Nuclear structure with exotic beams

Lecture 3:

Nuclear Reactions II

Transfer Reactions

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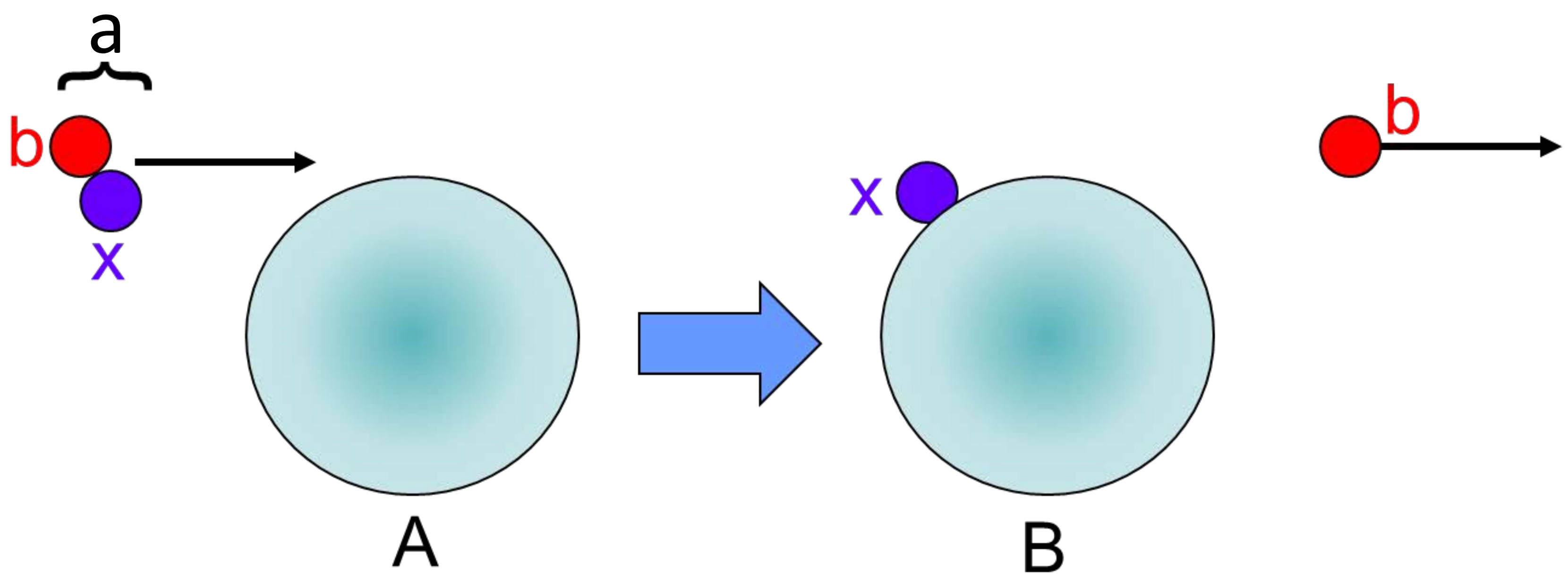
Re-arrangement reactions

- $a+A \rightarrow b+B$ or $A(a,b)B$
- Nuclei are transformed, nucleons are exchanged ($b \neq a$, $B \neq A$)
- We'll focus on simple processes – "Direct" reactions
- We need to use some of what we learned about elastic scattering.

Direct transfer reactions



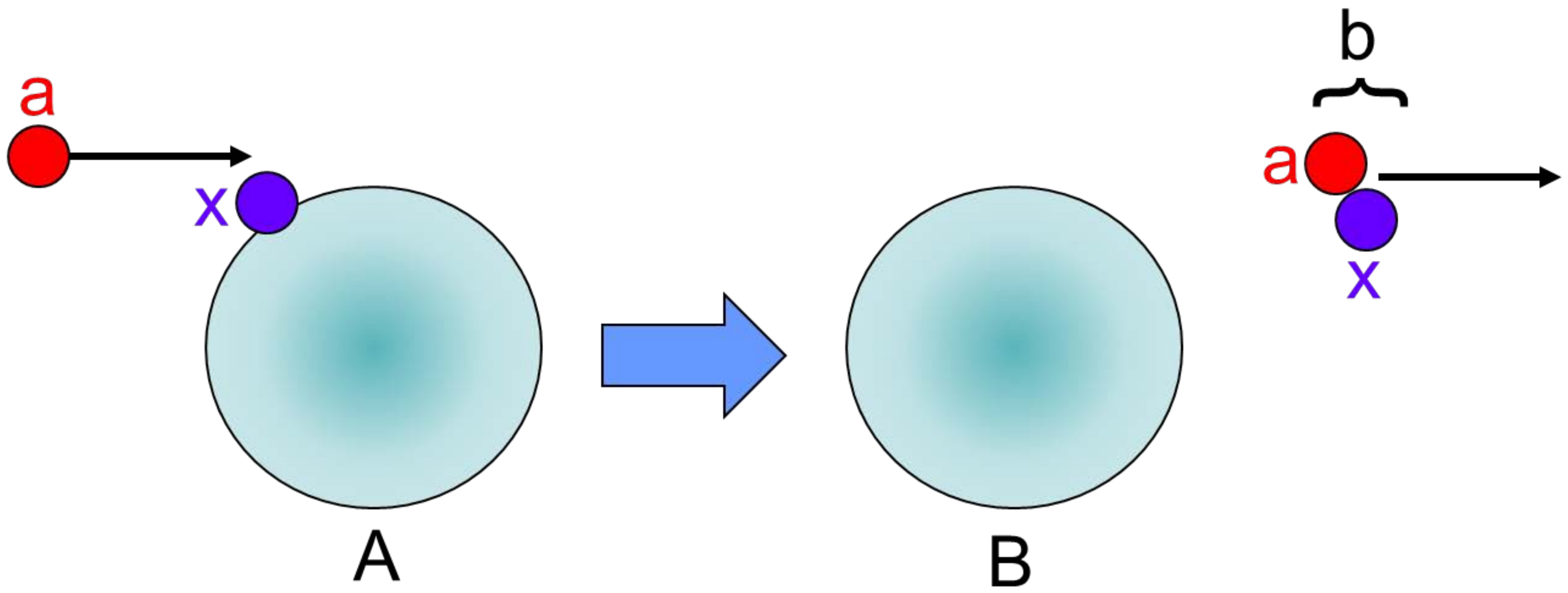
Adding nucleon(s) to A:

"x" is transferred from a to A, making B=A+x and b=a-x

Known as "Stripping"

x can be one or more nucleons

Direct transfer reactions



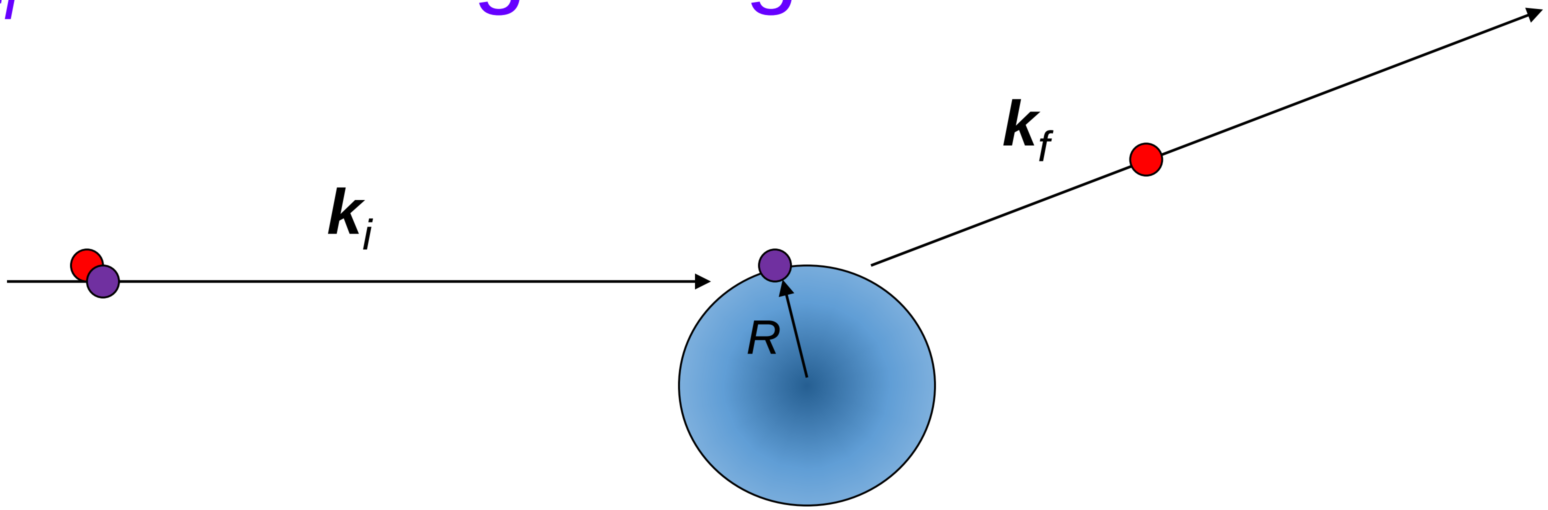
Removing nucleon(s) from A:
"x" is transferred from A to b, making B=A-x and b=a+x

