Theoretical study of the Isovector Monopole Resonance

Xavier Roca-Maza

Single Particle and collective motion from nuclear many-body correlations (PCM2025)

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WARNING:

THE MATERIAL PRESENTED HERE IS PART OF (MY) <u>WORK IN PROGRESS</u> ON THIS TOPIC

⇒**Theoretical results** obtained with the **charge-exchange** version of the **HFBCS-QRPA** code based on **Skyrme Energy Density Functional** (see **arXiv:2102.06562**)

⇒One of Skyrmes employed: **SAMi = Skyrme Aizu Milano**

Motivation

Direct **experimental evidence** for the **IVGMR:**

→ **($\pi^{\pm}, \pi^{0}$)** reaction [Errell 1984-1986, Irom 1986]

→ **(^{7}Li,^{7}Be)** reaction [Nakayama 1999]

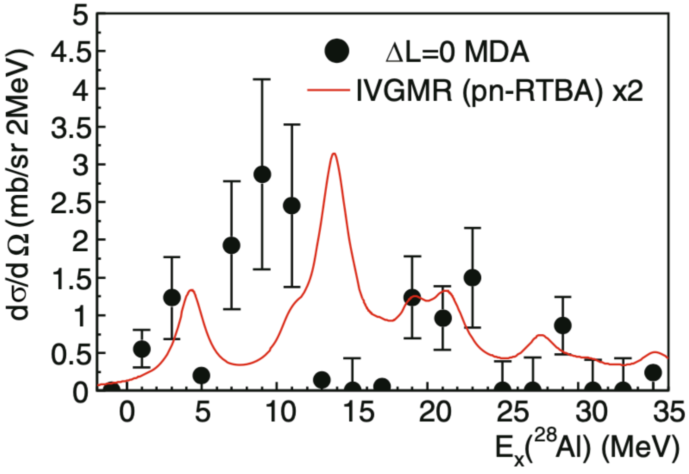
difficulties on the interpretation of the data (background).

Recent efforts

→**(^{10}C,^{10}B*)** reaction [Sasamoto 2012]

→**(^{10}Be,^{10}B*)** reaction [Scott 2017]: **IVGMR identified (MDA) in ^{28}Al ($\Delta T_z = +1$)**

^{28}**Al → strength is fragmented**, not easy to identify the resonance.



Heavier nuclei (connection collective mode properties with the nuclear Equation of State) ⇒ **more intense ^{10}Be** or **^{10}C beams** [1]**.**

Important also for **ground-state isospin-mixing** $\langle T | T+1 \rangle \neq 0$ in **heavy N=Z** nuclei, like ^{100}Sn [Hamamoto and Sagawa 1993; Colò et al. 1995]

[1] Zegers, R.G.T. (2023). Excitation of Isovector Giant Resonances Through Charge-Exchange Reactions. In: Tanihata, I., Toki, H., Kajino, T. (eds) Handbook of Nuclear Physics

Charge-exchange Isovector Monopole (IVM):

Excitation operator [IVM as Isobaric Analog State (IAS) "overtone"]

The **excitation operator** used in the **literature** [1]:

