ハドロンの重力形状因子と質量分解

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Symmetric energy-momentum tensor

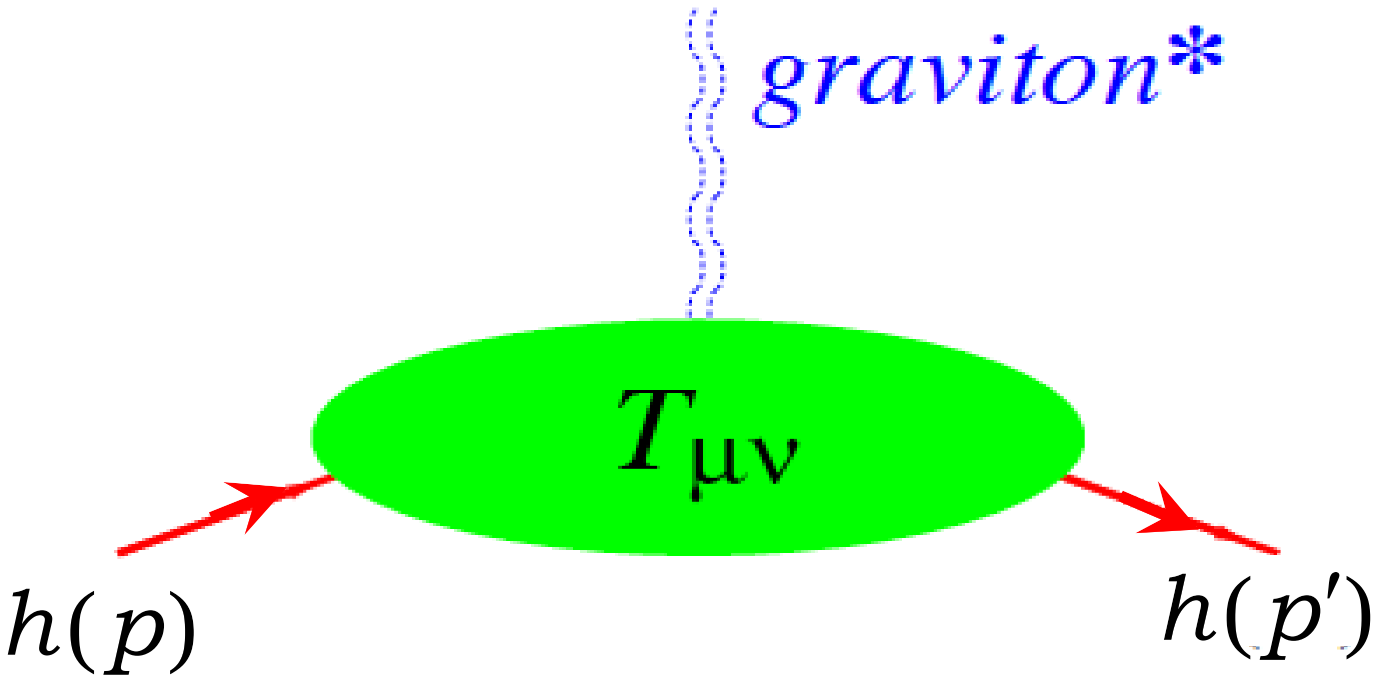
$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2 \;+\; (\text{ghost}) \;+\; (\text{gauge fix})$$

$$\left(= \underbrace{\frac{1}{2}\bar{\psi}\gamma^{\mu} i\partial^{\nu}\psi - F^{\mu\rho}\partial^{\nu}A_{\rho} + \frac{\eta^{\mu\nu}}{4}F^2 \;+\; (\text{ghost}) \;+\; (\text{gauge fix})}_{\sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)}\partial^\nu \phi_n - g^{\mu\nu}\mathcal{L}} + \partial_\lambda X^{[\lambda\mu]\nu} \right)$$

$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}(x)}\bigg|_{g^{\mu\nu}\to\eta^{\mu\nu}}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\partial_\mu T^{\mu\nu} = 0$$



$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^{2}$$

$$\equiv \quad T_q^{\mu\nu} \quad + \quad T_g^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)}\psi + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$\langle N(p')|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$A_q(0) + A_g(0) = 1$$

$$\langle N(p)|T^{\mu\nu}|N(p)\rangle = 2p^\mu p^\nu$$

$$\frac{1}{2}\Big(A_q(0) + B_q(0) + A_g(0) + B_g(0)\Big) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle N(p)\,S|J^i|N(p)\,S\rangle}{\langle N(p)\,S|N(p)\,S\rangle} = \frac{1}{2}S^i$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$

$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}\psi + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^{2} \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