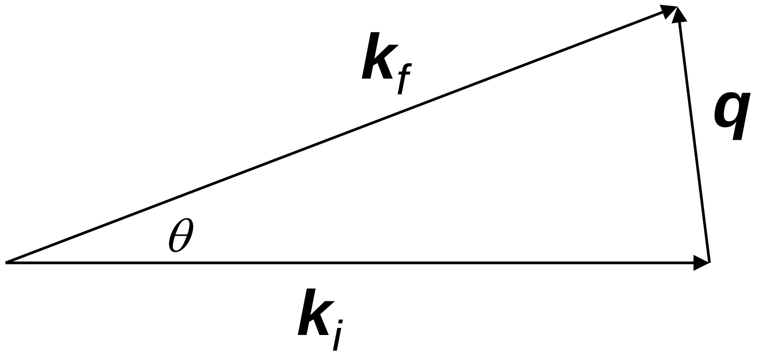
Known as "Pickup"
x can be one or more nucleons

Advantages to direct transfer

- It is Selective
  - Single-nucleon transfer preferentially populates simple states with strong “single-particle” character
  - Important for understanding the nature of single-particle levels, especially interesting now in the era of “modified shell structure” in exotic nuclei
  - Different reactions probe different amplitudes
- It is “Easy” to understand
  - Reaction mechanism is relatively simple – a single-step transition between two states
- The cross sections tend to be “large”
  - 1 to 10s of mb/sr for single particle stripping & pickup

Momentum Matching:
Larger $l_{transfer}$ means larger angle





$$q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos\theta$$

angular momentum of the transferred particle = $qR = l$, or $q = l/R$
This roughly determines the best angle for transfer:

$$\boxed{\theta_{\max} = \cos^{-1}\left(\frac{k_f^2 + k_i^2 - (l/R)^2}{2k_f k_i}\right)}$$

Early (*d,p*) theory and data from Phys. Rev. **80** (1950)

On Angular Distributions from (d, p) and (d, n) Nuclear Reactions

S. T. Butler*
Department of Mathematical Physics, University of Birmingham, Birmingham, England
October 30, 1950

$$\frac{d\sigma}{d\Omega} \propto \left| \int_{R_B}^{\infty} j_L(qr) u_{nl}(r) r dr \right|^2 \approx \left| j_L(qR_B) \right|^2$$

R_B is the "Butler radius"

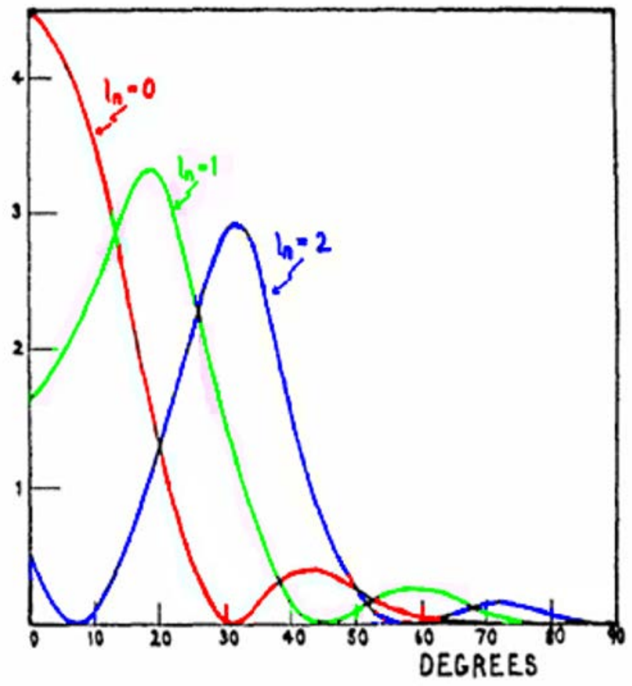


FIG. 1. Theoretical angular distributions for (d, p) and (d, n) reactions for different angular momentum transfers to the initial nucleus.

Angular Distributions of Protons from the Reaction $O^{16}(d, p)O^{17}$

Hannah B. Burrows
University of Liverpool, Liverpool, England
W. M. Gibson
University of Bristol, Bristol, England
and
J. Rotblat
Medical College of St. Bartholomew's Hospital, London, England
October 30, 1950

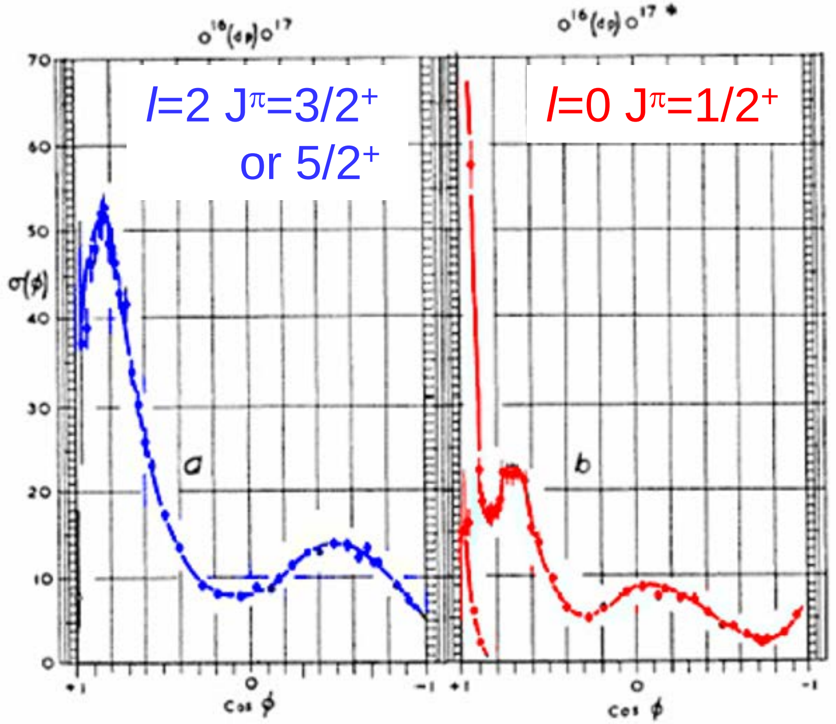


FIG. 1. $O^{16}(d, p)O^{17}$ angular distributions in the center-of-mass (c.m.) system: ϕ = c.m. angle, $\sigma(\phi)$ = c.m. differential cross section in arbitrary units. Curve *a* is for formation of O^{17} in the ground state, and curve *b* is for the 0.88-Mev excited state.

The shape tells you *l* – what about the rest?

I have calculated angular distributions resulting from such a stripping process by equating, at the nuclear surface, the exact wave function for a particle outside the nucleus to the interior wave function. After some simplification the resulting boundary equations can be solved in such a way that unknown properties of the nuclear wave functions affect the important parts of the distributions merely as a constant multiplying factor. The re-

(Butler, 1950)

...Known today as the "spectroscopic factor" *S*.

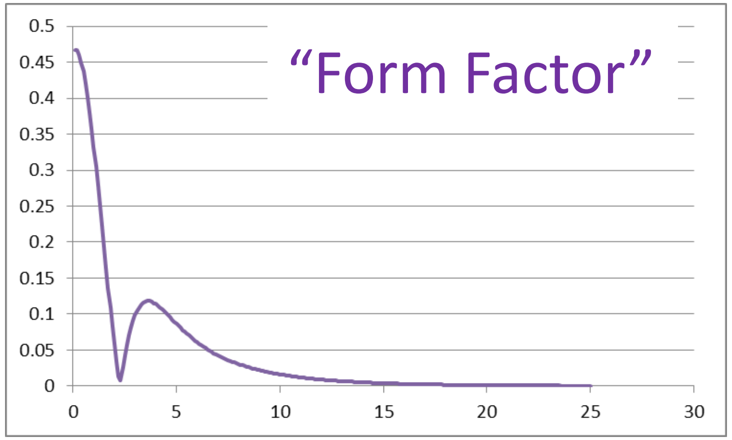
This contains all the interesting nuclear structure information

What does it mean and How do we get it?