$$\mathcal{O}_{\pm} = \sum_{i=1}^{A} r_i^2 Y_{00}(\Omega_i) t_{\pm}(i)$$

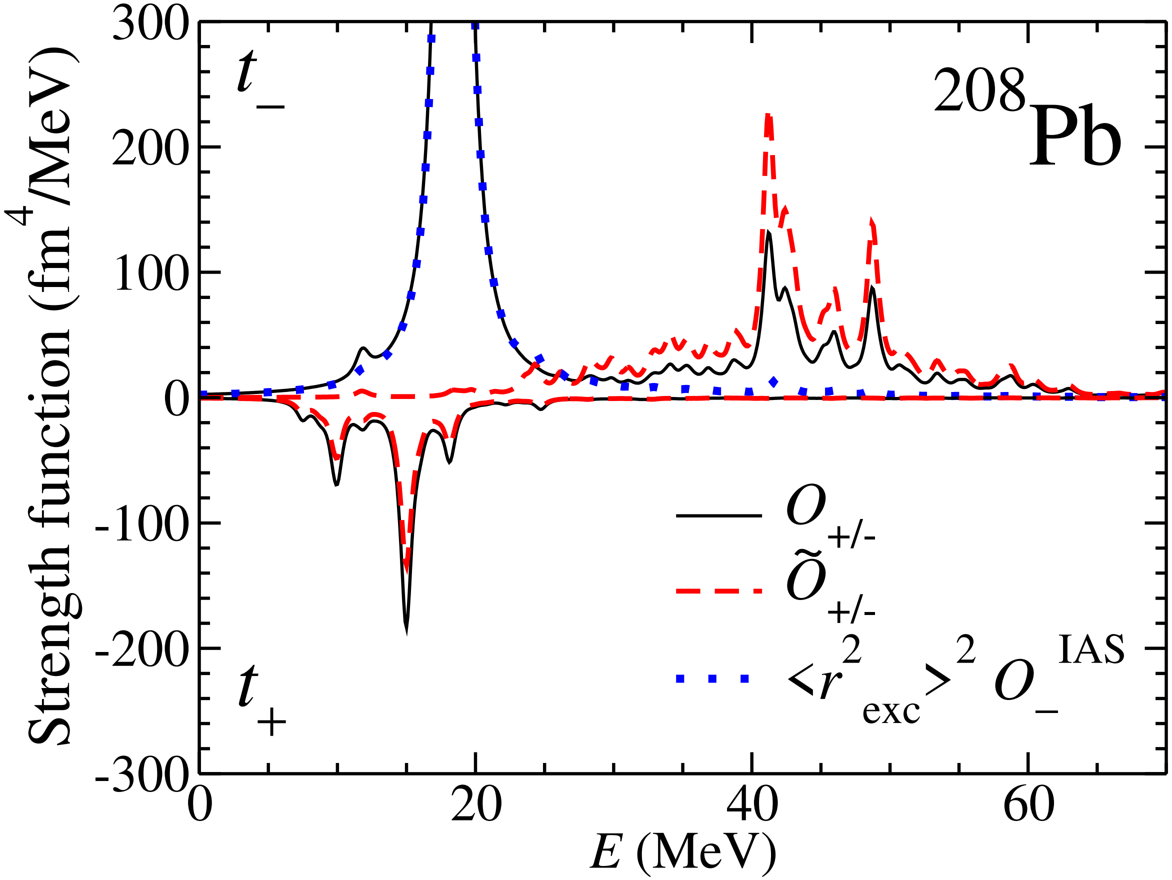
gets **large contribution** from **IAS**

$$\mathcal{O}_{-}^{\mathrm{IAS}} = \sum_{i=1}^{A} Y_{00}(\Omega_i) t_{-}(i)$$

Since **IAS (t_-)** in $N > Z$ **exhausts** most of the **NEWSR** $m_-(0) - m_+(0) = N - Z$

The IAS **transition density**

$$\delta\rho_{\mathrm{IAS}} \propto \rho_{\mathrm{exc}} \Rightarrow \int d\mathbf{r} \delta\rho_{\mathrm{IAS}} \tilde{\mathcal{O}}_{\pm} = 0$$



$$\tilde{\mathcal{O}}_{\pm} \equiv \sum_{i=1}^{A} \left(r_i^2 - \langle r^2_{\mathrm{exc}} \rangle \right) Y_{00}(\Omega_i) t_{\pm}(i)$$

New operator subtract the IAS contributions

[1] N. Auerbach and A. Klein, "A microscopic theory of giant electric isovector resonances," Nuclear Physics A **395**, 77–118 (1983).

$$\langle r^2_{\mathrm{exc}} \rangle \equiv \frac{1}{N - Z} \int d\mathbf{r} r^2 \rho_{\mathrm{exc}}$$

Charge-exchange Isovector Monopole:

Sum rules

Non energy weighted sum rule

$$m_-(0) - m_+(0) = \langle 0 | [\tilde{\mathcal{O}}_+, \tilde{\mathcal{O}}_-] | 0 \rangle$$

$$= \frac{1}{4\pi}\left(N\langle r_n^4\rangle - Z\langle r_p^4\rangle\right) + \frac{N-Z}{4\pi}\langle r_{\rm exc}^2\rangle^2\left(1 - 2\frac{N\langle r_n^2\rangle - Z\langle r_p^2\rangle}{(N-Z)\langle r_{\rm exc}^2\rangle}\right)$$

with $\mathcal{O}_\pm$: dominated by IAS

$m_-(0)$ is largely reduced by using the new operator: **about 60-70%**; while the $m_+(0)$ is only moderately reduced: **about 10-30%** (tested in ^{48}Ca, ^{90}Zr, ^{208}Pb)