angular momentum distribution

mass & energy distribution

$$\langle N(p')|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

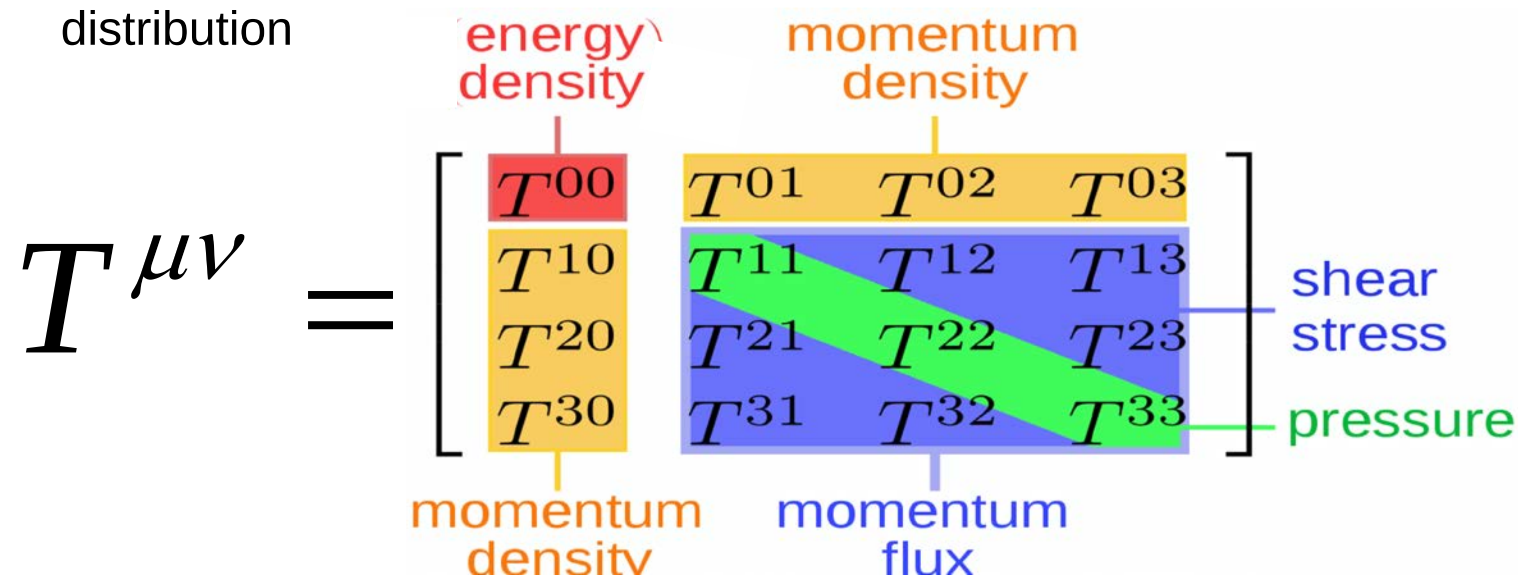
$$P = \frac{p+p'}{2}$$

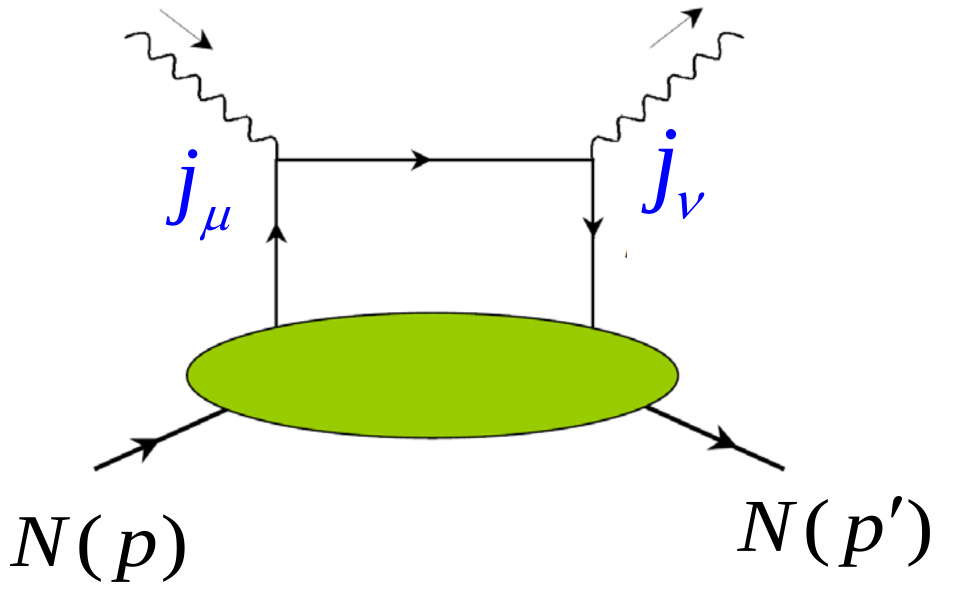
$$\Delta = p' - p$$

$$t = \Delta^2$$

force & pressure distribution

mass & pressure distribution



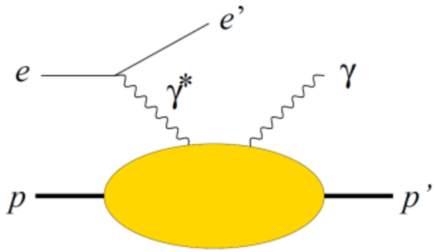


$$j_\mu(x)\, j_\nu(0) \sim \sum_i C_i(x) O_i(0)$$

$$C^q_{\mu\nu;\alpha\beta}(x) T_q^{\alpha\beta}$$

$$C^g_{\mu\nu;\alpha\beta}(x) T_g^{\alpha\beta}$$

DVCS



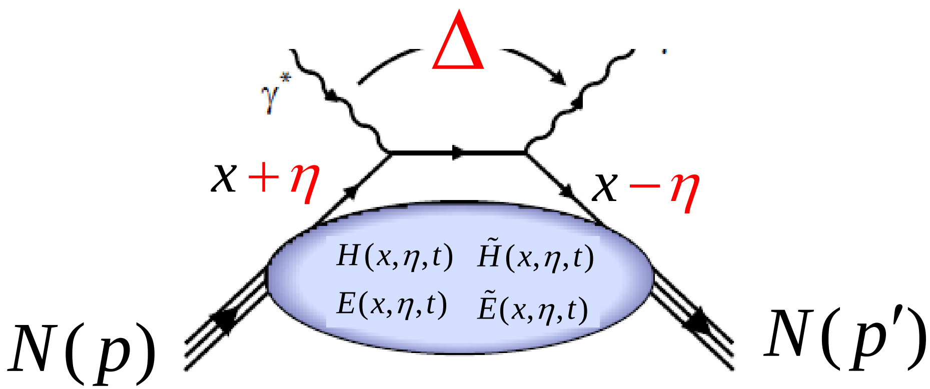
$$P = \frac{p + p'}{2}$$

JLab, HERMES, COMPASS, EIC

$$\int \frac{dz^-}{2\pi} e^{ixPz} \langle N(p') | \bar{\psi}(-\frac{z^-}{2}) \gamma^+ \psi(\frac{z^-}{2}) | N(p) \rangle = \frac{1}{P^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

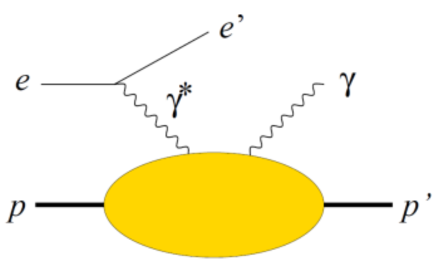
$$\int \frac{dz^-}{2\pi} e^{ixPz} \langle N(p') | \bar{\psi}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \psi(\frac{z^-}{2}) | N(p) \rangle = \frac{1}{P^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta P = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \left\langle N(p') \middle| \psi^\dagger(0) \psi(z^-) \middle| N(p) \right\rangle$$



DVCS

$$P = \frac{p+p'}{2}$$

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$$\int \frac{dz^-}{2\pi} e^{ixPz} \langle N(p') | \bar{\psi}(-\frac{z^-}{2}) \gamma^+ \psi(\frac{z^-}{2}) | N(p) \rangle = \frac{1}{P^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

