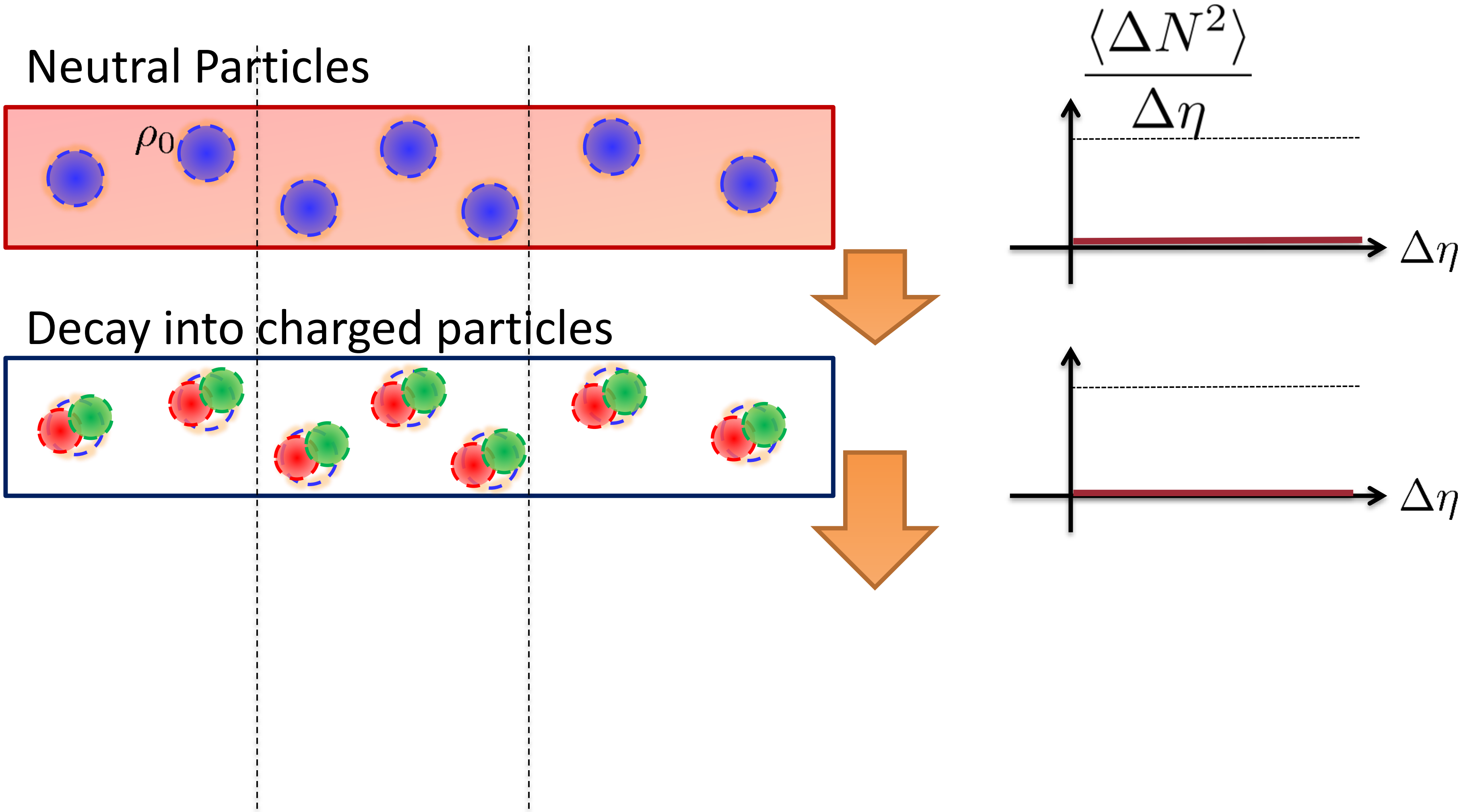
- Large ambiguity in the experimental analysis of higher-order cumulants.