How do we "measure" *S* ??

- *S* is *not* an experimental observable, so you *cannot* "measure" it.
- Does that mean *S* is meaningless, as some might claim?
- I think no – meaningful values of *S* can be deduced from comparisons between measured cross sections and the predictions of nuclear reaction models. (Typical is the Distorted Wave Born Approximation or DWBA).
- But then – *S* is *model dependent*, so *caveat emptor*.
- We can try to deduce *absolute* or *relative* values of *S*.

One-page summary of the DWBA for transfer: (*d,p*)

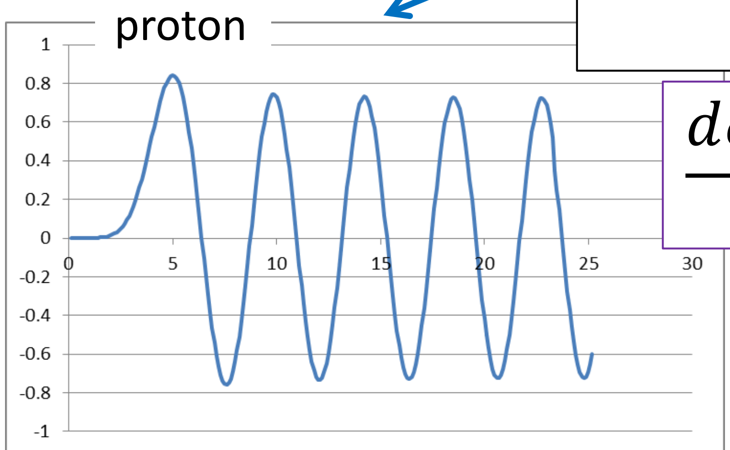


(or "single-particle overlap" for *B=A+x*)

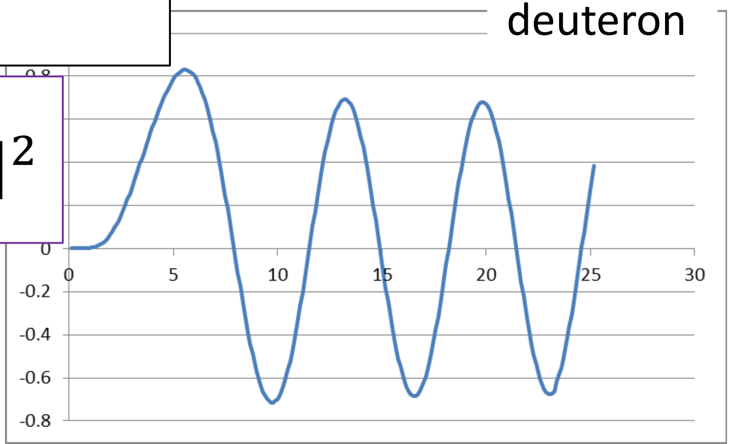
Matrix element with nuclear structure

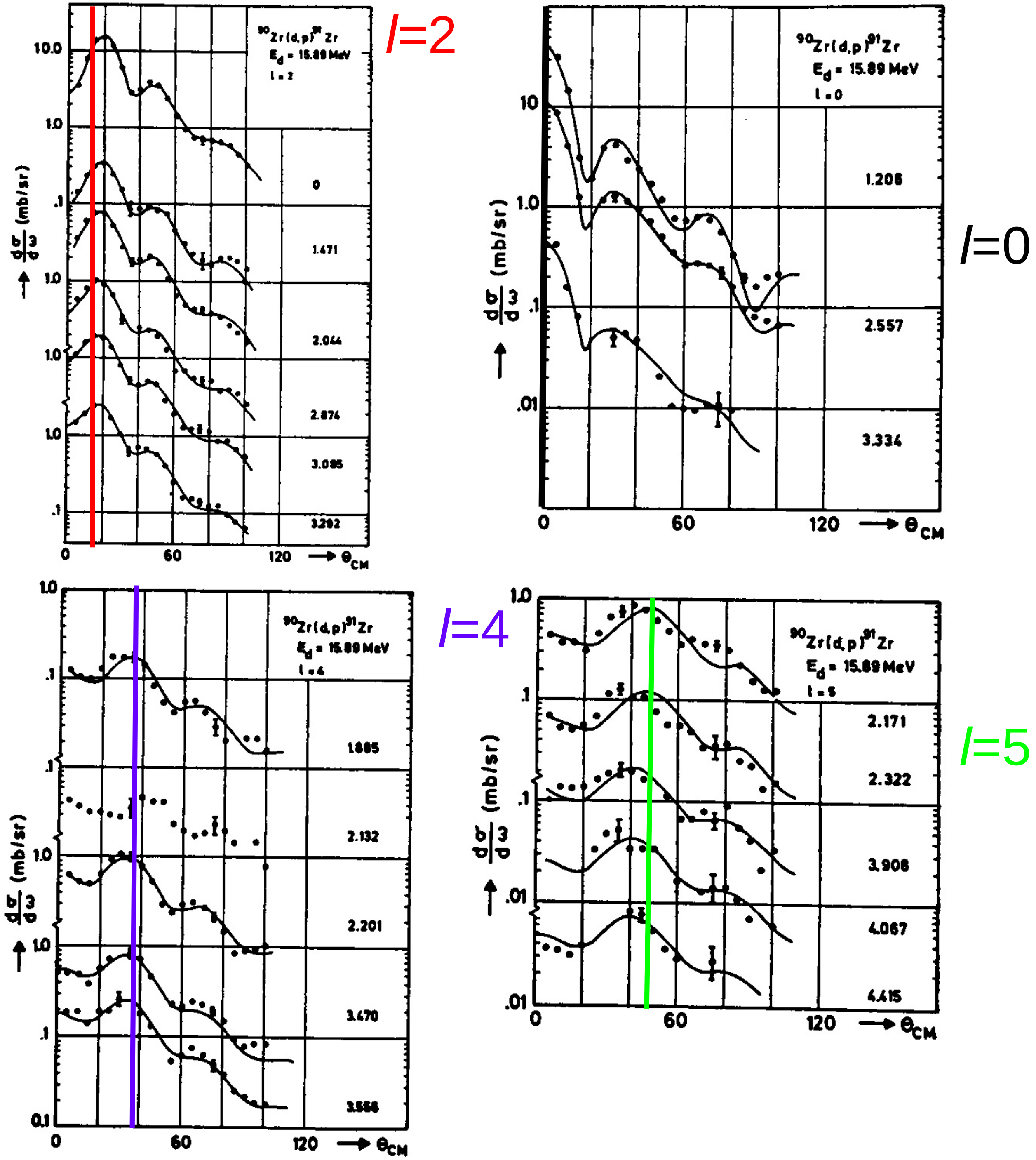
$$T_{DWBA} = J \int d^3r_b \int d^3r_a \, \chi^-(\overrightarrow{k_f},\overrightarrow{r_b}) < bB|V|aA > \chi^+(\overrightarrow{k_i},\overrightarrow{r_a})$$

Distorted waves from OM – many different incident and outgoing angular momenta



$$\frac{d\sigma_{DWBA}}{d\Omega} = |T_{DWBA}|^2$$





Neutron stripping: $^{90}Zr(d,p)^{91}Zr$ (Q=4.97 MeV)

(*d*,*p*) is the prototypical direct-transfer reaction. ^{2}H is simple and loosely bound.