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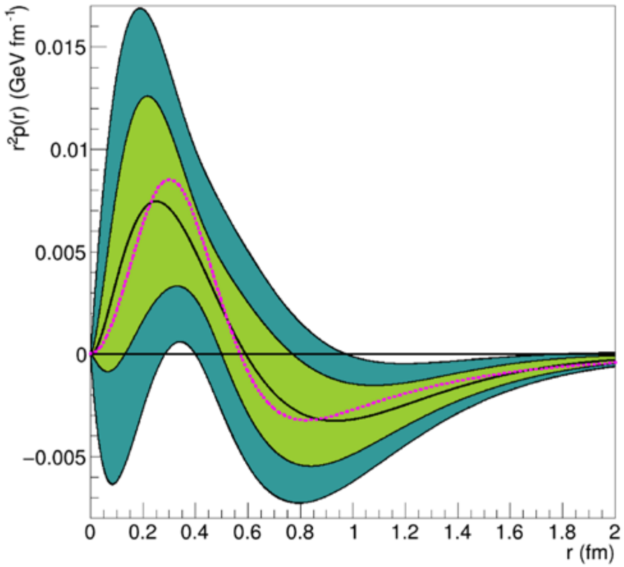
$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

$$\left(t = \Delta^2 \to 0 \,, \quad \eta = \frac{-\Delta \cdot n}{2P \cdot n} \to 0 \right)$$

$$H^q(x,0,0) = q(x)$$

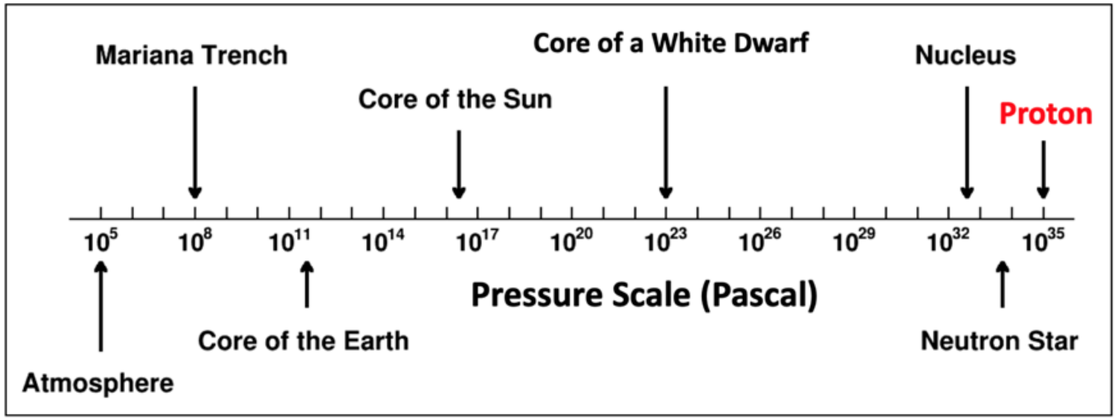
$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t)$$

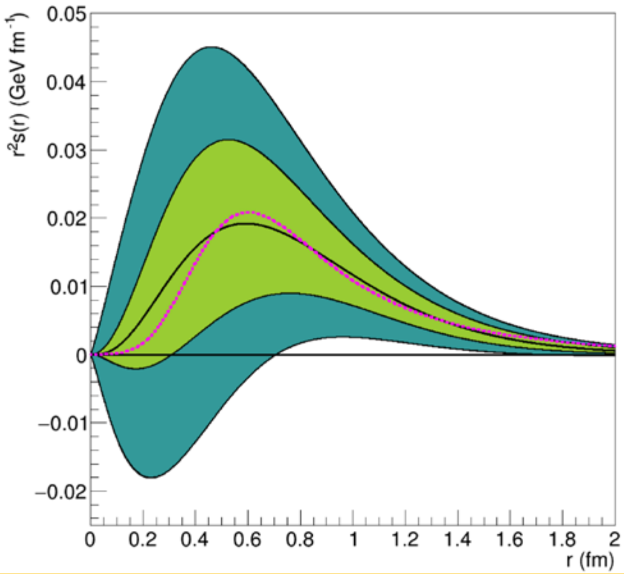
$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t)\,, \qquad \int_{-1}^{1} dx\, x E^q(x,\eta,t) = B_q(t) - 4\eta^2 D_q(t)$$

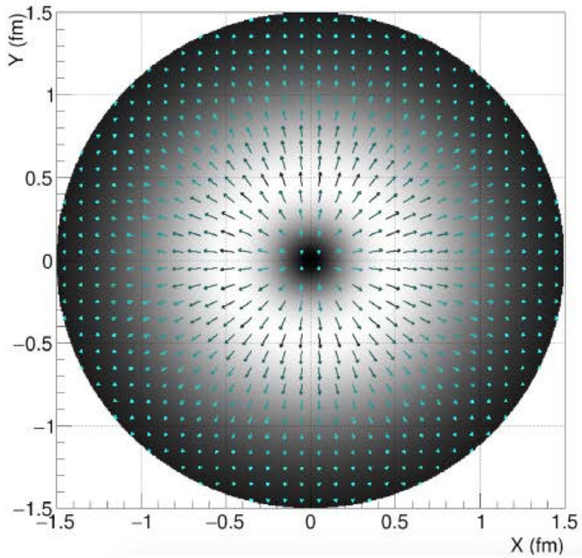


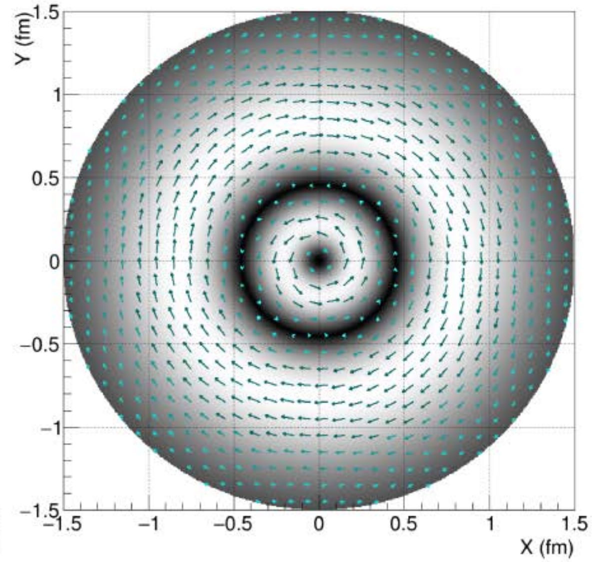
V. D. Burkert et al, Nature 557 ('18) 396

