Dynamical description of hydrodynamic fluctuations in high-energy nuclear collisions

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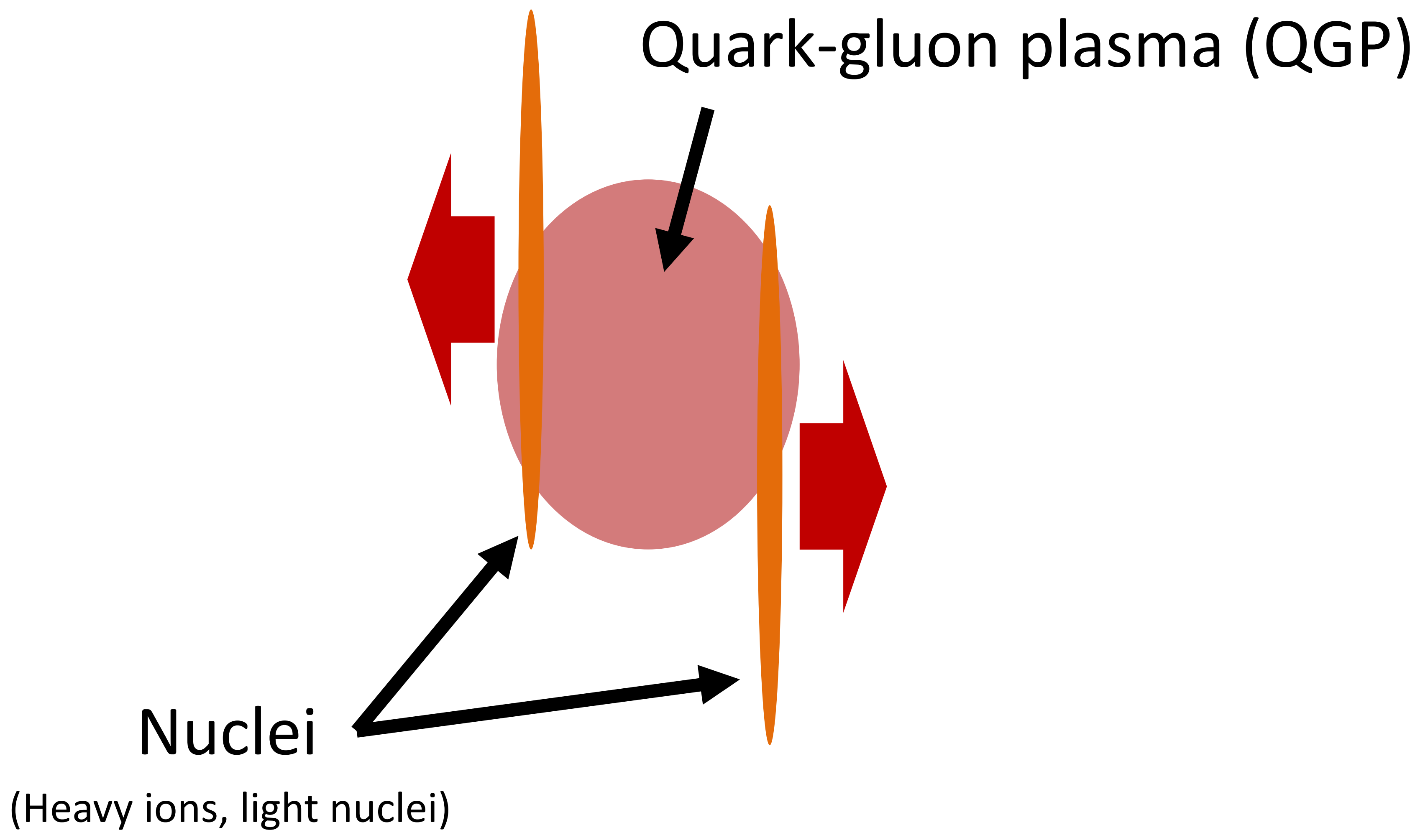
DYNAMICAL DESCRIPTION OF HIGH-ENERGY NUCLEAR COLLISIONS

High-energy nuclear collisions



High-energy nuclear collisions

Purpose?

→ **Equilibrium properties of QGP**

Equation of state

Critical point, first-order phase transition, …

Transport properties

Viscosity, diffusion, relaxation time, CME, CVE, …

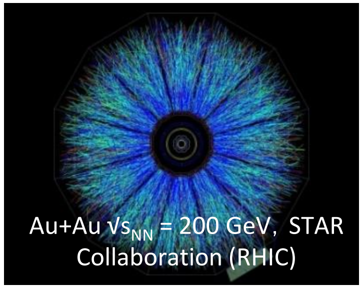
Stopping power for jets/mini-jets, …

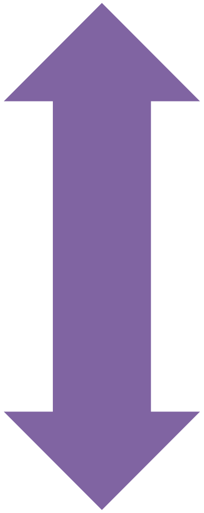
etc.

Dynamical models

Experiments

- Created matter is *small* and *short-lived*
- Matter *expands* in relativistic velocities
- Observed quantities are just *hadron momentum*