Energy weighted sum rule

$$m_+(1) + m_-(1) = \frac{1}{\pi}\frac{\hbar^2}{2m}A\langle r^2\rangle(1 + \kappa + \eta_N + \tilde{\eta}_C)$$

Skyrme

$$\kappa \equiv \frac{4}{A\langle r^2\rangle\rho_0}\left(\frac{m}{m^*_{IV}} - 1\right)\int d\mathbf{r}\, r^2\rho_n\rho_p$$

$$\eta_N \equiv \frac{1}{2A\langle r^2\rangle\rho_0}\left(\frac{m}{m^*_{IV}} - 1\right)\int d\mathbf{r}\, r^2\left(\rho_n - \rho_p\right)^2$$

$$\tilde{\eta}_C \equiv \frac{m}{2\hbar^2 A\langle r^2\rangle}\int d\mathbf{r}\left(r^2 - \langle r^2_{\rm exc}\rangle\right)^2\left(\rho_n - \rho_p\right)U_C$$

Enhance. fac.	^{48}Ca	^{90}Zr	^{208}Pb
κ	0.2438	0.2798	0.30071
η_N	0.0035	0.0019	0.0063
η_C	0.3090	0.4152	2.0825
$\tilde{\eta}_C$	0.0866	0.1125	0.58321

comes from D.C. $\frac{1}{4\pi}\sum_{ij}(r_i^2 - \langle r^2_{\rm exc}\rangle)(r_j^2 - \langle r^2_{\rm exc}\rangle)[t_+(i), [\mathcal{H}, t_-(j)]]$

Non charge-exchange Isovector Monopole:

Excitation operator [center-of-mass (CM) important]

Correction $\langle r^2_{\rm exc} \rangle$ will not produce any effect since it is a constant and $[\mathscr{H}, T_z] = 0$

$$\tilde{\mathscr{O}}_z = \sum_{i=1}^{A} \left(r_i^2 - \langle r^2_{\rm exc} \rangle \right) Y_{00}(\Omega_i) t_z(i) = \mathscr{O}_z$$

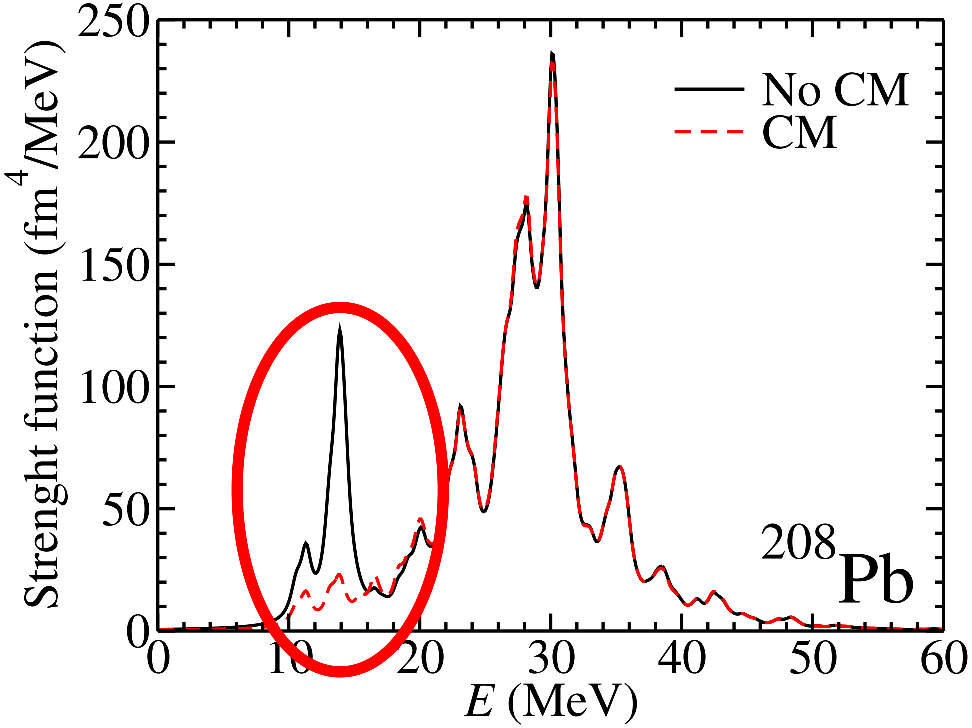
But center-of-mass correction will produce an effect