V. D. Burkert et al, 2303.08347









$$\langle N(p')|T^{ik}|N(p)\rangle \sim \left(\Delta^i\Delta^k - \delta^{ik}\mathbf{\Delta}^2\right)D(t), \qquad \left\langle T^{ij}\right\rangle(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\,p(r)$$

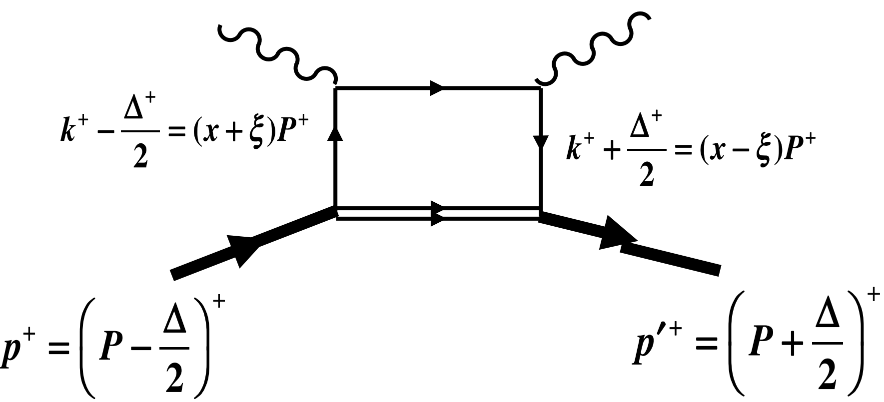
Kumano, Song, Teryaev, PRD97, 014020 ('18), 1711.08088

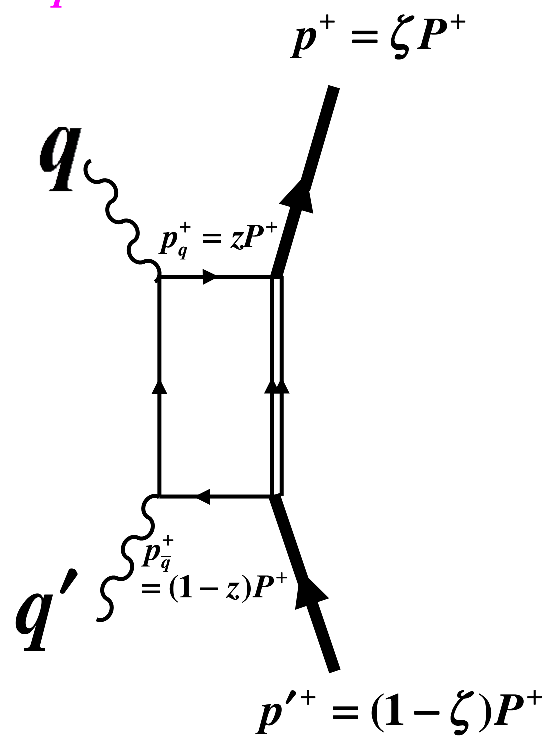
$$\text{GPD:}\quad H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}e^{ixP^+y^-}\left\langle\pi^0(p')\left|\bar\psi(-y/2)\gamma^+\psi(y/2)\right|\pi^0(p)\right\rangle\Big|_{y^+=0,\vec y_\perp=0},\qquad P^+=\frac{(p+p')^+}{2}$$

$$\text{GDA:}\quad \Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\left\langle\pi^0(p)\pi^0(p')\left|\bar\psi(-y/2)\gamma^+\psi(y/2)\right|0\right\rangle\Big|_{y^+=0,\vec y_\perp=0}$$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

s-t crossing





$\gamma\gamma^* \to \pi^0\pi^0$

Kumano, Song, Teryaev, PRD97, 014020 ('18) ,1711.08088

Spacelike gravitational form factors and radii for pion

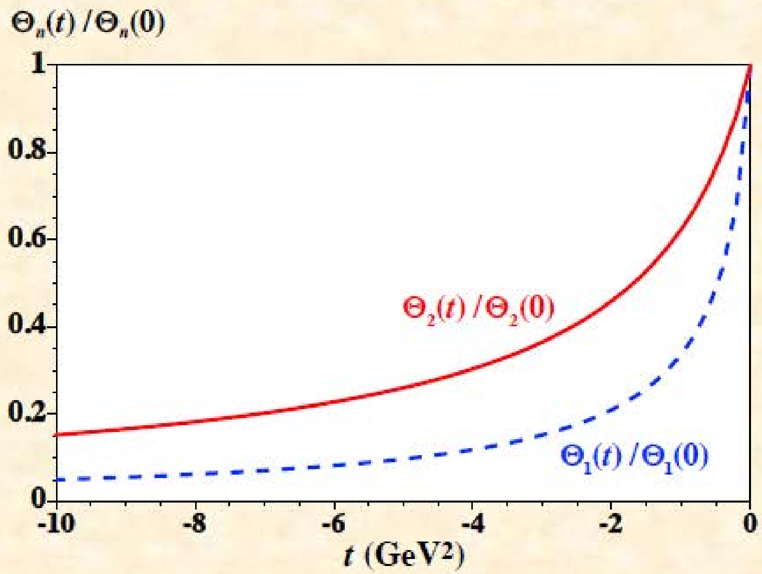
$$F(s)=\Theta_1(s),\ \Theta_1(s),\quad F(t)=\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r)=\frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}}F(q)=\frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