Colored lines indicate θ_{max} estimated from matching calculation

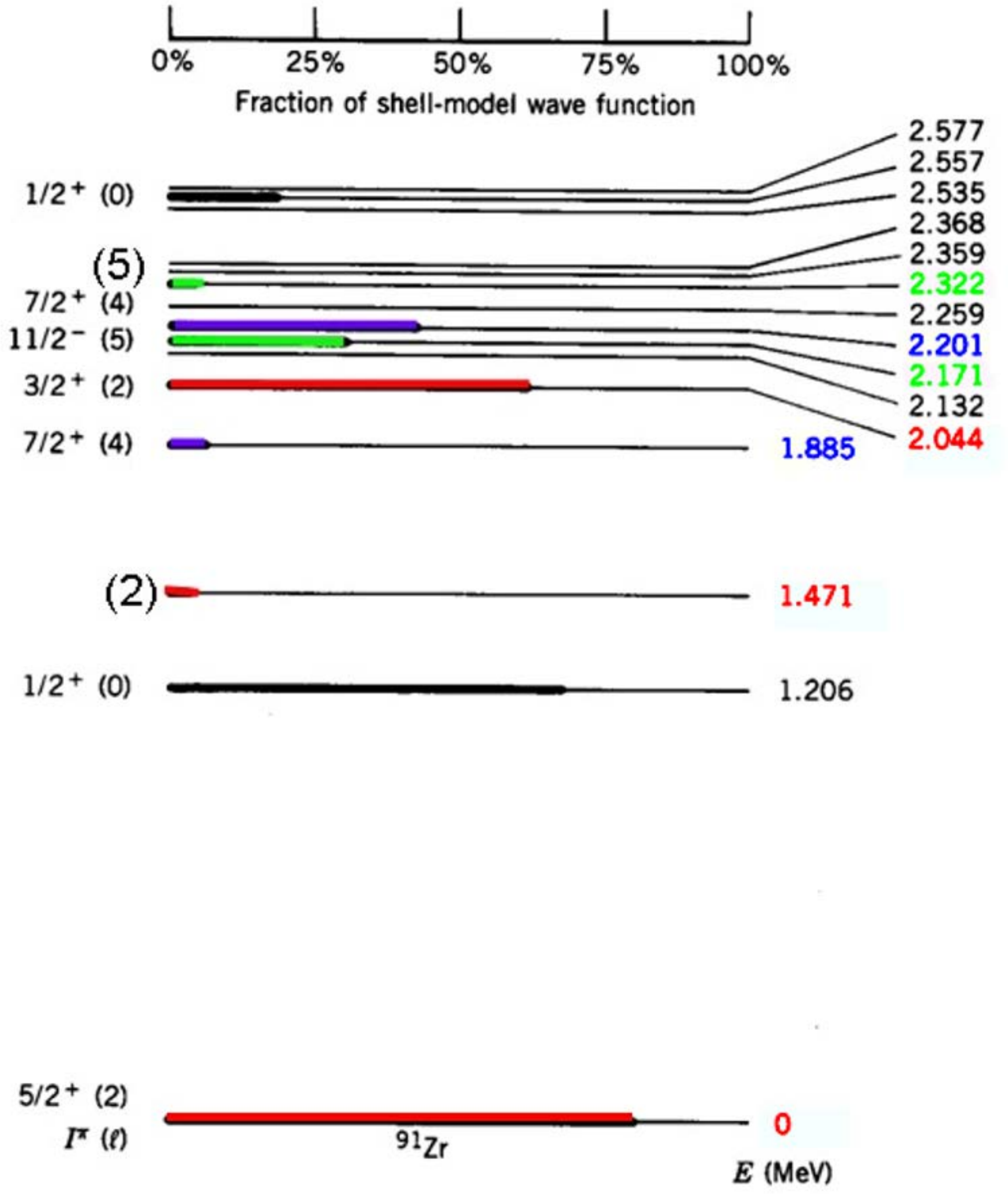
The normalization between curves and data is interpreted as the Spectroscopic Factor

H. P. Block et al, NPA 273 142 (1976).

Interpretation of *S*

- *S* reflects the overlap between the initial and final states; $d\sigma/d\Omega \propto S$
- *S* "measures" orbital vacancies (# of holes) for stripping, or orbital occupancies (# of particles) for pickup.
- McFarlane and French (Rev. Mod. Phys. **32**, 1960):
  - $\#Holes = \Sigma C^2 S \times \frac{2J_F+1}{2J_i+1}$ (adding or "stripping")
  - $\#Particles = \Sigma C^2 S_i$ (removing or "pickup")
  - Sum is over all states that could have a particle in the orbital of interest
- Connection to resonances: $S_i = \gamma^2{}_i/\gamma^2{}_{SP}$ ("Schiffer's anzatz")

Spectroscopic factors from $^{90}Zr(d,p)^{91}Zr$



Neutron orbitals of interest:

1g7/2 : l=4
2d5/2 : l=2
2d3/2 : l=2
3s1/2 : l=0
1h11/2: l=5

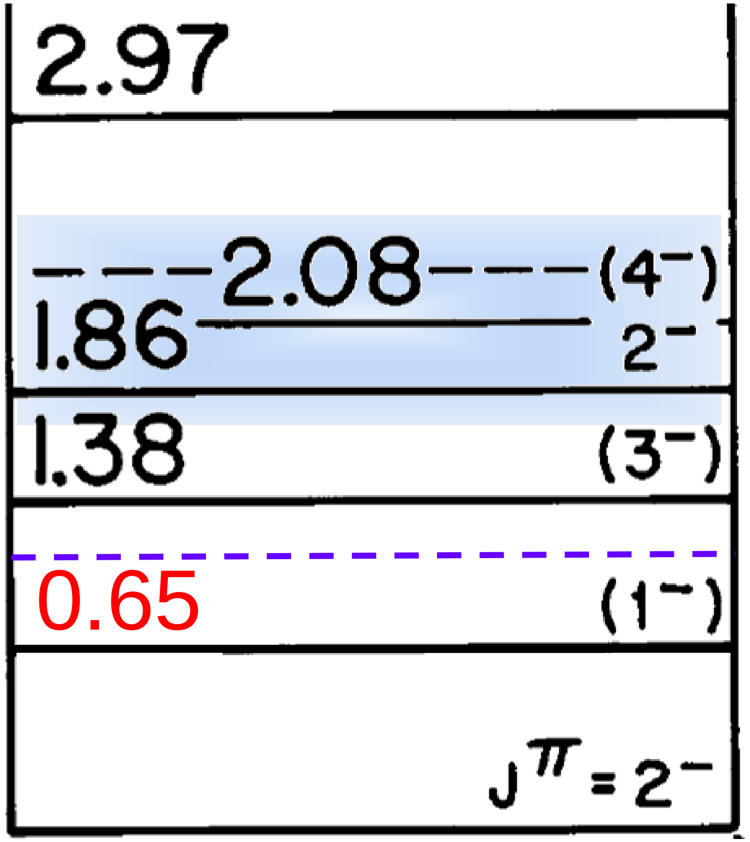
No single state exhausts the total shell-model strength!

We can use this to deduce the order of single-particle orbitals

Example: ^{14}B (Lightest particle-stable N=9 nucleus)

$n + {}^{13}B_{g.s.}(3/2^-)$

We are interested in single-neutron *sd* states (recall first lecture and Talmi and Unna)



2^-_2 state is broad (Γ~1 MeV)