$$\mathscr{O}_z^{\rm CM} = 2\frac{Z}{A}\sum_{i=1}^{N} r_i^2 Y_{00}(\Omega_i) - 2\frac{N}{A}\sum_{i=1}^{Z} r_i^2 Y_{00}(\Omega_i)$$

~ 5% on EWSR;

~ 20% on IEWSR;

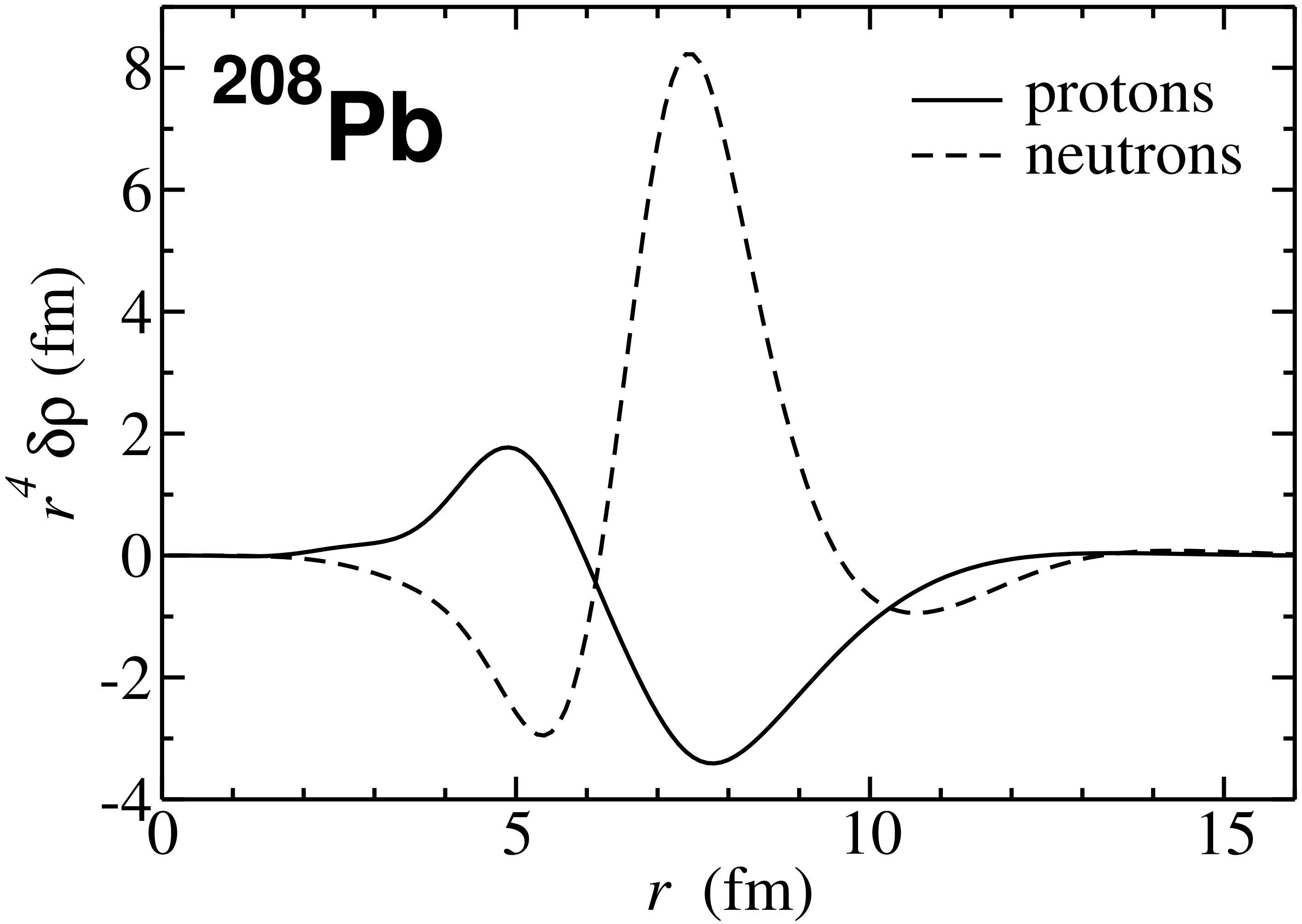
~ 10% NEWSR



Non charge-exchange Isovector Monopole:

Transition density $\int d^3r\,\delta\rho_{\rm IVM}\mathcal{O}^{\rm CM}_{\rm IVM} = \sqrt{4\pi}\int dr\,r^4\,\delta\rho_{\rm IVM}$

Isovector-like transition density



Non charge-exchange Isovector Monopole:

Sum rules

Non energy weighted sum rule (not analytic)

$$m_z(0) = \langle \mathcal{O}^\dagger \mathcal{O} \rangle_{\mathrm{RPA}} \neq \langle \mathcal{O}_z^\dagger \mathcal{O}_z \rangle_{\mathrm{HF}}$$

Energy weighted sum rule (κ defined in previous slides)

$$m_z(1) = \frac{4}{\pi} \frac{\hbar^2}{2m} \frac{NZ}{A} \langle r^2 \rangle (1+\kappa)$$