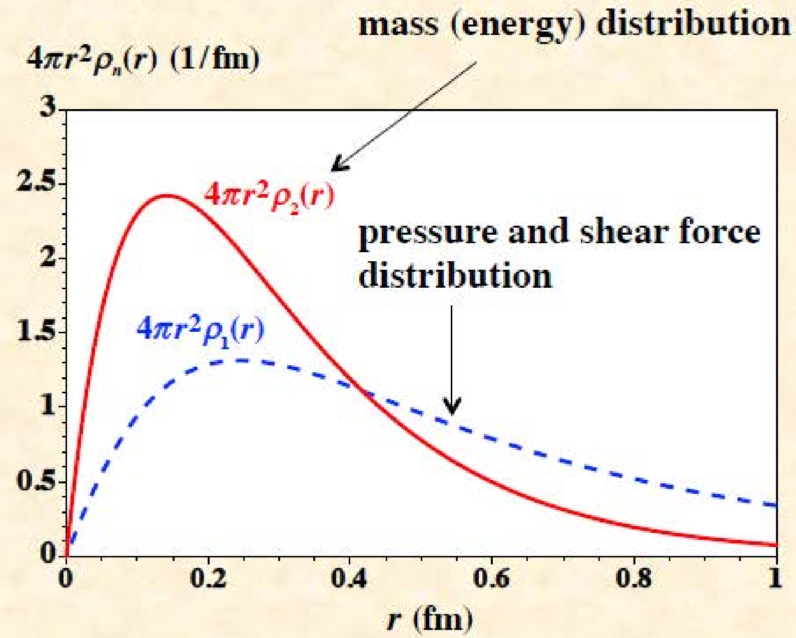
This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2\rangle_{\text{mass}}}=0.32\sim 0.39\ \text{fm},\quad \sqrt{\langle r^2\rangle_{\text{mech}}}=0.82\sim 0.88\ \text{fm}$$

← First finding on gravitational radius from actual experimental measurements

$$\Leftrightarrow \sqrt{\langle r^2\rangle_{\text{charge}}}=0.672\pm 0.008\ \text{fm}$$





$$\Theta_2(t)=4A^\pi(t),\quad \Theta_1(t)=-D^\pi(t)$$

Spacelike gravitational form factors and radii for pion

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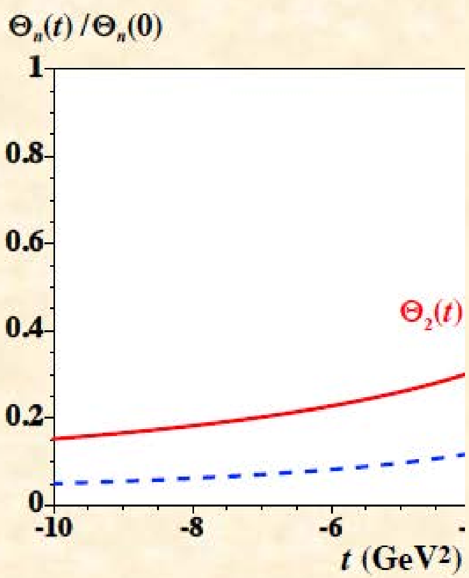
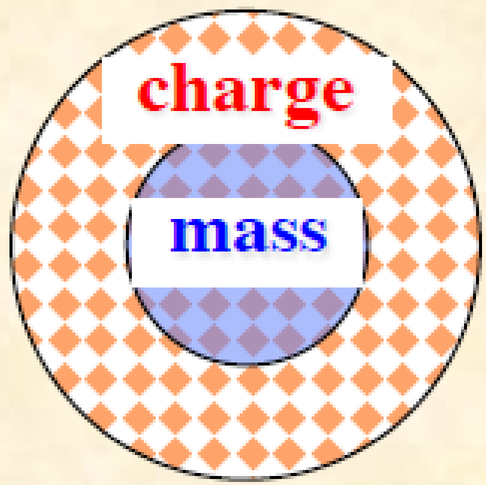
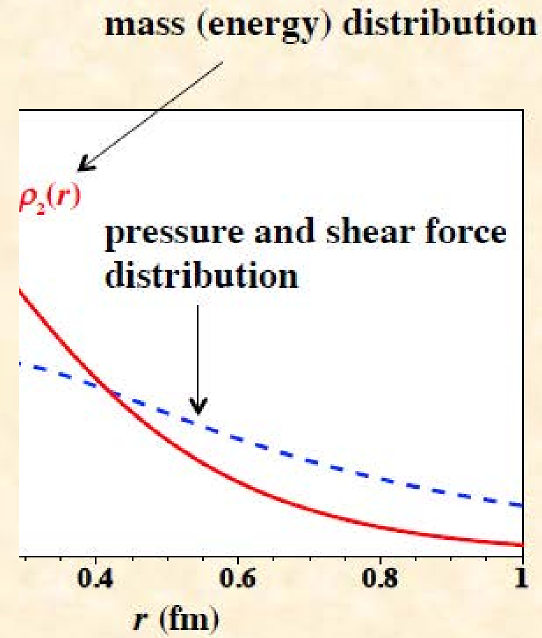






$$\Theta_2(t)=4A^{\pi}(t),\qquad \Theta_1(t)=-D^{\pi}(t)$$

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle} = M\left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \boxed{\int d^3x \psi^\dagger\left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta\right)\psi \ + \cdots} + \boxed{\int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right) \ + \cdots}$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: –0.2, 0.2

$$\int_0^1 dx x\left[q(x,\mu) + \bar{q}(x,\mu)\right] = A_q(t=0,\mu)$$

$$A_q\left(t=0,\mu_0=1.3\text{GeV}\right) = 0.613$$

global QCD analysis at NNLO

CT18 (MMHT2014, NNPDF)

$$A_q(0,\mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0,\mu_0) + n_f\left(A_q(0,\mu_0) - 1\right)}{4C_F + n_f}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}} + \cdots$$

$$\bar{C}_q(0,\mu = 1\text{ GeV}) = -\bar{C}_g(0,\mu = 1\text{ GeV}) = -0.180 \pm 0.003$$

NNLO QCD [KT, JHEP03, 013 ('23)]

$$A_q(\mu_0) = \int_0^1 dx\, x\left[q(x,\mu_0) + \bar{q}(x,\mu_0)\right]$$

global QCD analysis at NNLO

$$A_q\left(\mu_0 = 1.3\,\mathrm{GeV}\right) = 0.613$$

CT18

(MMHT2014,NNPDF)

$$\langle N(p)|\, m\bar{\psi}\psi \,|N(p)\rangle = \langle N(p)|\, m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \,|N(p)\rangle = 2M\left(\sigma_{\pi N} + \sigma_s\right)$$

$$\sigma_{\pi N} = \frac{1}{2M}\langle N(p)|\, \frac{m_u + m_d}{2}\left(\bar{u}u + \bar{d}d\right) |N(p)\rangle = 59.1 \pm 3.5\,\mathrm{MeV}$$

Hoferichter, Elvira, Kubis, Meißner, PRL115, 092301

$$\sigma_s = \frac{1}{2M}\langle N(p)|\, m_s \bar{s}s \,|N(p)\rangle = 45.6 \pm 6.2\,\mathrm{MeV}$$