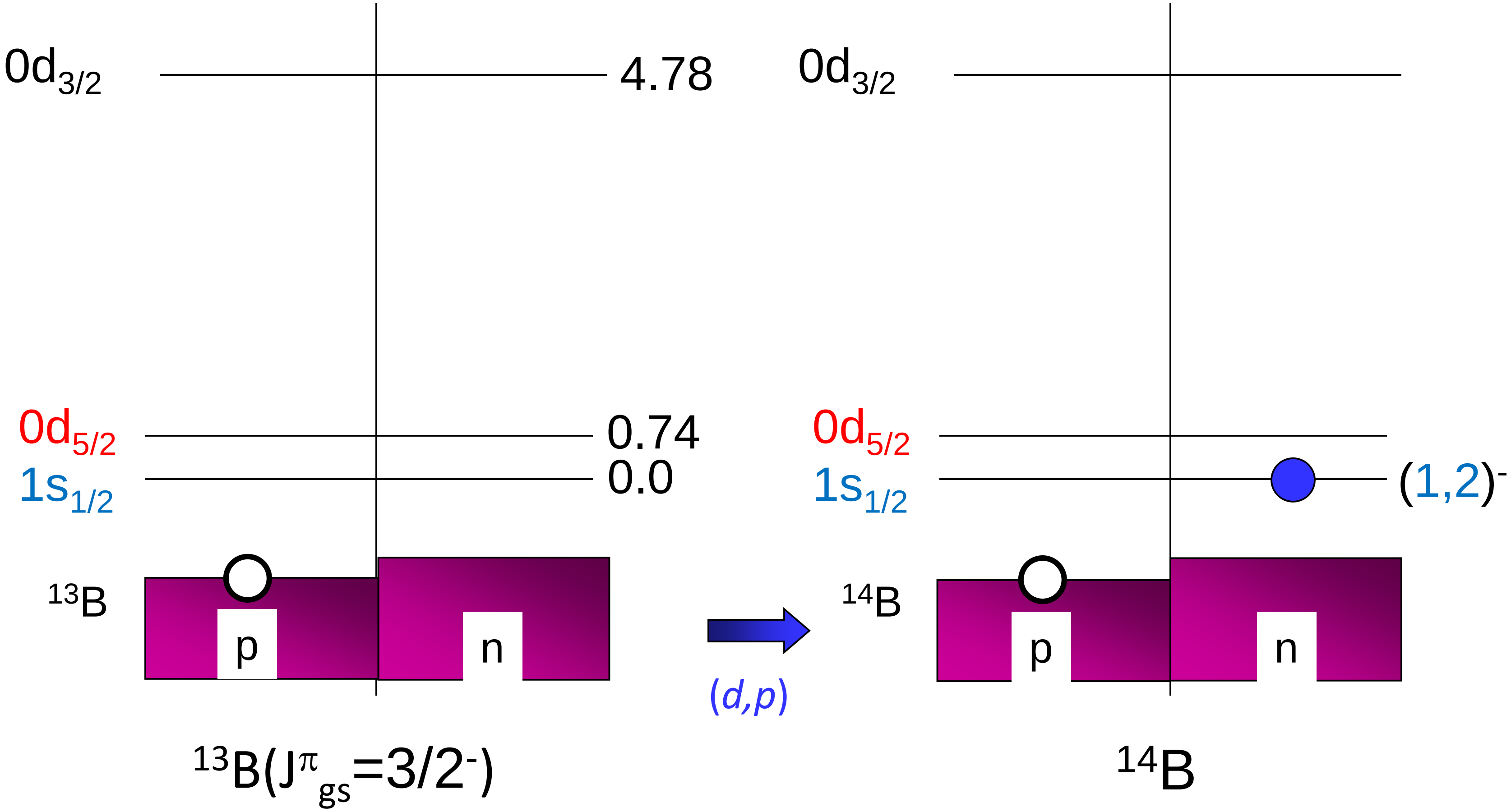
S_n=0.969 MeV

^{14}B

From most recent TUNL A=14 compilation (1991)

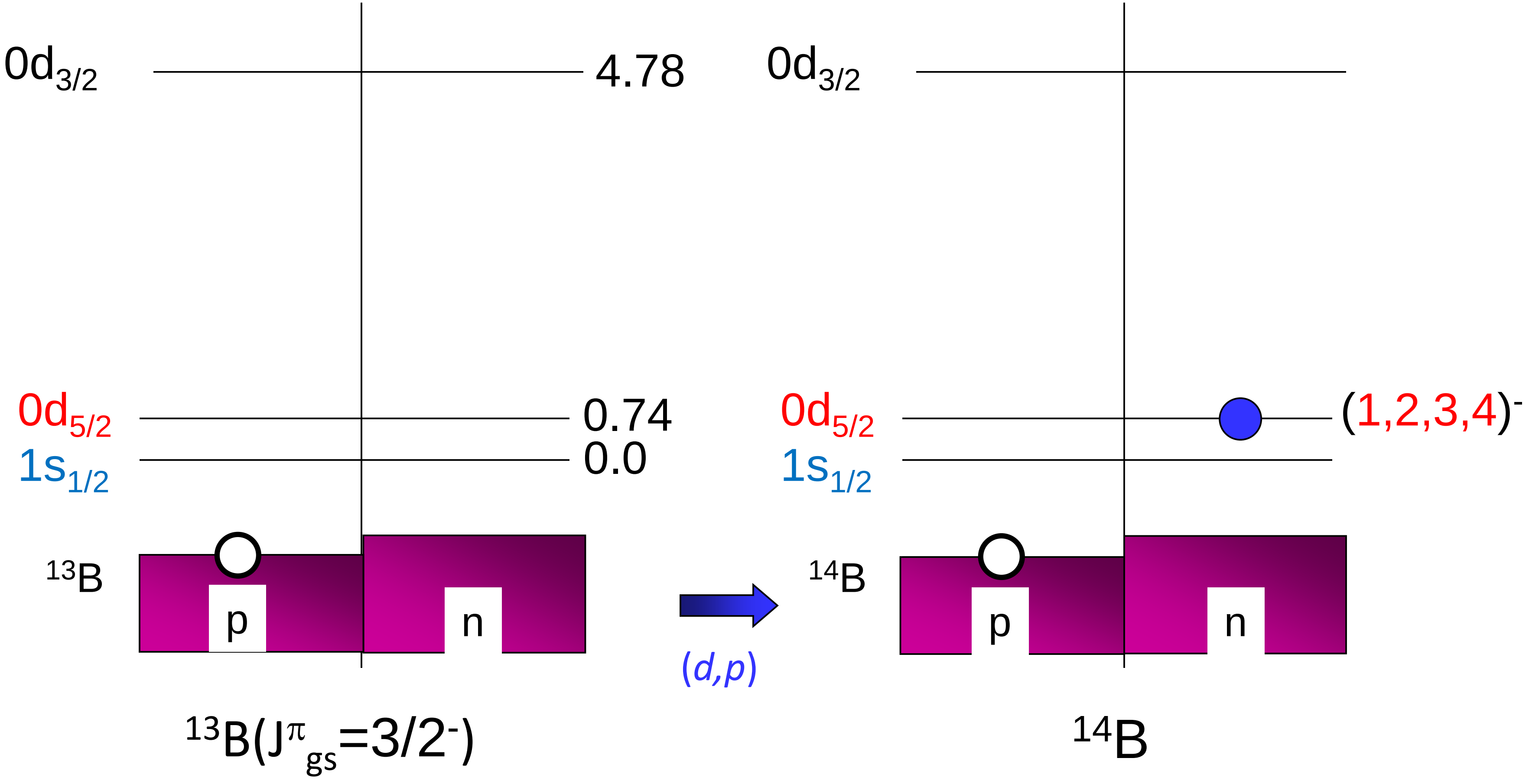
Most information is from $^{14}C(^7Li,^7Be)^{14}B$ and analogies to the ^{12}B spectrum.
More recent ^{14}Be β-decay work suggests positive-parity levels not shown here

Simple picture for $^{13}B(d,p)^{14}B$



(*d*,*p*) populates single-neutron states in ^{14}B
(schematic picture)

Simple picture for $^{13}B(d,p)^{14}B$



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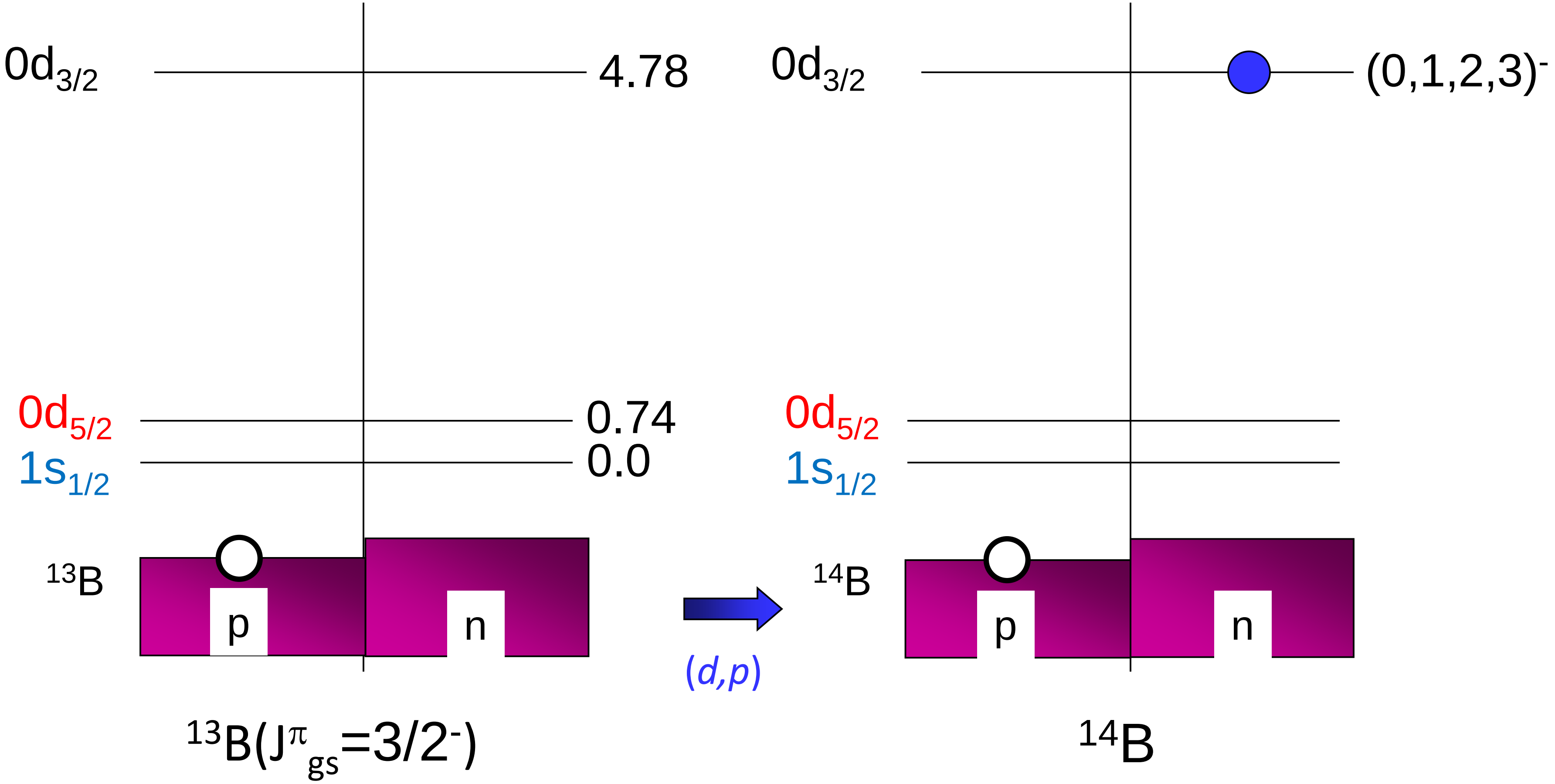
Simple picture for $^{13}B(d,p)^{14}B$