Inverse energy weighted sum rule (dielectric theorem)

$$\frac{1}{m_z(-1)} = 2 \frac{\partial^2 \langle \mathcal{H} \rangle}{\partial \langle \mathcal{O}_z \rangle^2} \bigg|_{\lambda=0}$$

$$\langle \mathcal{O}_z^{\mathrm{CM}} \rangle = \frac{1}{\sqrt{\pi}} \frac{NZ}{A} \int d\mathbf{r} r^2 \left(\rho_n - \rho_p \right) \propto \frac{NZ}{A} \langle r^2 \rangle^{1/2} \Delta r_{np}$$

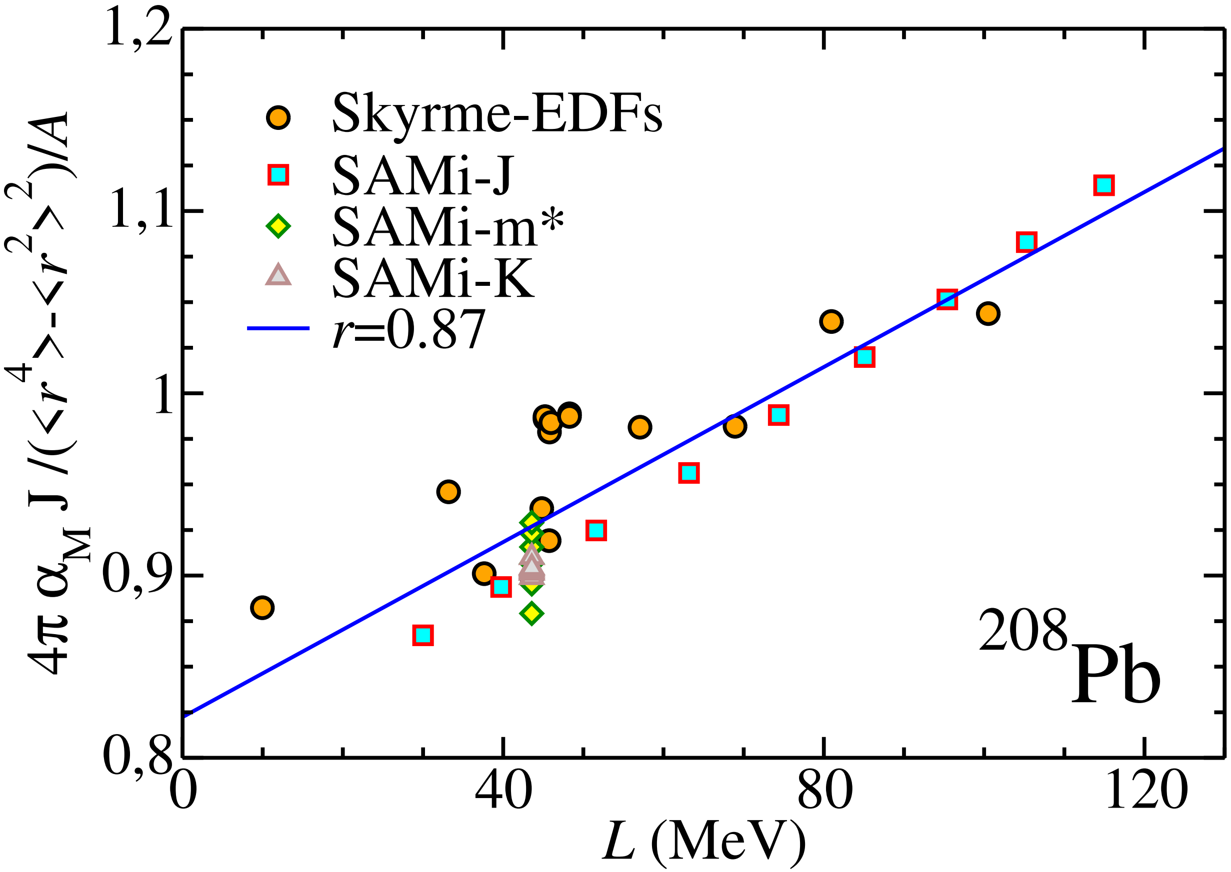
Variation of **excitation operator** due to variation of **isovector density** $\rho_{\mathrm{IV}} \equiv \rho_n - \rho_p \Rightarrow$ **polarizability** $\alpha_M = 2m_z(-1)$ proportional to the **second derivative** of the expectation value of the **energy density** with respect to $\rho_{\mathrm{IV}} \Rightarrow$ **Symmetry energy**