Alexandrou, et al., PRD102, 054517

$$\langle N(p)|T^{00}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(0}p^{0)} + \bar{C}_{q,g}(0)M\eta^{00}\right]u(p)$$

$$\left\langle \hat{H}_{q,g}\right\rangle = \frac{\langle N|\int d^3x T^{00}_{q,g}|N\rangle}{\langle N|N\rangle} = M\left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \boxed{\int d^3x \psi^\dagger\left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta\right)\psi \ + \cdots} + \boxed{\int d^3x \frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right) \ + \cdots}$$

$$M = M_q + M_g$$

M_q: 0.4, M_g: 0.6

$$M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$

$A_{q,g}(0)$: 0.6, 0.4; $\bar{C}_{q,g}(0)$: –0.2, 0.2

$$M = \left(M_q - M_m\right) + M_m + M_g$$

$M_q - M_m$: 0.3, M_m: 0.1, M_g: 0.6

$$M_m = \left\langle \int d^3x\, m\bar{\psi}\psi \right\rangle = \frac{\sigma_{\pi N} + \sigma_s}{M}M$$

M_m: 0.1

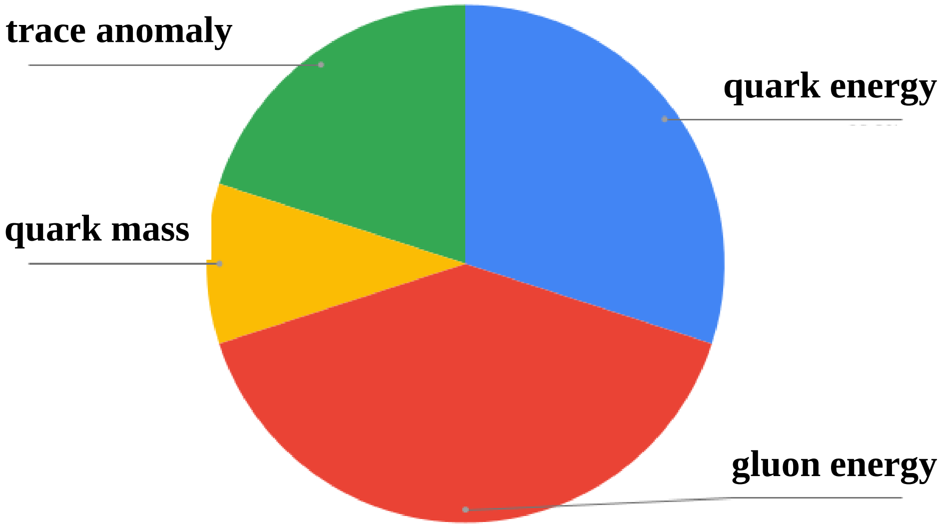
$$M = \left(M_q - M_m - \Delta M_q\right) + M_m + \left(M_g - \Delta M_g\right) + \left(\Delta M_q + \Delta M_g\right)$$

$M_q - M_m - \Delta M_q$: 0.3, M_m: 0.1, $M_g - \Delta M_g$: 0.4, $\Delta M_q + \Delta M_g$: 0.2

$$\Delta M_q + \Delta M_g = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)\right\rangle = \frac{1}{4}\left(M - M_m\right)$$

$\Delta M_q + \Delta M_g$: 0.2

Ji's proton mass decomposition



$$\langle \pi(p) | T_{q,g}^{00} | \pi(p) \rangle = 2A_{q,g}^{\pi}(0) p^0 p^0 + 2M^2 \bar{C}_{q,g}^{\pi}(0) \eta^{00}$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle \pi | \int d^3x T_{q,g}^{00} | \pi \rangle}{\langle \pi | \pi \rangle} = M \left(A_{q,g}^{\pi}(0) + \bar{C}_{q,g}^{\pi}(0) \right)$$

$$\hat{H} = \hat{H}_q + \hat{H}_g = \boxed{\int d^3x \psi^\dagger \left(-i\boldsymbol{D} \cdot \boldsymbol{\alpha} + m\beta \right) \psi \; + \cdots} + \boxed{\int d^3x \frac{1}{2} \left(\boldsymbol{E}^2 + \boldsymbol{B}^2 \right) \; + \cdots}$$

$$M = M_q + M_g \qquad M_{q,g} = \left(A_{q,g}^{\pi}(0) + \bar{C}_{q,g}^{\pi}(0) \right) M$$

$$A_q^{\pi}(\mu_0) = \int_0^1 dx\, x \left[q^{\pi}(x,\mu_0) + \bar{q}^{\pi}(x,\mu_0) \right]$$

global QCD analysis at NLO

$$A_q^{\pi}(\mu_0 = 1.3\,\mathrm{GeV}) = \begin{cases} 0.70 \pm 0.02 & \text{JAM ('18)} \\ 0.81 \pm 0.16 & \text{xFitter ('20)} \\ 0.61 \pm 0.08 & \text{JAM ('21)} \end{cases}$$

$\bar{C}_q^{\pi}(0, \mu = 1\,\mathrm{GeV}) = -0.04 \pm 0.02$ NNLO QCD with NLO input [KT, JHEP03, 013 ('23)]

$$\langle \pi(p) | m\bar{\psi}\psi | \pi(p) \rangle$$

$$= M_{\pi}^2 + O(6\%)$$

χPT

Gasser, Leutwyler, Annals Phys. 158, 142

Colangelo, Gasser, Leutwyler, PRL86, 5008

$$\langle \pi(p) | T^{00}_{q,g} | \pi(p)\rangle = 2A^{\pi}_{q,g}(0)\, p^0 p^0 + 2M^2 \bar{C}^{\pi}_{q,g}(0)\,\eta^{00}$$

$$\left\langle \hat{H}_{q,g} \right\rangle = \frac{\langle \pi | \int d^3x\, T^{00}_{q,g} | \pi\rangle}{\langle \pi|\pi\rangle} = M\left(A^{\pi}_{q,g}(0)+\bar{C}^{\pi}_{q,g}(0)\right)$$

$$\hat H = \hat H_q + \hat H_g = \boxed{\int d^3x\, \psi^\dagger\left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha}+m\beta\right)\psi \;+\cdots} + \boxed{\int d^3x\, \frac{1}{2}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right) \;+\cdots}$$