- Fluctuations observed in HIC are not in equilibrium.
- Plenty of information encoded in rapidity window dependences

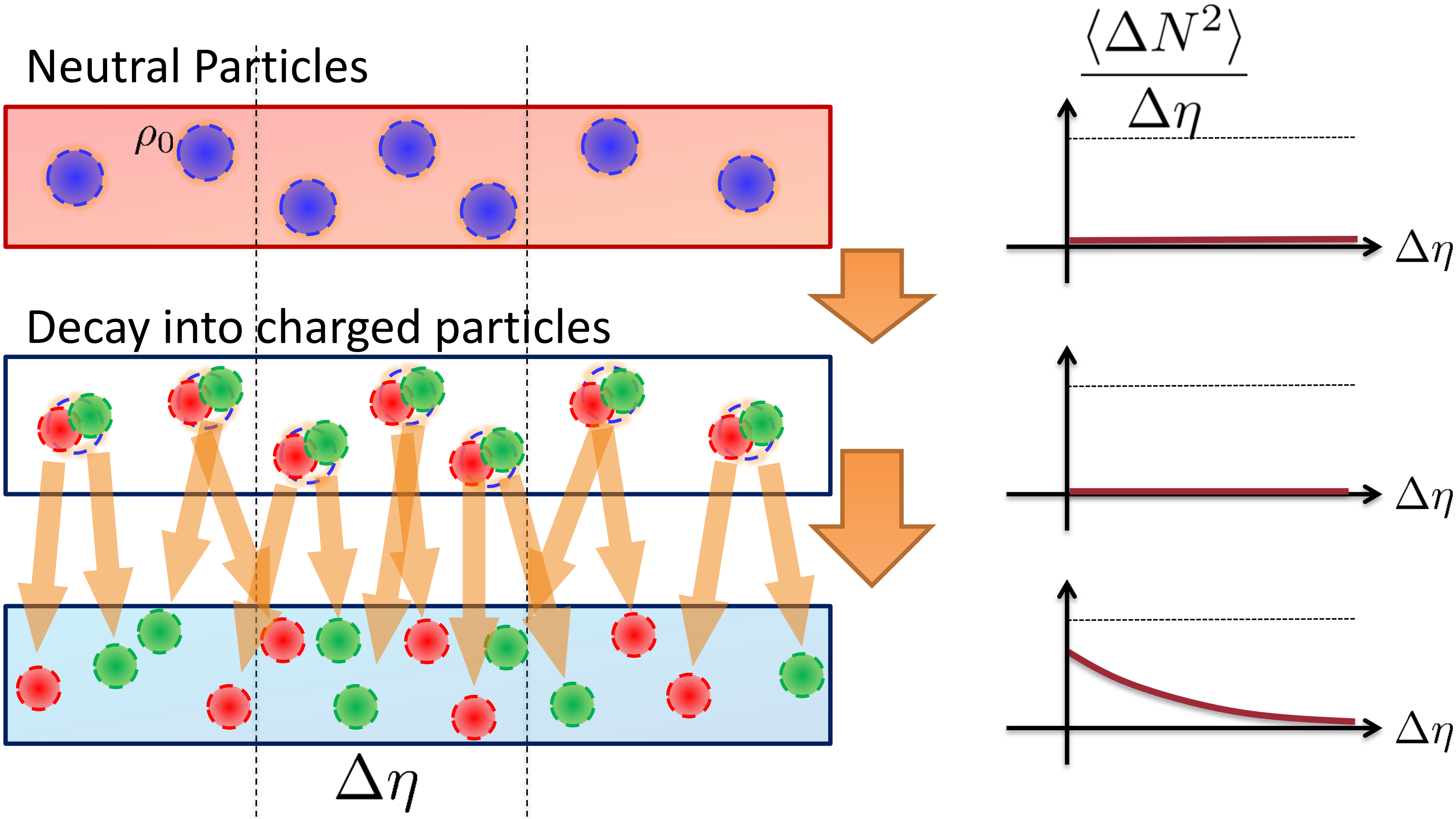
- 2nd-order cumulant (correlation function) already contains interesting information.

- Future
  - Evolution of higher-order cumulants around the critical point / 1st transition
  - combination to momentum (model-H)
  - more realistic model (dimension, Y dependence, …)

Resonance Decay



Resonance Decay



The larger $\Delta\eta$, the slower diffusion.