(d,p) populates single-neutron states in ^{14}B
(schematic picture)

$\nu(sd)$ states in ^{14}B with (d,p) (ignore the $0d_{3/2}$ orbital)

$$\psi(2_1^-) = \alpha_2 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \beta_2 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

$$\psi(2_2^-) = -\beta_2 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \alpha_2 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

2-

$$\psi(1_1^-) = \alpha_1 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \beta_1 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

$$\psi(1_2^-) = -\beta_1 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \alpha_1 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

1-

$$\psi(3_1^-) = \alpha_3 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

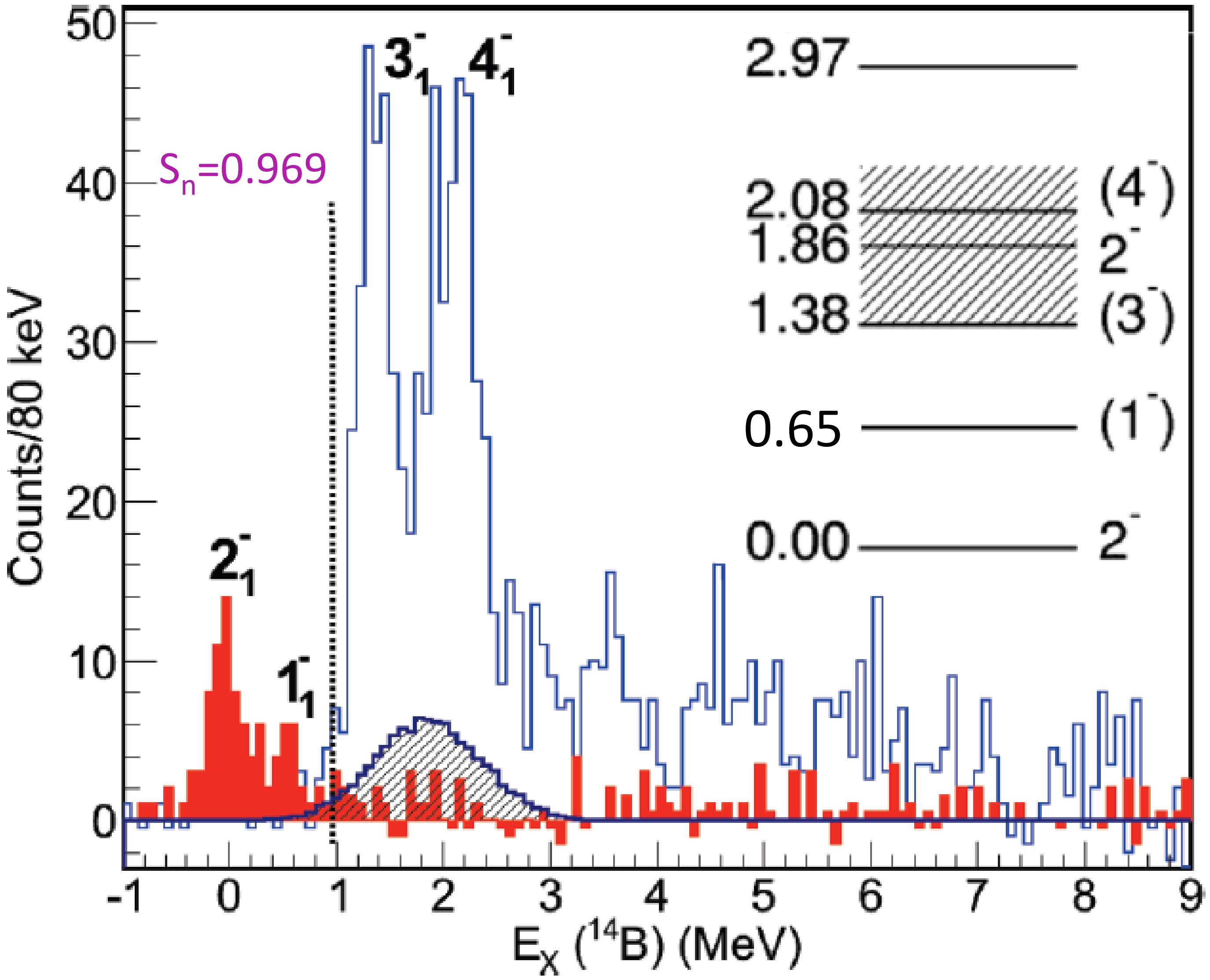
3-

$$\psi(4_1^-) = \alpha_4 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

4-

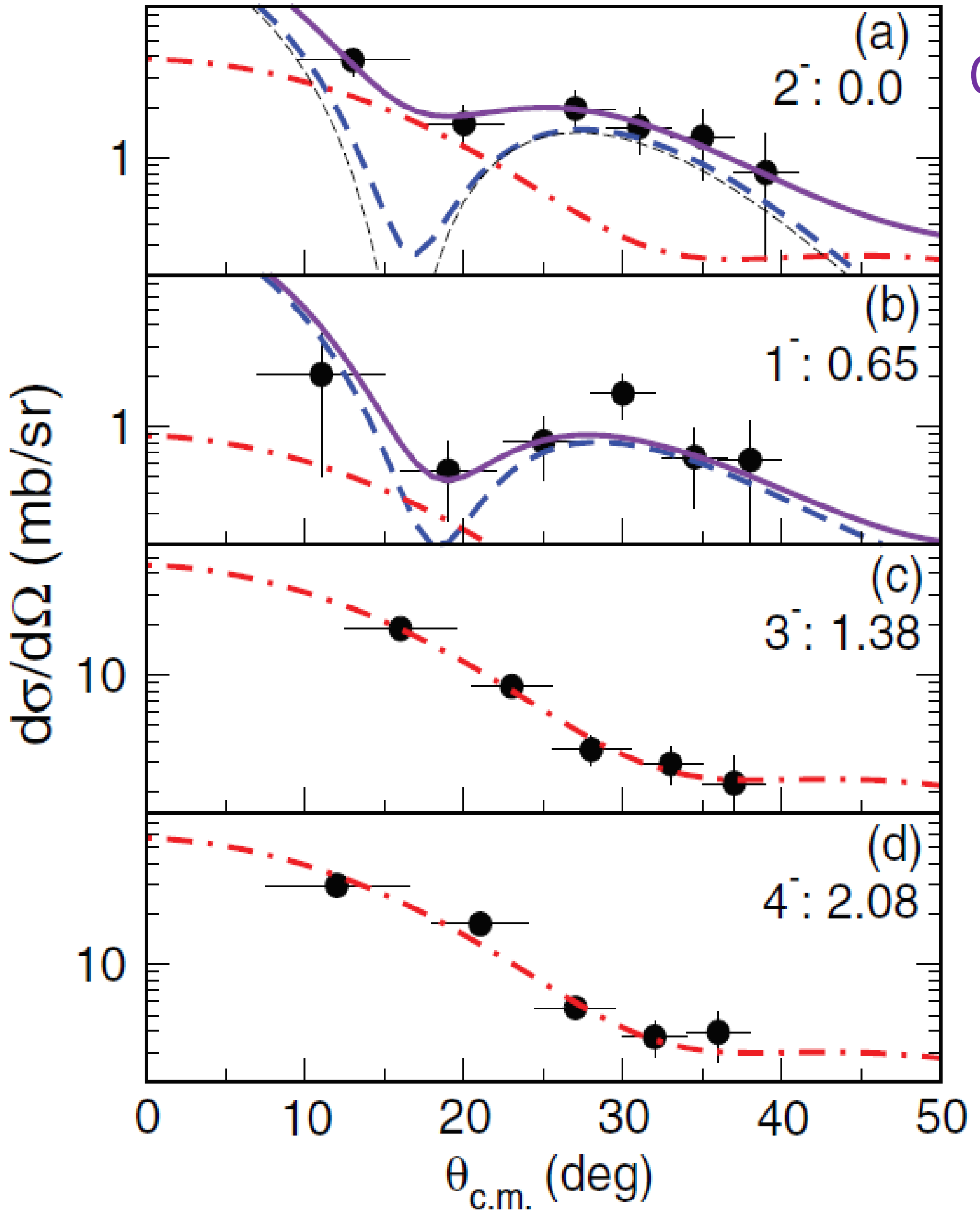
(d,p) spectroscopic factors give us the α's and the β's