$$M = \underset{0.6}{M_q} + \underset{0.4}{M_g}$$

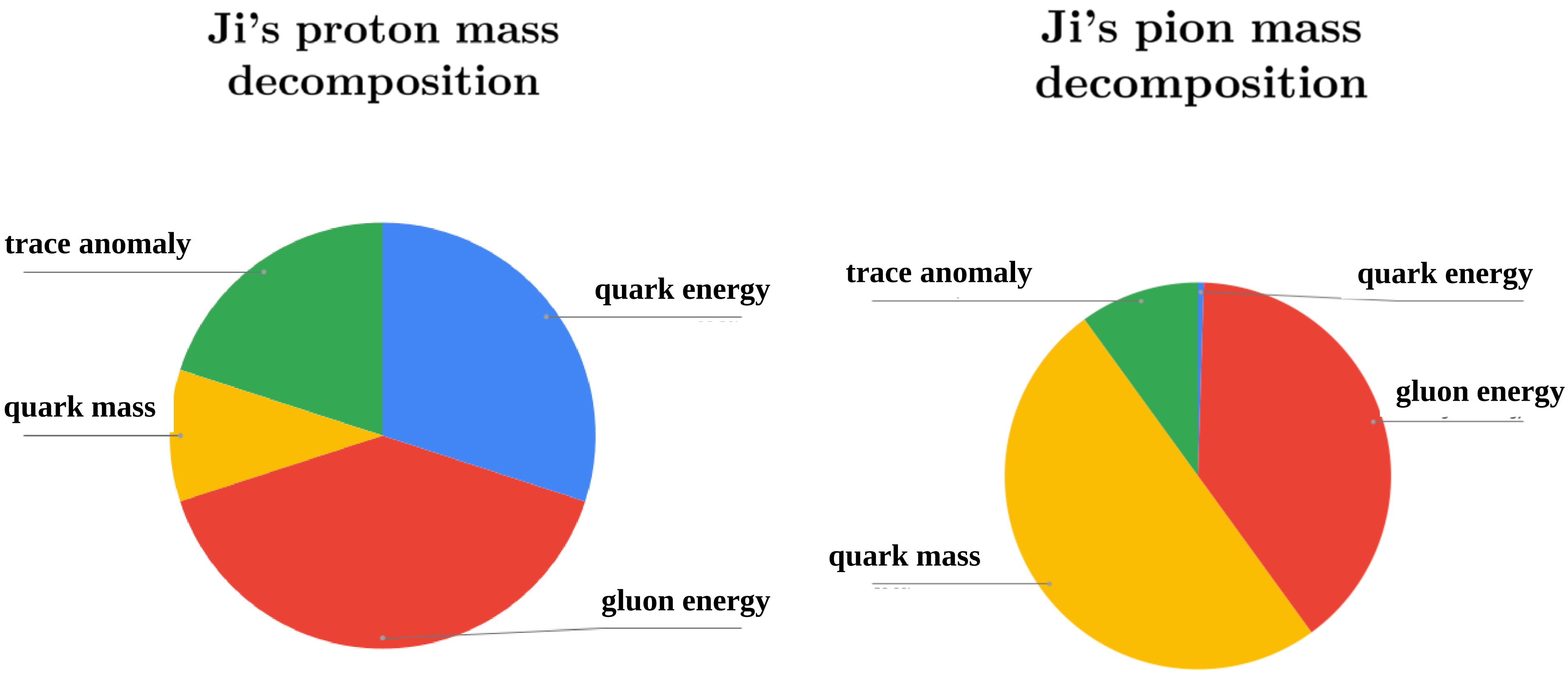
$$M_{q,g} = \left(\underset{0.6,\ 0.4}{A^{\pi}_{q,g}(0)} + \underset{-0.0,\ 0.0}{\bar{C}^{\pi}_{q,g}(0)}\right)M$$

$$M = \underset{0.1}{\left(M_q - M_m\right)} + \underset{0.5}{M_m} + \underset{0.4}{M_g}$$

$$\underset{0.5}{M_m} = \left\langle \int d^3x\; m\bar{\psi}\psi \right\rangle = \frac{1}{2}M$$

$$M = \underset{0.0}{\left(M_q - M_m - \Delta M_q\right)} + \underset{0.5}{M_m} + \underset{0.4}{\left(M_g - \Delta M_g\right)} + \underset{0.1}{\left(\Delta M_q + \Delta M_g\right)}$$

$$\underset{0.1}{\Delta M_q + \Delta M_g} = \frac{1}{4}\left\langle \int d^3x \left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)\, m\bar{\psi}\psi\right)\right\rangle = \frac{1}{4}\left(M - M_m\right)$$


Ji's proton mass decomposition
trace anomaly
quark energy
quark mass
gluon energy
Ji's pion mass decomposition
trace anomaly
quark energy
gluon energy
quark mass

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)M\eta^{\mu\nu}\right]u(p)$$

$$\eta_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle$$

$$M^2 = \tilde{M}_q^2 + \tilde{M}_g^2$$

$$\tilde{M}^2_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M^2$$

	$\tilde{M}_q^2$	$\tilde{M}_g^2$	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
nucleon	–0.2	1.2	0.6, 0.4	–0.2, 0.2
pion	0.5	0.5	0.6, 0.4	–0.04, 0.04

$$\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)M\eta^{\mu\nu}\right]u(p)$$

$$\eta_{\mu\nu}\langle N(p)|T^{\mu\nu}_{q,g}|N(p)\rangle = 2M^2\left(A_{q,g}(0)+4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle$$

$$M^2 = \tilde{M}_q^2 + \tilde{M}_g^2 \qquad \tilde{M}^2_{q,g} = \left(A_{q,g}(0)+4\bar{C}_{q,g}(0)\right)M^2$$

–0.2	1.2	0.6, 0.4	–0.2, 0.2	nucleon
0.5	0.5	0.6, 0.4	–0.04, 0.04	pion

$$\hat{H} = \hat{H}_q + \hat{H}_g = \boxed{\int d^3x\,\psi^\dagger\left(-i\boldsymbol{D}\cdot\boldsymbol{\alpha}+m\beta\right)\psi \;+\cdots} + \boxed{\int d^3x\,\frac{1}{2}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right) \;+\cdots}$$

$$M = M_q + M_g \qquad M_{q,g} = \left(A_{q,g}(0)+\bar{C}_{q,g}(0)\right)M$$

0.4	0.6	0.6, 0.4	–0.2, 0.2	nucleon
0.6	0.4	0.6, 0.4	–0.0, 0.0	pion

① **mass decomposition**

Ji, PRD52 271 ('95)