Dynamical models are needed

✓ Numerical simulation framework

✓ *Quantitative* description of whole reaction

✓ Pre-equilibrium/Hydro/Hadron gas stages

Equilibrium properties of QGP

Modern dynamical model

"Standard structure" of dynamical models

Hadronic cascades

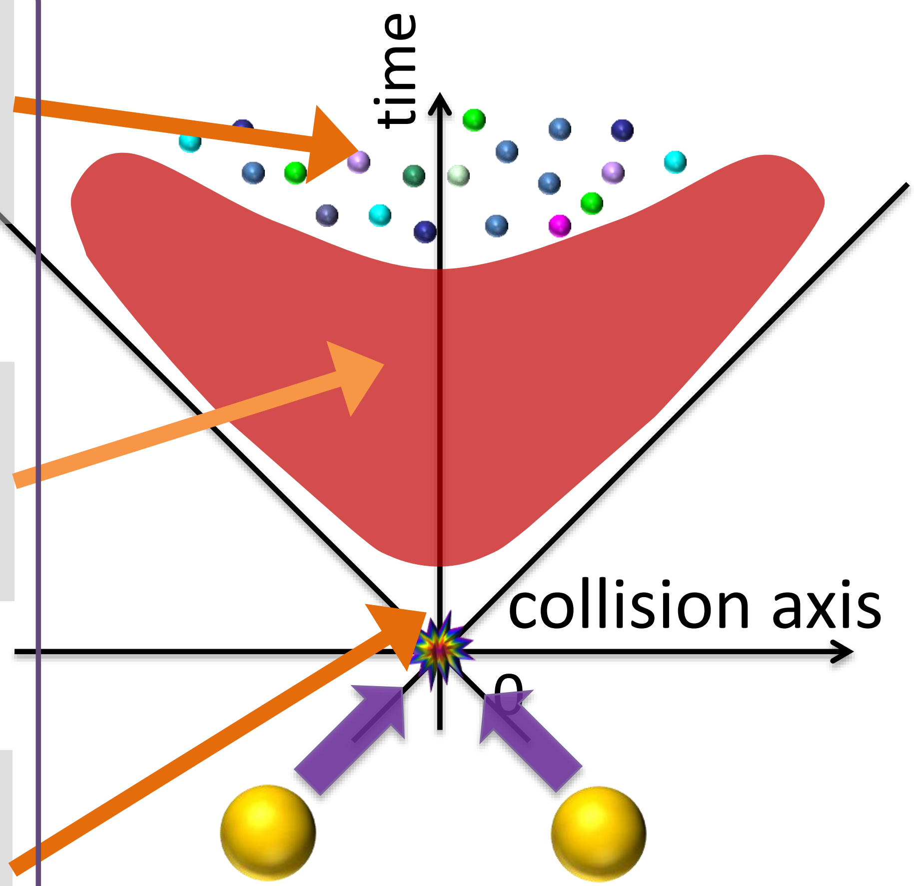
Non-equilibrium dynamics
of hadron gas

Relativistic Hydrodynamics

Spacetime evolution
of thermalized matter

Initial state model

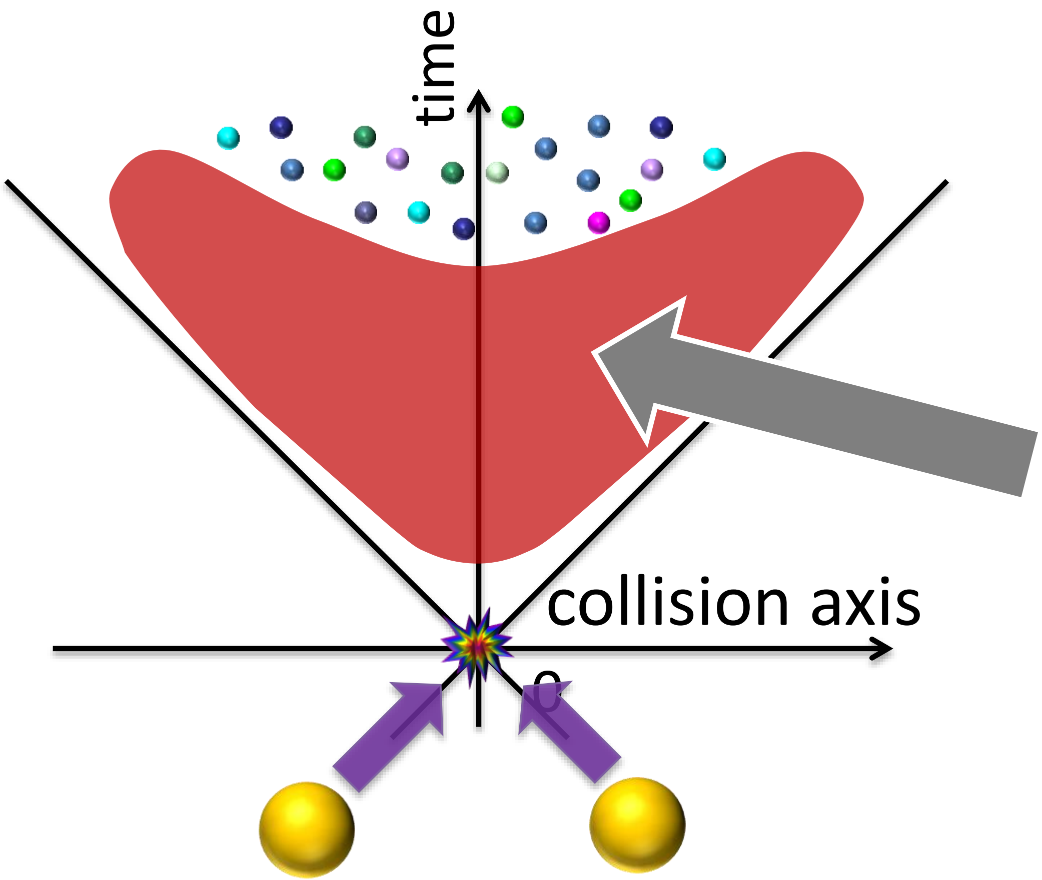
Initial state and
pre-equilibrium dynamics