^{14}B Excitation-energy spectrum from HELIOS



Bedoor et al., PRC **88** 011304 (2013)

$^{13}B(d,p)^{14}B$ angular distributions



Blue: L=0

Red: L=2

Violet: L=0 + L=2

$2^-(0.00)$: $S_0=.71$ $S_2=.17$

$1^-(0.65)$: $S_0=0.96$ $S_2=.06$

$3^-(1.38)$: $S_2=1.00$ (fixed)

$4^-(2.08)$: $S_2=1.00$

OMPs fit 30 MeV $d+^{12}C$, $p+^{12,13}C$ elastic scattering

Bedoor et al., PRC **88** 011304 (2013)

Sum rules with simple, pure states:

All S = 1.0 and $(2J_I+1=4)$

$$\#holes = 1 \times \frac{5}{4} + 1 \times \frac{3}{4} = 2\ (s_{1/2})$$

$$\#holes = 1 \times \frac{5}{4} + 1 \times \frac{3}{4} + 1 \times \frac{7}{4} + 1 \times \frac{9}{4} = 6\ (d_{5/2})$$

$$J_F = \quad 2 \qquad 1 \qquad 3 \qquad 4$$

Sum rules with observed states:

Measured values of S ($2J_I$+1=4)

$$\#holes = \boxed{0.71} \times \frac{5}{4} + \boxed{0.96} \times \frac{3}{4} = 1.6\ (s_{1/2})$$

$$\#holes = \boxed{0.17} \times \frac{5}{4} + \boxed{0.06} \times \frac{3}{4} + 1 \times \frac{7}{4} + 1 \times \frac{9}{4} = 4.3\ (d_{5/2})$$

$$J_F = \quad 2 \qquad\qquad 1 \qquad\qquad 3 \qquad\qquad 4$$

We're missing two states!

Recovering the unobserved strength

$$\psi(2_1^-) = \alpha_2 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \beta_2 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

$$\psi(2_2^-) = -\beta_2 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \alpha_2 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

2^-

$$\psi(1_1^-) = \alpha_1 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \beta_1 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

$$\psi(1_2^-) = -\beta_1 \nu(1s_{1/2})\pi(0p_{3/2})^{-1} + \alpha_1 \nu(0d_{5/2})\pi(0p_{3/2})^{-1}$$

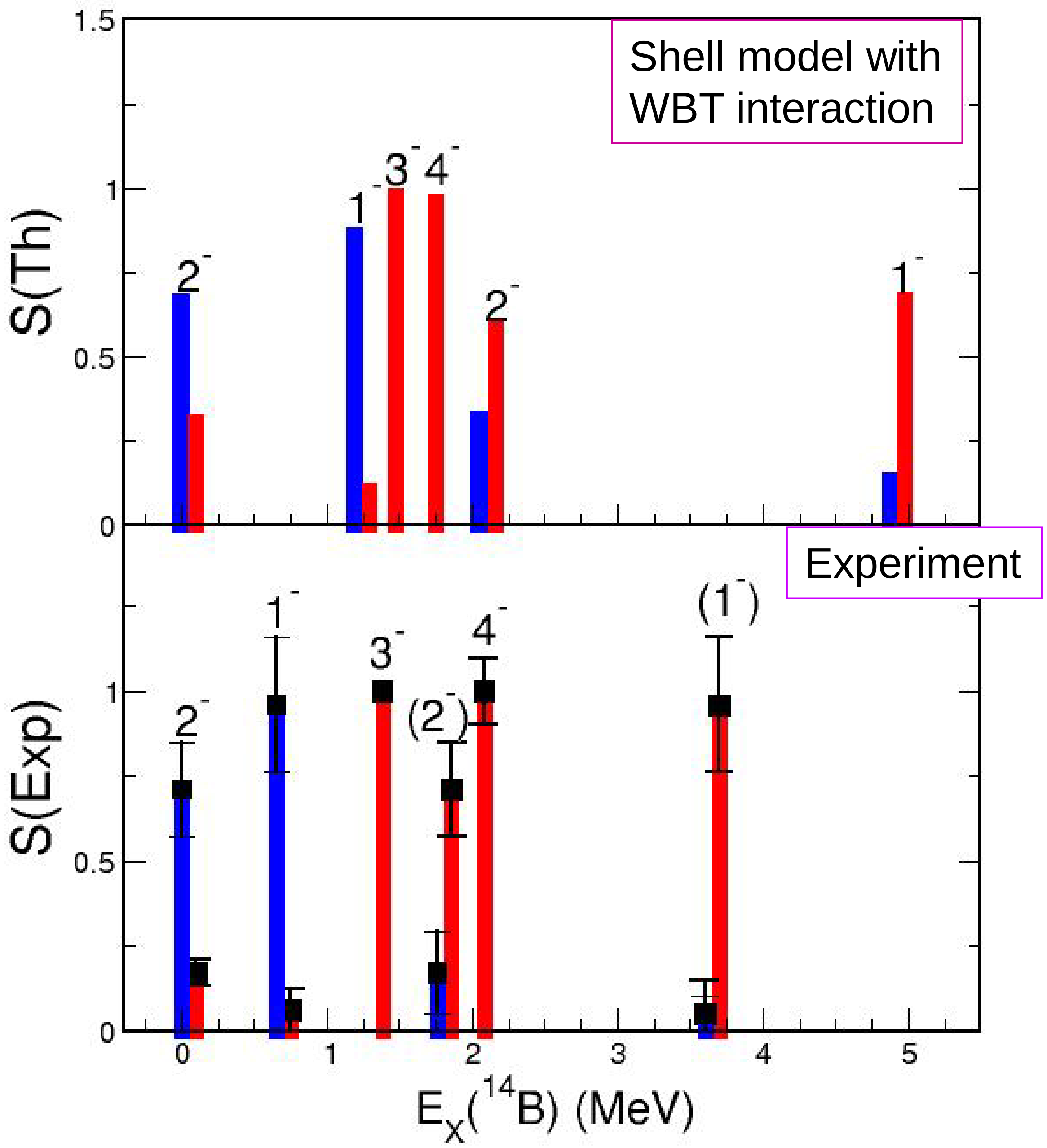
1^-

Assume $\psi(2^-_2)$ and $\psi(1^-_2)$ are the orthogonal partners of $\psi(2^-_1)$ and $\psi(1^-_1)$. We already know α_2, β_2 and α_1, β_1, so we can then guess the spectroscopic factors for $\psi(2^-_2)$ and $\psi(1^-_2)$.

Then:

Experimentally, $\Sigma S(2J_f+1)/4 = 5.9$ ($L=2$) and 1.9 ($L=0$),

Close to the sum-rule values of 6.0 and 2.0.