Non charge-exchange Isovector Monopole:

Polarizability (α_M) and Equation of State ($S(\rho) = J - L\frac{\rho_0 - \rho}{3\rho_0} + \mathcal{O}[\rho^2]$)

Assuming macroscopic **Droplet Model** [1] or **Hydrodynamic model** [2] for guidance:

$$\alpha_M = \frac{A}{4\pi J}\left(\langle r^4\rangle - \langle r^2\rangle^2\right)\left(1 + \frac{7}{3}\frac{L}{J}A^{-1/3}\right)$$



SAMi-J family:
$J \in [27,35]$ MeV

26 % change ⇒ 25%

SAMi-K family:
$K_0 \in [230,260]$ MeV

10% change ⇒ 0.5 %

SAMi-m* family:
$m^*_{IS}/m \in [0.6,0.85]$

30% change ⇒ 5 %

[1] J. Meyer, P. Quentin, and B.K. Jennings, "The isovector dipole mode: A simple sum rule approach," Nuclear Physics A **385**, 269–284 (1982).

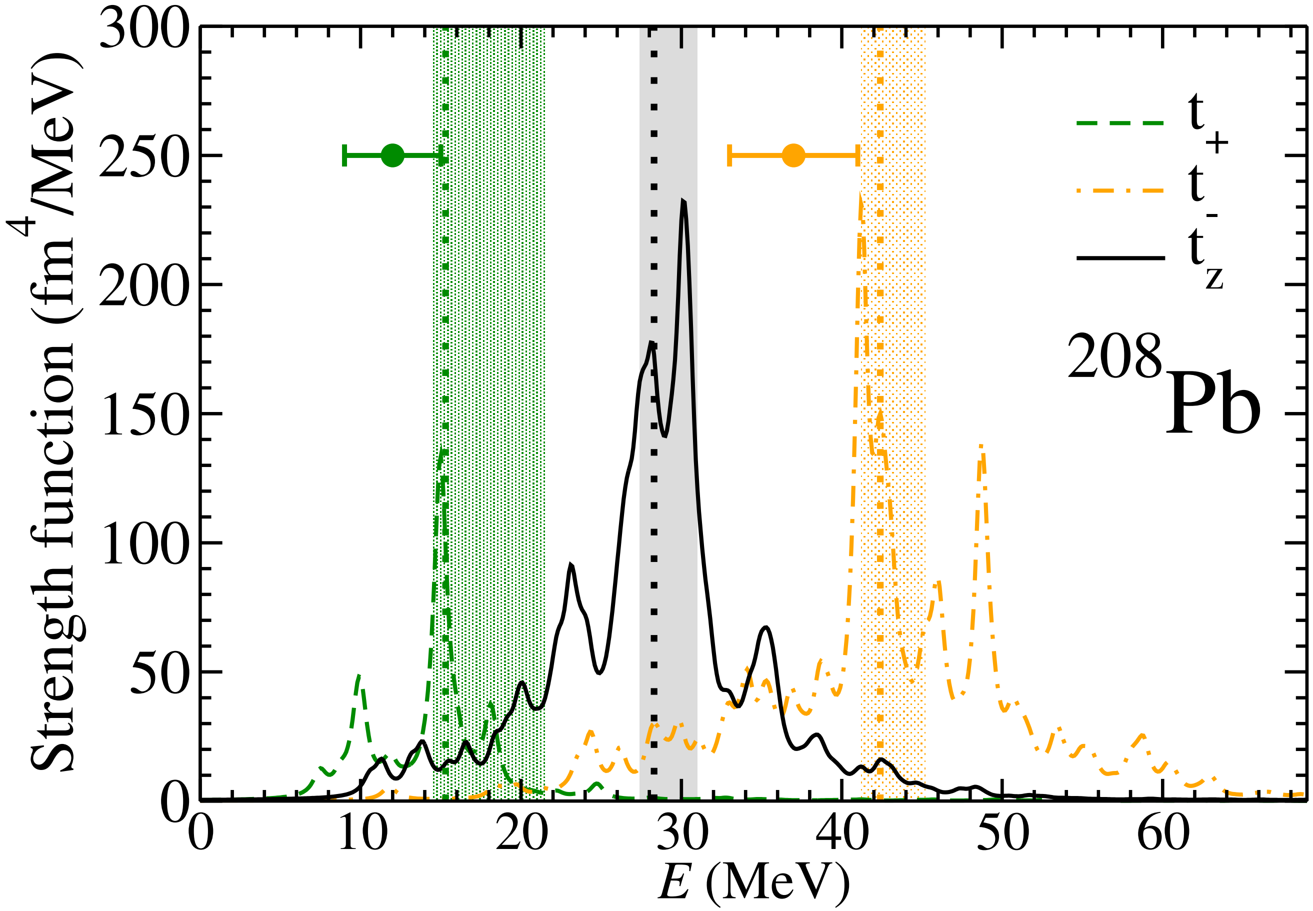
[2] J.D. Bowman, E. Lipparini, and S. Stringari, "Isovector monopole excitation energies," Physics Letters B **197**, 497–499 (1987).

Isovector Monopole:

Strength function (exp data from pion charge-exchange [1])