Brief history of dynamical models

- Ideal hydrodynamics (～2001)　完全流体　P. F. Kolb *et al.*; D. Teaney *et al.*; P. Huovinen *et al.*; T. Hirano *et al.*
- Hybrid model (hydro + cascade) (～2001)　A. Dumitru *et al.*; D. Teaney *et al.*; T. Hirano *et al.*
  ideal hydro + hadronic transport
- Viscosity (2008+)　粘性流体　H. Song *et al.*; S. Ryu *et al.*
  second-order causal hydrodynamics
- Event-by-event fluctuations (2011+)　M. Gyulassy *et al.*; C. E. Aguiar *et al.*; Y.-Y. Ren, *et al.*; Z. Qiu *et al.*; H. Petersen *et al.*; K. Werner *et al.*; B. H. Alver *et al.*; B. Schenke *et al.*; A. K. Chaudhuri; P. Bozek *et al.*; L. Pang *et al.*; H. Zhang *et al.*; T. Hirano *et al.*
  Monte-Carlo initial conditions
- Jet-induced medium response (2012+)　Y. Tachibana, T. Hirano
  Back reaction to hydrodynamics
- Hydrodynamic fluctuations (2014+)　揺動流体　K. Murase, T. Hirano
  Thermal fluctuations of hydrodynamics
- Critical dynamics (Recent)　M. Bluhm, M. Nahrgang *et al.*; M. Sakaida *et al.*; S. Wu *et al.*
- Dynamical initialization (Recent)　C. Shen *et al.*; Y. Akamatsu *et al.*; Y. Kanakubo *et al.*
- etc.　(Classical Yang-Mills eq, Anisotropic hydro, Chiral magneto hydrodynamics, Non-eq chiral fluid dynamics, Hydro+, …)

Hydrodynamic fluctuations



Hydrodynamic fluctuations

= Thermal fluctuations of hydro fields

WHY HYDRODYNAMIC FLUCTUATIONS?

Fluctuations in heavy-ion collisions

- **Final observables**
  - – flow coefficients v_n, etc.

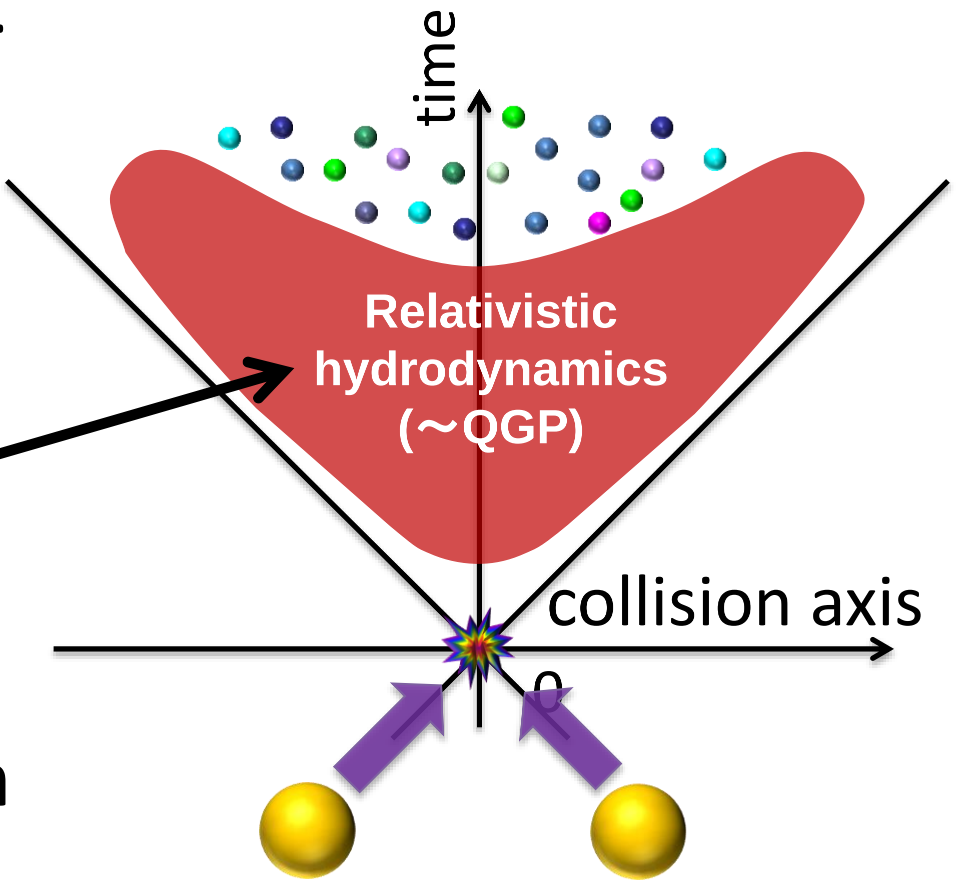
Matter response

EoS, η, ζ, τ_R, etc.

Additional fluctuations

hydro fluctuations

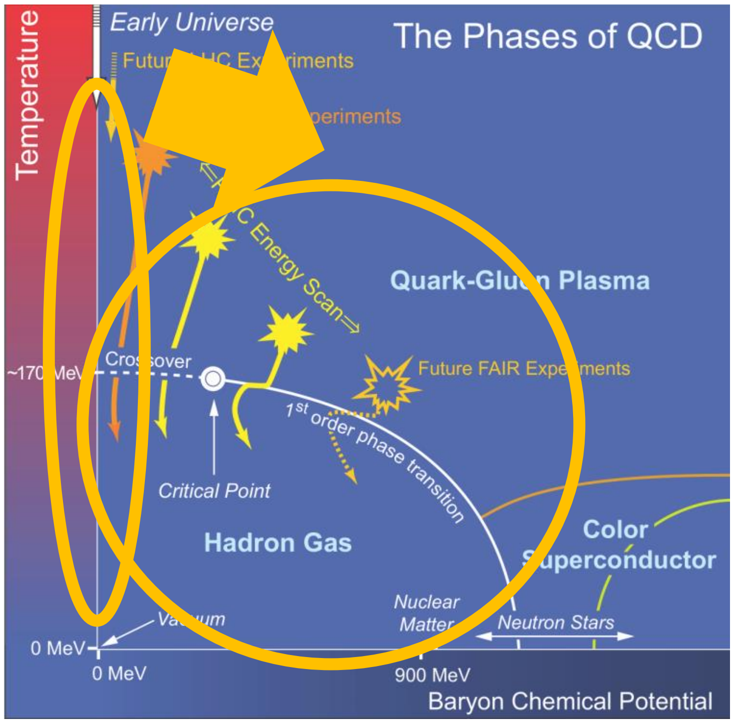
jets/mini-jets, etc



- **Initial state fluctuation**
  - – nucleon distribution,
  - – gluon/color fluctuations, etc.