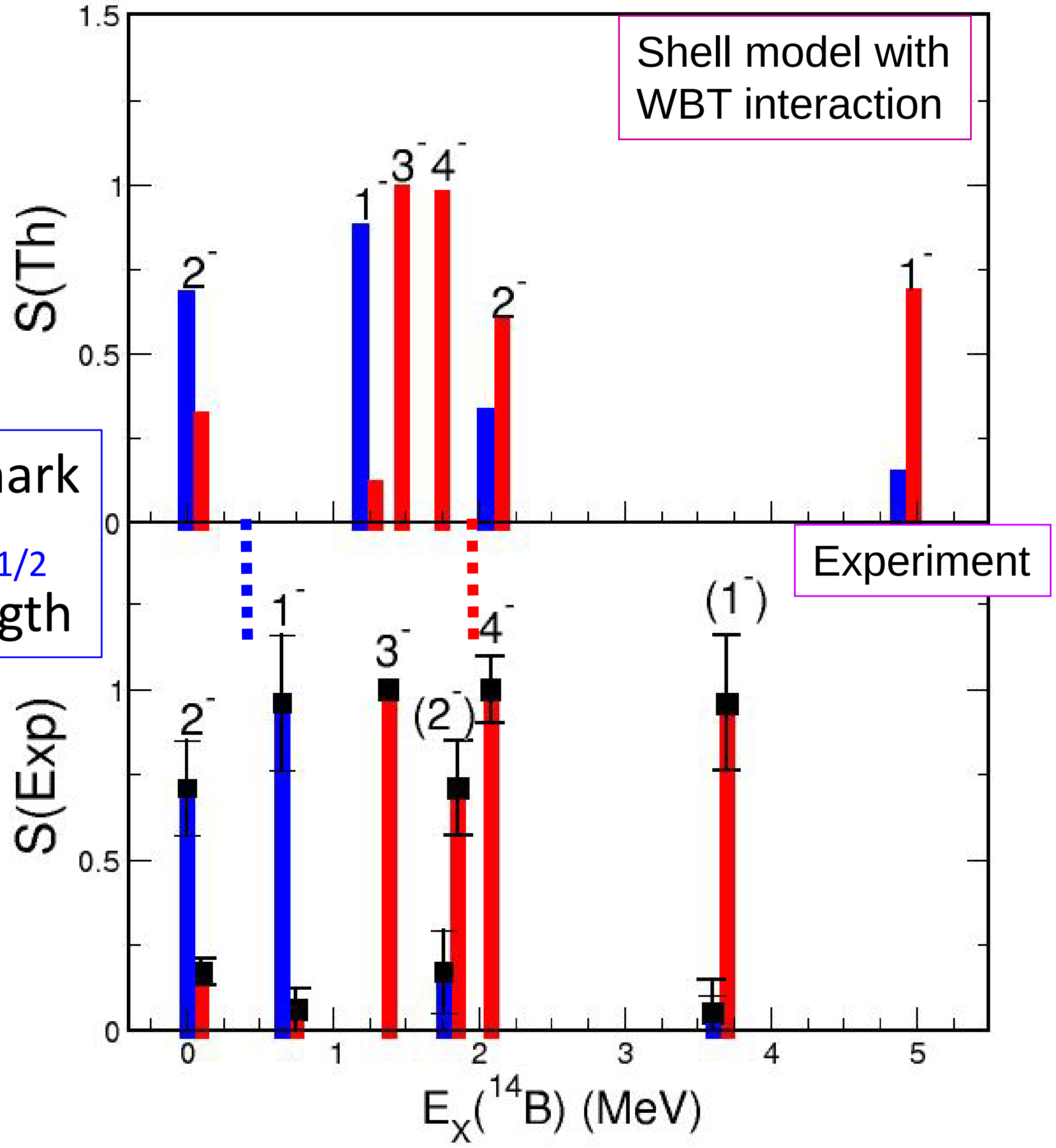
$^{13}B(d,p)^{14}B$ Spectroscopic factors

Excitation energies and relative spectroscopic factors from the shell model

Blue: L=0, $1s_{1/2}$
Red: L=2, $0d_{5/2}$

$^{13}B(d,p)^{14}B$ Spectroscopic factors

Dashed lines mark centroids of $1s_{1/2}$ and $0d_{5/2}$ strength

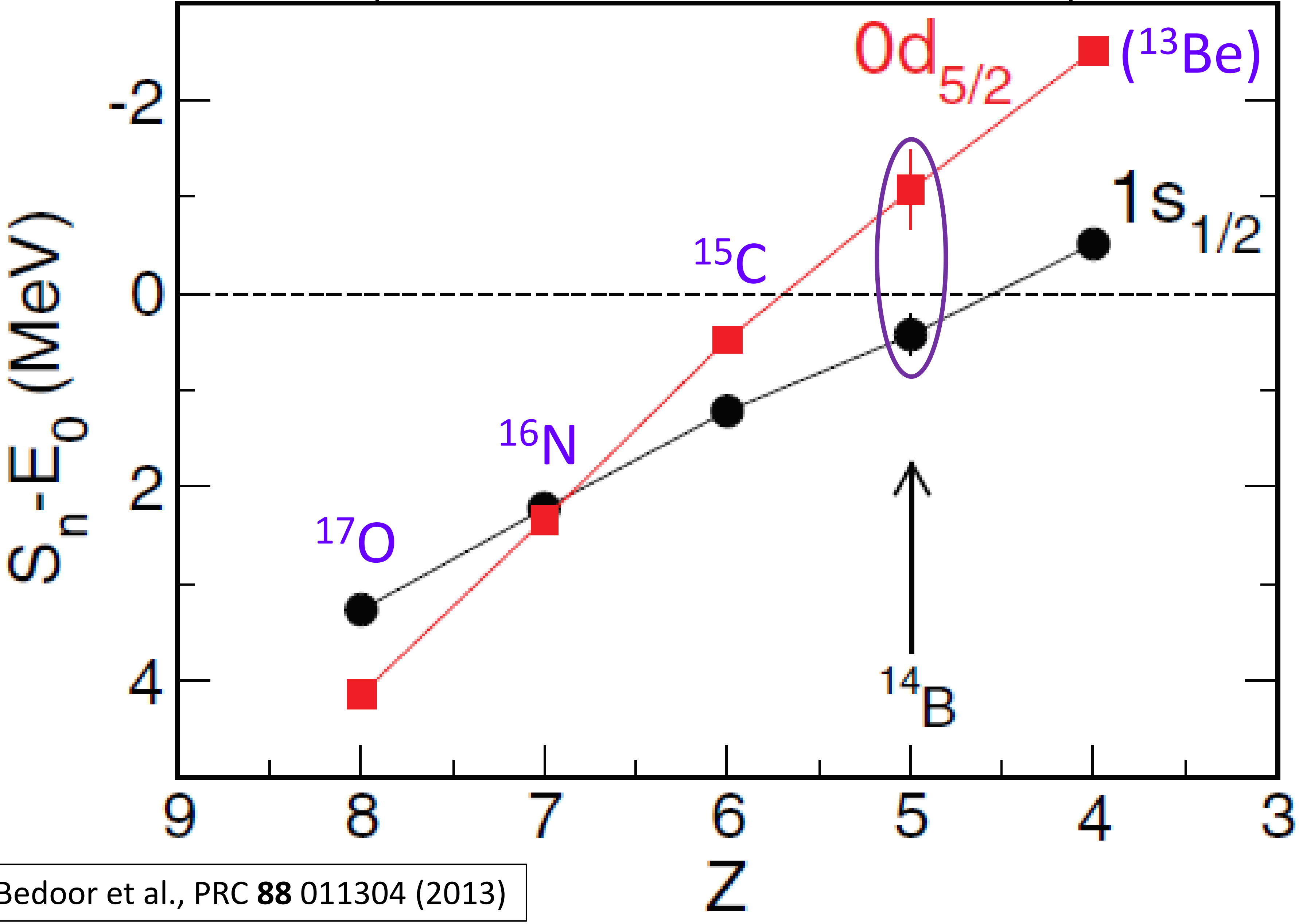


Excitation energies and relative spectroscopic factors from the shell model

Blue: L=0, $1s_{1/2}$

Red: L=2, $0d_{5/2}$

Evolution of E($1s_{1/2}$) and E($0d_{5/2}$) for N=9 with Z: (Remember Talmi and Unna)



Bedoor et al., PRC **88** 011304 (2013)

Conclusions

- Scattering and transfer reactions can tell us a great deal about nuclear structure.
- We have to combine information from many different places to gain understanding.
- We must not forget that much of what we think we "know" we actually don't – we surmise in the context of models, so we should be careful about our claims.
- Next time: techniques.