$E_{+}^{\mathrm{cent.}} = 15.3$ MeV $E_{-}^{\mathrm{cent.}} = 42.4$ MeV

$E_{z}^{\mathrm{cent.}} = 28.3$ MeV



Shaded region
$E_{\mu}^{\mathrm{const.}} - E_{\mu}^{\mathrm{scal.}}$

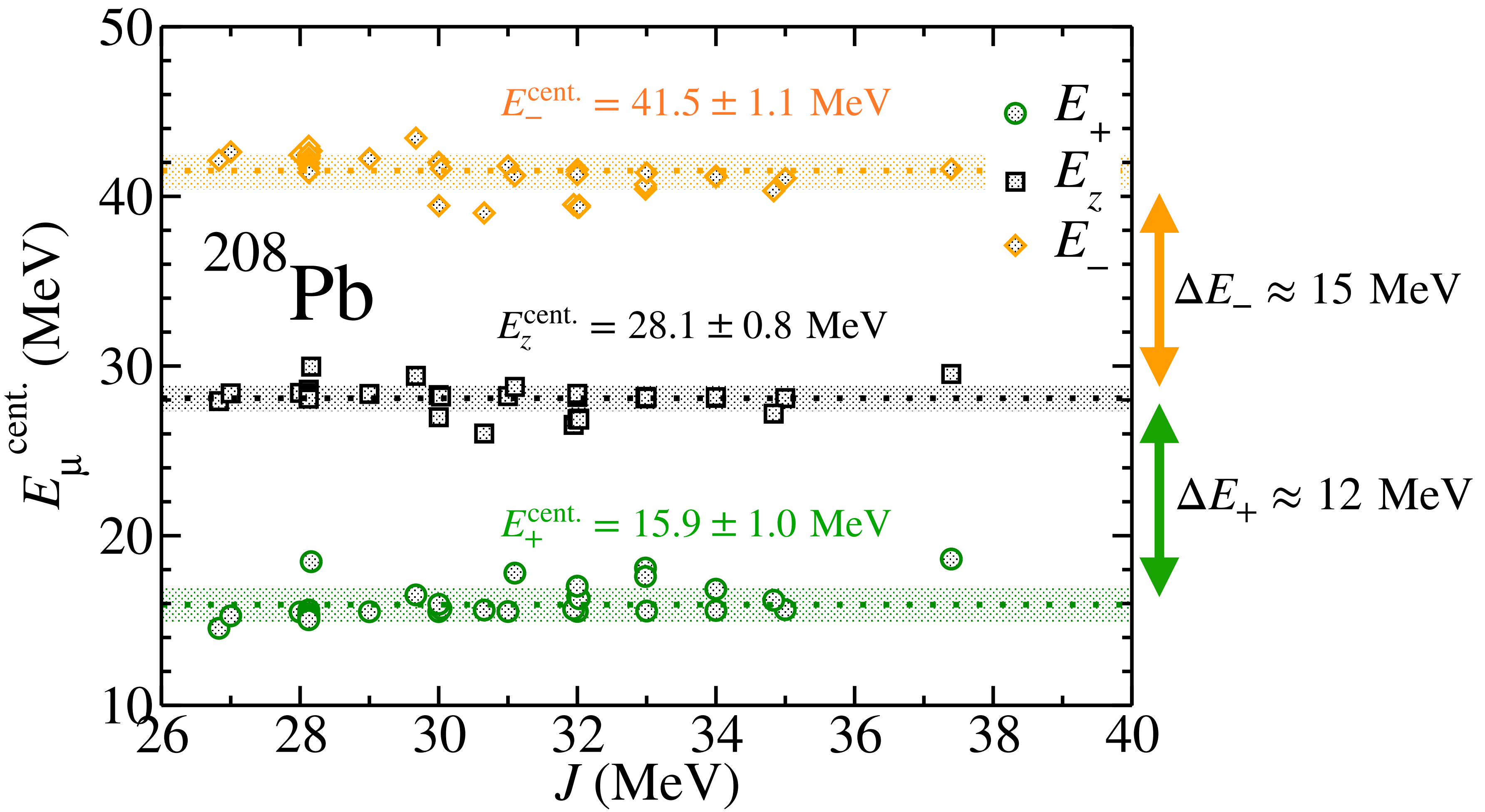
Differences in the **excitation energies** essentially depend on the **Coulomb displacement energy** ΔE_C and the **symmetry potential**
$\Delta E_T \sim 25(N-Z)/A$ MeV
(isospin splitting between $T \pm 1$ and T components of the IVM [2])

[1] A. Erell et al., Phys. Rev. C34 (1986) 1822;
[2] R. Leonardi, Phys. Rev. C **14**, 385 (1976).

Isovector Monopole:

Excitation energy systematics (model independent?)

$$\Delta E_{\pm} \approx \Delta E_C \mp \Delta m_{np} - \Delta E_T \approx 19 \pm 1.2 - 5.5 \approx {}^{15}_{12}\ \text{MeV}$$



Conclusions

The **IAS** effects must be **subtracted** from the excitation operator to **better isolate** the **charge-exchange IVM**

The **center of mass (CM)** must be **subtracted** from the excitation operator to **better isolate** the **non change-exchange IVM**

The **subtraction of the CM** is **important** to access information on the **symmetry potential** through the study of the **monopole polarizability** α_M

The **energies** of the three **IVM** modes are **quite different** ⇒ competing effects of the **Coulomb** and **symmetry** potentials.

Excitation energies are, to a good extent, **model independent (?)**