QCD critical point search

Search of QCD critical point and 1st order phase transition



Schematic phase diagram of QCD
[taken from the 2007 NSAC Long Range Plan]

Dynamical models
for *high-energy* collisions
(Hydro + cascade + ...)

Needed extensions

- EoS modeling
- critical fluctuations
- dynamical initialization
- dynamical core-corona separation

Dynamical models
for *lower-energy* collisions?

Hydrodynamics

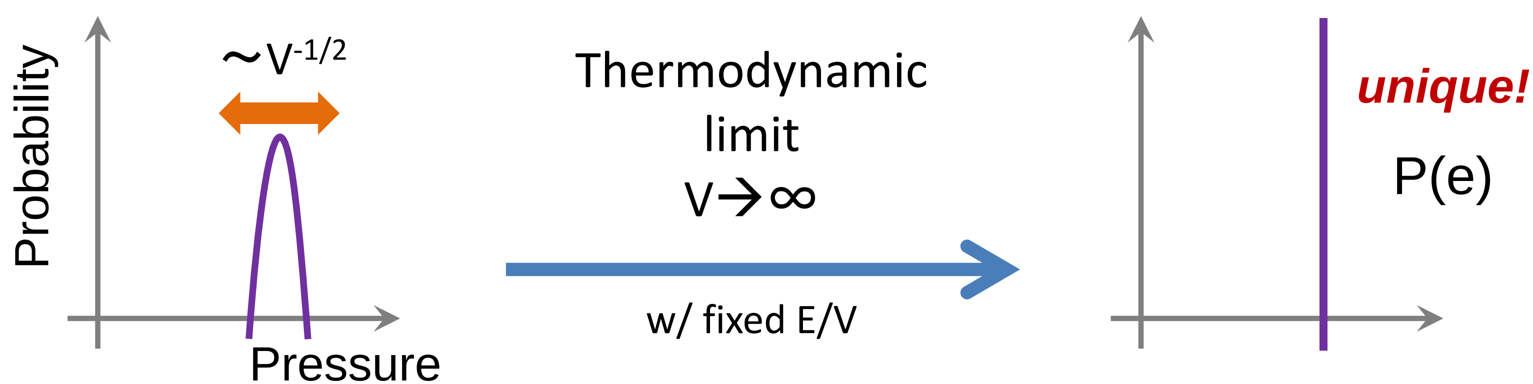
Hydro = Basically, conservation laws

e.g. $\partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu\nu} = e u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$

"Hydrodynamics" works iff…

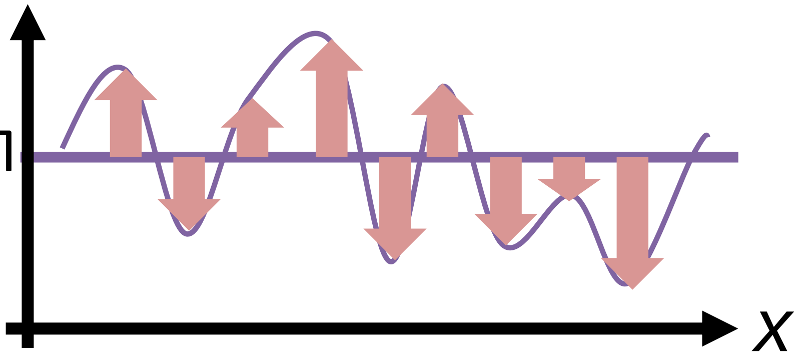
fluxes T^{ij} (∼ pressure, stress, etc.) ***are uniquely determined by conserved densities*** $T^{0\mu}$ (∼e, u^μ).

e.g. **EoS** & **Constitutive eqs.** near equilibrium



Hydrodynamic fluctuations

Thermal fluctuations of fluid fields



spontaneous field fluctuations of fluid fields such as $\pi^{\mu\nu}$, Π, etc. at each t and each x

c.f. L. D. Landau and E. M. Lifshitz, Fluid Mechanics (1959)

Fluctuation-dissipation relation (FDR)

Magnitude of *fluctuations* $\delta\pi$, etc.
is determined by *dissipation* η, etc. (and temperature T)

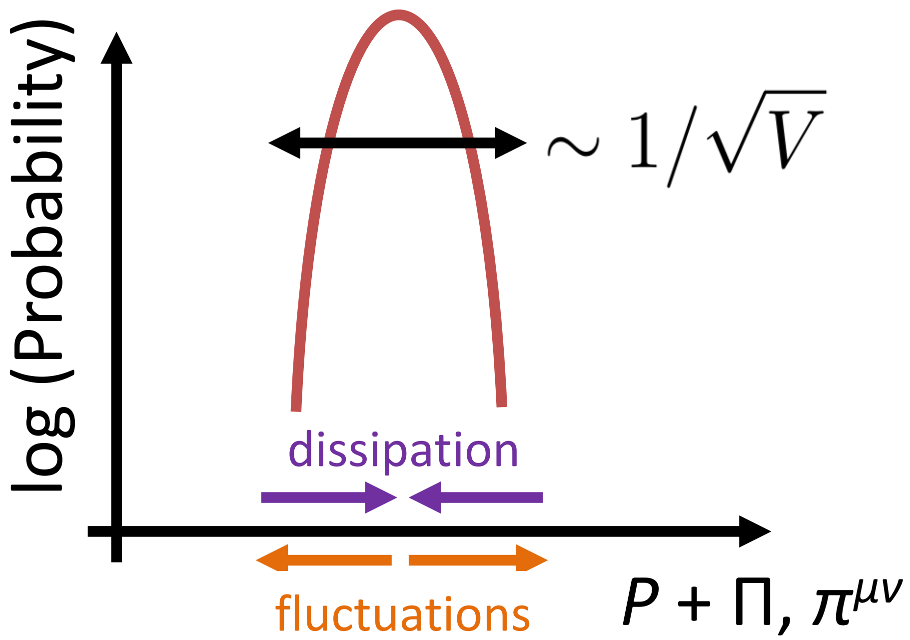
$$\langle \delta\pi^{\mu\nu}(x)\delta\pi^{\alpha\beta}(x')\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x-x')$$

$\eta \neq 0$ ⇒ $\delta\pi \neq 0$

Fluctuation-dissipation theorem

Thermal distribution of fluid fields

$$\ln \mathrm{Pr} \sim S \sim V \cdot \left(s_{\mathrm{eq}} - \frac{\tau_\Pi \Pi^2}{2T\zeta} - \frac{\tau_\pi \pi^{\mu\nu}\pi_{\mu\nu}}{4T\eta} \right)$$



FDR = Balance of fluctuations and dissipation to maintain the thermal distribution

Hydrodynamic fluctuations

FDR: $\langle \delta\pi^2 \rangle \simeq 4T\eta \, / \, \Delta t \, \Delta x^3$ (Δt, Δx: typical length/time scale)

Water in a glass

$\eta \simeq 10^{-3}$ [Pa sec] ($\simeq 50\, s$)

$T \simeq 300$ [K] ($\simeq 3 \times 10^{-9}$ [MeV])

$\Delta x \simeq 10^{-3}$ [m]

$\Delta t \simeq 10^{-1}$ [sec]

⇓

$\delta\pi \simeq 4 \times 10^{-8}$ [Pa]

$P \simeq 10^{5}$ [Pa]

⇓

$\delta\pi \, / \, P = 4 \times 10^{-13}$

QGP in heavy-ion collisions

$\eta \simeq 0.1\, s \simeq 0.2$ [fm^{-3}]

$T \simeq 300$ [MeV]

$\Delta x \simeq 1$ [fm]

$\Delta t \simeq 1$ [fm]

⇓

$\delta\pi \simeq 2 \times 10^{2}$ [MeV/fm^3]

$P \simeq 4 \times 10^{3}$ [MeV/fm^3]

⇓

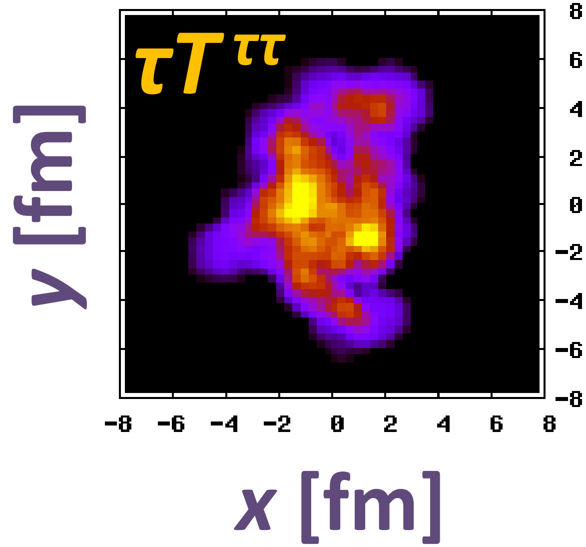
$\delta\pi \, / \, P = 0.05$

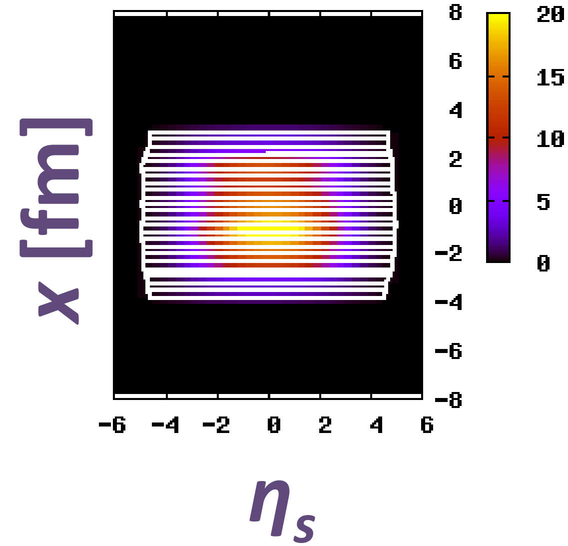
Hydrodynamic fluctuations are not negligible in QGP

Hydrodynamic evolution

without HF

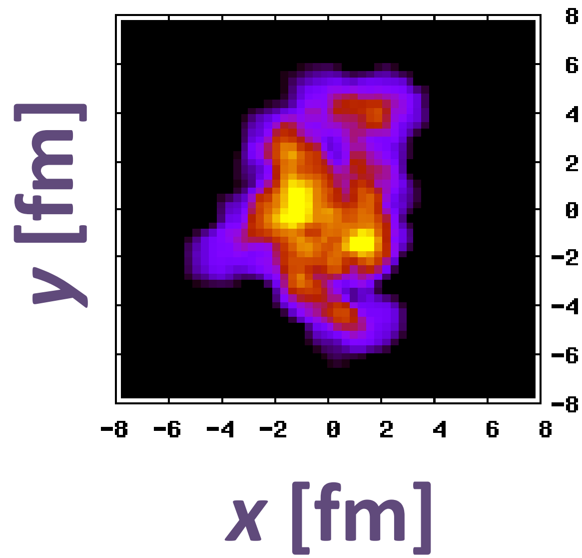
conventional
2^{nd}-order
viscous hydro

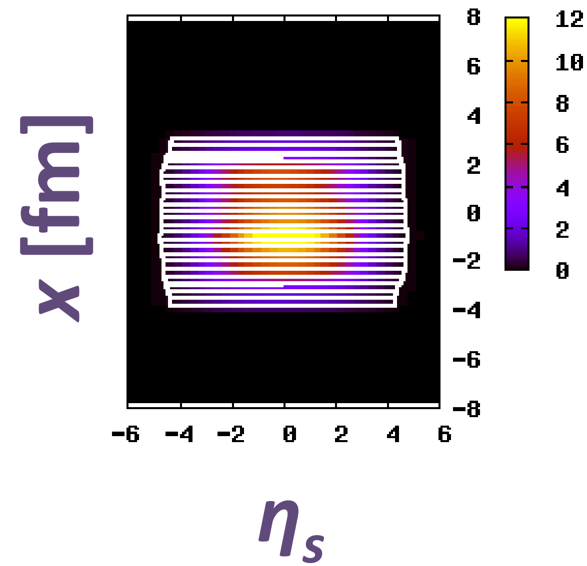




with HF

2^{nd}-order
fluctuating hydro





DISCUSSIONS

1. Smeared fluctuating hydrodynamics

- Noise terms with autocorrelations ~ $\delta^{(4)}(x-x')$
- Hydrodynamic eqs are non-linear

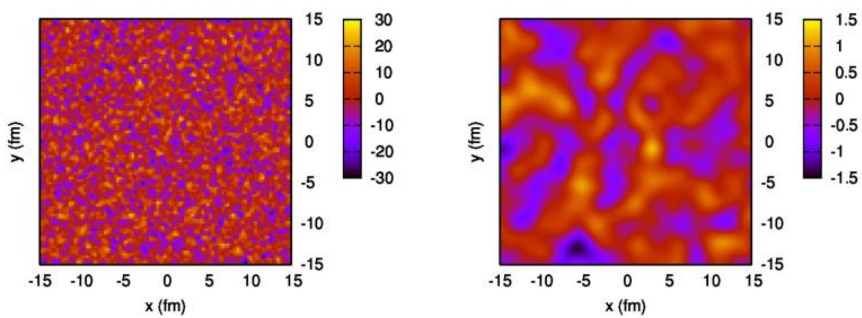
→ Continuum limit is non-trivial

Regularization: cutoff length λ (or **cutoff momentum Λ**=1/λ)

~ coarse-graining scale > microscopic scale

Noise terms $w(x)$ are smeared by,
e.g., Gaussian of width $\sigma = \lambda$

$$w(\boldsymbol{x})^{\sigma} = \frac{1}{(2\pi\sigma^2)^{3/2}} \int d^3x' \exp\left(-\frac{(\boldsymbol{x}-\boldsymbol{x}')^2}{2\sigma^2}\right) w(\boldsymbol{x}').$$



(a) no smearing (b) Gaussian ($\sigma = 1$fm)

2. Stochasitic Integrals

Structure of 2nd order fluctuating hydrodynamics

$$\begin{cases} dU = f_1(U,\Gamma)d\tau, \\ d\Gamma = f_2(U,\Gamma)d\tau + g_2(U)\circ \underline{dB}, \end{cases}$$

noise

$U = (e, u^\mu, n_i)$ $\quad \Gamma = (\Pi, \pi^{\mu\nu}, \nu_i^\mu)$ $\quad \tau$: proper time

→ **No difference between Ito/Stratonovich SDE**

$$g_2(U)\circ dB = g_2(U)\cdot dB + \frac{1}{2}\left[\frac{\partial g_2}{\partial U}\underline{dU} + \underline{\frac{\partial g_2}{\partial \Gamma}d\Gamma}\right] dB$$

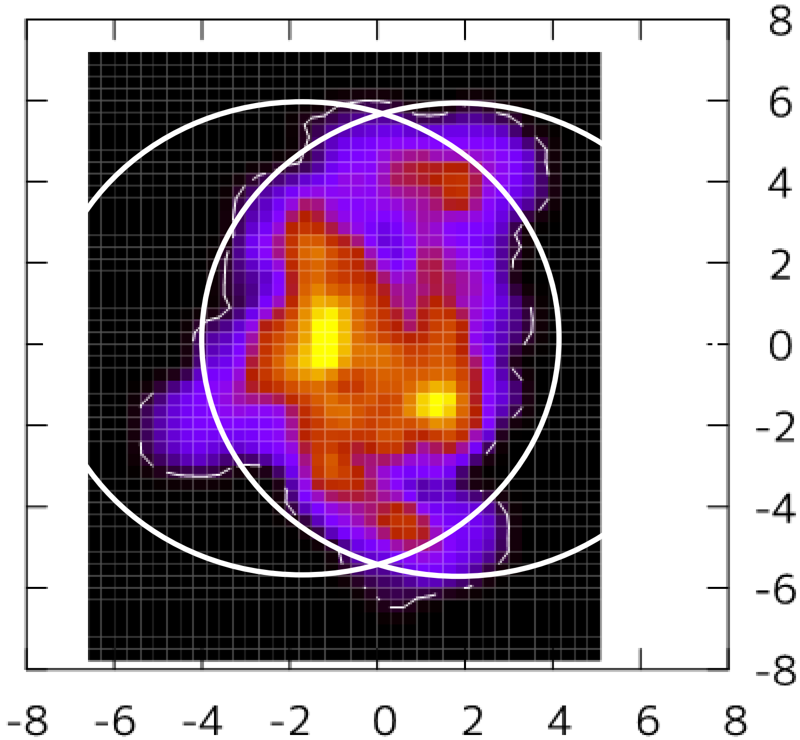
Stratonovich product $\qquad$ $d\tau$ $\qquad$ 0

$$= g_2(U)\cdot dB.$$

Ito product

3. FDR under background evolution

Matter created in nuclear collisions
is not ***static*** and ***homogeneous***



Au+Au 200GeV MC-KLN
(*x*-*y* plane)

Usual FDR relies on
the linear-response
of ***global equilibrium***
to small perturbations

How is FDR modified in
inhomogeneous and
non-static matter?

3. FDR under background evolution

Differential form FDR for simplified IS

$$\langle \xi_\Pi(x)\xi_\Pi(x')\rangle = \left(2 + \tau_\Pi \mathrm{D}\ln\frac{T\zeta}{\tau_\Pi} - \tau_\Pi\theta\right) T\zeta\delta^{(4)}(x-x'),$$

$$\langle \xi_\pi^{\mu\nu}(x)\xi_\pi^{\alpha\beta}(x')\rangle = 2\left[\left(2 + \tau_\pi \mathrm{D}\ln\frac{T\eta}{\tau_\pi} - \tau_\pi\theta\right)\Delta^{\mu\nu\alpha\beta} + \tau_\pi\mathcal{D}\Delta^{\mu\nu\alpha\beta}\right] T\eta\delta^{(4)}(x-x'),$$

$$\begin{aligned}\langle \xi_i^{\mu}(x)\xi_j^{\alpha}(x')\rangle &= -2T\kappa_{ij}\Delta^{\mu\alpha}\delta^{(4)}(x-x') \\ &\quad - \Delta^{\mu\alpha}[K_{ij}^{\mathrm{A}}(x)\mathcal{D} - K_{ij}^{\mathrm{A}}(x')\mathcal{D}']\delta^{(4)}(x-x') \\ &\quad + \sum_{k=1}^{n}\left\{-\Delta^{\mu\alpha}\left[\tau_{ik}\mathrm{D}T\kappa_{kj} - (\mathrm{D}\tau_{ik})T\kappa_{kj} - \tau_{ik}\theta T\kappa_{kj}\right]^{\mathrm{S}} - K_{ij}^{\mathrm{S}}\mathcal{D}\Delta^{\mu\alpha}\right\}\delta^{(4)}(x-x'),\end{aligned}$$

where $K_{ij}^{\mathrm{S/A}}(x) = \sum_{k=1}^{n} T(x)(\tau_{ik}(x)\kappa_{kj}(x) \pm \tau_{jk}(x)\kappa_{ki}(x))/2$, and $[\circ_{ij}]^{\mathrm{S}} = (\circ_{ij} + \circ_{ji})/2$.

$$\mathcal{D}\Delta^{\mu\nu\alpha\beta} = \Delta^{\mu\nu}{}_{\kappa\lambda}\Delta^{\alpha\beta}{}_{\gamma\delta}\mathrm{D}\Delta^{\kappa\lambda\gamma\delta},$$
$$\mathcal{D}\Delta^{\mu\alpha} = \Delta^{\mu}{}_{\kappa}\Delta^{\alpha}{}_{\gamma}\mathrm{D}\Delta^{\kappa\gamma}.$$

complicated expression due to the tensor structure…

Essential structure

$$\langle \xi(x)\xi(x')\rangle = \left(2 + \tau_R\mathrm{D}\ln\frac{T\kappa}{\tau_R} - \tau_R\theta\right) T\kappa\delta^{(4)}(x-x').$$

new modification terms ∝ relaxation time

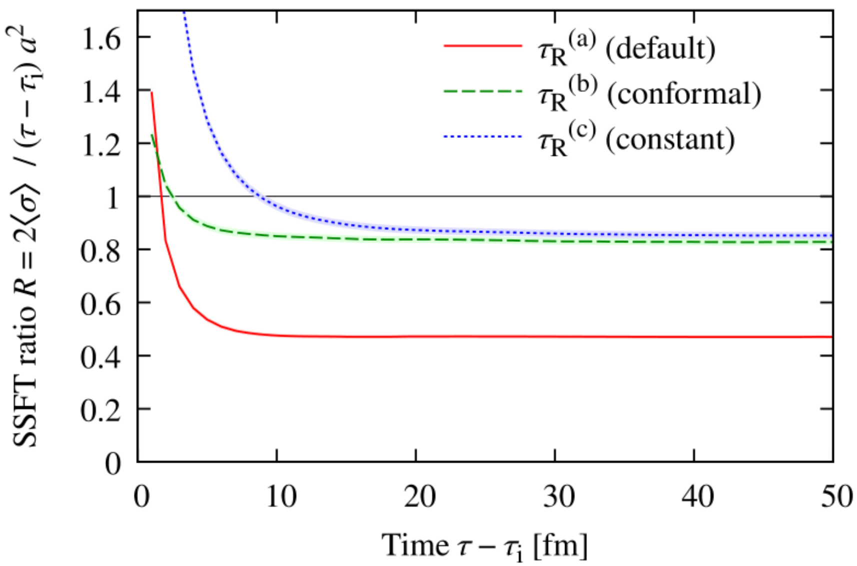
3. FDR under background evolution

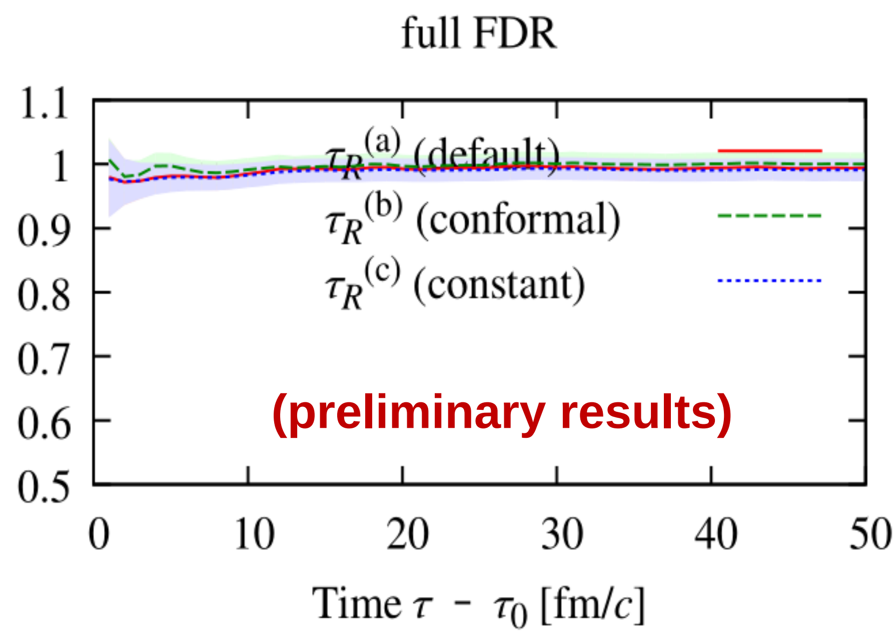
SSFT breaking by relaxation times was actually artifact.

With the FDR modification, SSFT recovers:

$$\langle \xi(x)\xi(x') \rangle = \left(2 + \tau_R \mathrm{D} \ln \frac{T\kappa}{\tau_R} - \tau_R \theta \right) T\kappa \delta^{(4)}(x - x').$$

$R \neq 1$ **SSFT breaking**





3. FDR under background evolution

When background is non-static / non-uniform,
FDR have **corrections:**

$$\langle \xi(x)\xi(x') \rangle = \left[2 + \underline{\tau_R \mathrm{D} \ln \frac{T\kappa}{\tau_R} - \tau_R \theta} \right] T\kappa \delta^{(4)}(x - x')$$

K. Murase, Ph.D. Thesis (The University of Tokyo), Sec. 4.4 (2015)

Otherwise, **Fluctuation theorem (FT)** from non-equilibrium statistical mechanics is broken in 2nd-order fluctuating hydrodynamics

T. Hirano, R. Kurita, K. Murase, arXiv:1809.04773

λ-dependence of EoS and transport coefficients

- e.g. Decomposition of energy density in equilibrium

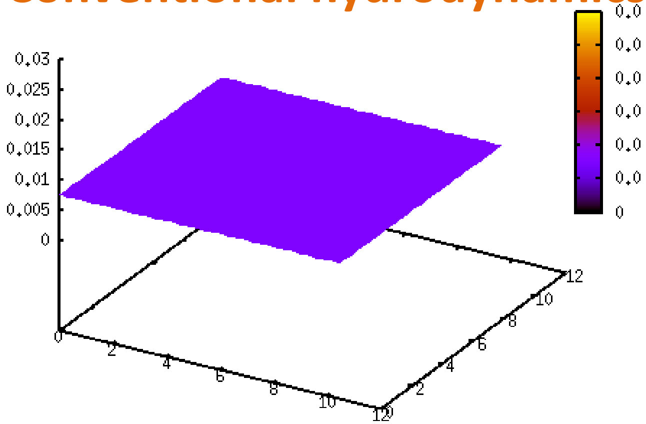
$$\langle T^{00} \rangle = e = \langle e_\lambda \rangle + \langle (e_\lambda + P_\lambda)\boldsymbol{u}_\lambda^2 + \pi_\lambda^{00} \rangle$$

e: internal energy in ordinary sense

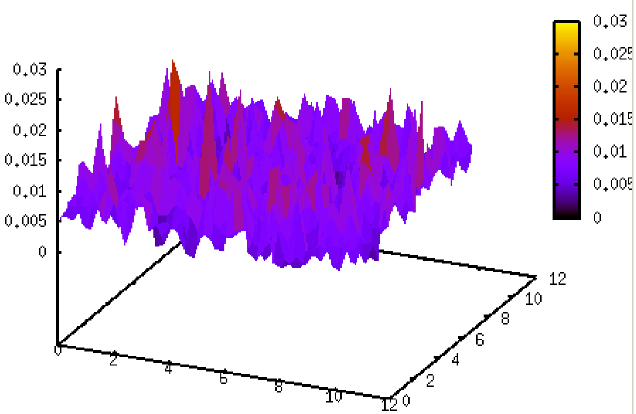
$\langle e_\lambda \rangle$: "internal" energy

$\langle (e_\lambda + P_\lambda)\boldsymbol{u}_\lambda^2 + \pi_\lambda^{00} \rangle$: "kinetic" energy

Global equilibrium in Conventional hydrodynamics



Global equilibrium in Fluctuating hydrodynamics



4. Renormalization of EoS/viscosity

λ-dependence of EoS and transport coefficients

- e.g. Decomposition of energy density in equilibrium

$$\langle T^{00}\rangle = e = \langle e_\lambda\rangle + \langle (e_\lambda+P_\lambda)\boldsymbol{u}_\lambda^2 + \pi_\lambda^{00}\rangle$$

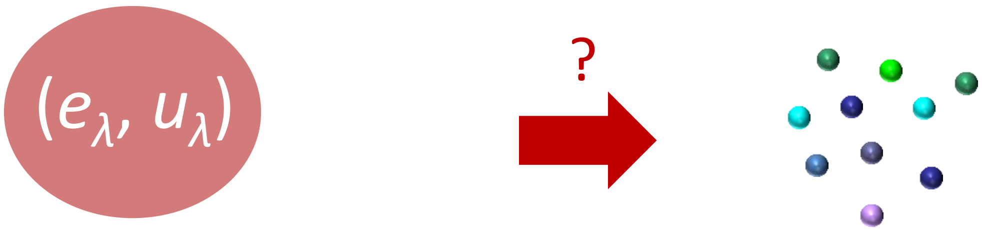
internal energy in ordinary sense — "internal" energy — "kinetic" energy

- **Every "macroscopic" quantities (e_λ, u_λ, etc.) are redefined for each cutoff λ.**
- **Macroscopic relations such as viscosity, EoS, etc. should be renormalized not to change the bulk properties (k→0, ω→0)**
- **Additional terms in hydrodynamic eqs: "long-time tail"**

P. Kovtun, et al., Phys. Rev. D68, 025007 (2003); P. Kovtun, et al., Phys. Rev. D84, 025006 (2011); P. Kovtun, J. Phys. A45, 473001 (2012); Y. Akamatsu, et al., Phys. Rev. C95, 014909 (2017); Y. Akamatsu, et al., Phys. Rev. C97, 024902 (2018)

5. Other renormalization

- **Cooper-Frye formula**:
  How to sample hadrons from $(e_\lambda, u_\lambda^\mu, \pi_\lambda^{\mu\nu}, \Pi_\lambda)$?



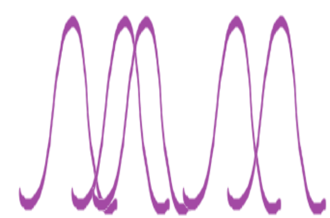
- **Initialization model (from partons, etc.)**:
  Width and shape of smearing kernel should match with those in fluctuating hydrodynamics?

Larger λ

larger internal energy e_λ
smaller initial flow u_λ

Renormalization?

Smaller λ



smaller internal energy e_λ
larger initial flow u_λ

SUMMARY

Summary

- ***Hydrodynamic fluctuations*** are thermal fluctuations of fluid fields whose power is determined by *FDR*
- ***Many interesting topics on dynamical modeling of hydrodynamic fluctuations***:
  - FDR corrections for causal hydro
  - Fluctuation-theorem
  - Renormalization of EoS/transport properties
  - Renormalization of initial condition
  - Renormalization of $f(p)$ in Cooper-Frye