Lorce, Moutarde, Trawinski, EPJC79, 89 ('19)

Metz, Pasquini, Rodini, PRD102, 114042 ('20)

Ji, Liu, Schafer, NPB971, 115537 ('21)

② **pressure**

$$-\bar{C}_{q,g}\frac{M}{V}$$

Lorce, EPJC78, 120 ('18)

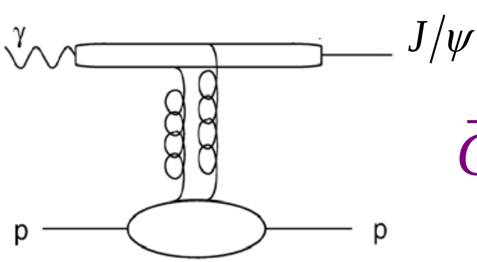
Liu, PRD104, 076010 ('21)

③ **nucleon's transverse spin sum rule**

Hatta, KT, Yoshida, JHEP 02 ('13) 003

$$J_{q,g}=\frac{1}{2}(A_{q,g}+B_{q,g})+\frac{p^3}{2(p^0+M)}\bar{C}_{q,g}$$

④ $\gamma p \to J/\psi\ p$ **near threshold** JLab, EIC



$$\bar{C}_g\ \left(=-\bar{C}_q\right)$$

Y. Hatta, D. Yang, PRD98, 074003

Y. Hatta, A. Rajan, D. Yang, PRD100, 014032

nucleon

$\bar{C}_q(0, \mu \sim 0.4\ \text{GeV}) = 0.25$ **Bag model** [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$\bar{C}_q(0, \mu = 2\ \text{GeV}) \approx -0.11$ **Phenomenological** [Lorce, EPJC78, 120 ('18)]

$\bar{C}_q(0, \mu \sim 0.63\ \text{GeV}) = 0.014$ **Instanton** [Polyakov, Son, JHEP09, 156 ('18)]

$\bar{C}_q(0, \mu = 1\ \text{GeV}) = -0.021 \pm 0.008$ **LCSR** [Azizi,Ozdem, EPJC80, 104 ('20)]

$\bar{C}_q(0, \mu \to \infty) \simeq -0.15$ **Trace anomaly** [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$\bar{C}_q(0, \mu = 1\ \text{GeV}) = -0.180 \pm 0.003$

$\bar{C}_q(0, \mu = 2\ \text{GeV}) = -0.163 \pm 0.003$

NNLO QCD [KT, JHEP03, 013 ('23)]

pion

$\bar{C}_q^{\pi}(0, \mu = 1\ \text{GeV}) = -0.04 \pm 0.02$ **NNLO QCD with NLO input** [KT, JHEP03, 013 ('23)]

$$\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)M\eta^{\mu\nu}\right]u(p)$$

$$\eta_{\mu\nu}\langle N(p)|T_{q,g}^{\mu\nu}|N(p)\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle = \eta_{\mu\nu}\langle N|T_q^{\mu\nu}|N\rangle + \eta_{\mu\nu}\langle N|T_g^{\mu\nu}|N\rangle$$

$$M^2 = \tilde{M}_q^2 + \tilde{M}_g^2 \qquad \tilde{M}_{q,g}^2 = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M^2$$

	$\tilde{M}_q^2$	$\tilde{M}_g^2$	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
nucleon	–0.2	1.2	0.6, 0.4	–0.2, 0.2
pion	0.5	0.5	0.6, 0.4	–0.04, 0.04

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$\eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{\psi}\psi \qquad \left(\beta(g) \equiv \mu\frac{dg}{d\mu},\ \gamma_m(g) = -\frac{\mu}{m}\frac{dm}{d\mu}\right)$$

$\eta_{\mu\nu}T_{q,g}^{\mu\nu}$

1&2-loop Hatta, Rajan, KT, JHEP 12 ('18) 008

3-loop (& all orders) KT, JHEP 01 ('19) 120

4-loop Ahmed, Chen, Czakon, JHEP 01 ('23) 077

trace anomaly separately for q, g

Hatta, Rajan, KT, JHEP 12 ('18) 008

$$\eta_{\mu\nu}T_q^{\mu\nu} = m\bar{\psi}\psi + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{\psi}\psi + \frac{1}{3}n_f F^2\right)$$

$$\eta_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{\psi}\psi - \frac{11}{6}C_A F^2\right)$$

$$\eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{\psi}\psi$$

$$C_A = N_c\ , \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$

trace anomaly separately for q, g $\overline{\mathrm{MS}}$ scheme Hatta, Rajan, KT, JHEP

$$\eta_{\mu\nu}T_q^{\mu\nu} = m\bar{\psi}\psi + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar{\psi}\psi + \frac{1}{3}n_f F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)m\bar{\psi}\psi + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54}\right)F^2\right]$$

$$\eta_{\mu\nu}T_g^{\mu\nu} = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar{\psi}\psi - \frac{11}{6}C_A F^2\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)m\bar{\psi}\psi + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54}\right)F^2\right]$$

$$\eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{\psi}\psi \qquad C_A = N_c\,, \quad C_F = \frac{N_c^2 - 1}{2N_c}$$

trace anomaly separately for q, g $\overline{\text{MS}}$ scheme

KT, JHEP01 ('19)120

$$\eta_{\mu\nu}T_q^{\mu\nu}=m\bar\psi\psi+\frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F m\bar\psi\psi+\frac{1}{3}n_f F^2\right)+\left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{61C_A}{27}-\frac{68n_f}{27}\right)-\frac{4C_F^2}{27}\right)m\bar\psi\psi+\left(\frac{17C_An_f}{27}+\frac{49C_Fn_f}{54}\right)F^2\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9}-\frac{8305}{729}\right)C_F^2-\frac{2}{243}(864\zeta(3)+1079)C_AC_F\right)-\frac{8}{729}(972\zeta(3)+143)C_AC_F^2\right.\right.$$
$$\left.+\left(\frac{32\zeta(3)}{9}+\frac{6611}{729}\right)C_A^2C_F-\frac{76}{243}C_Fn_f^2+\frac{8}{729}(648\zeta(3)-125)C_F^3\right\}m\bar\psi\psi$$
$$\left.+\left\{n_f\left(\left(\frac{52\zeta(3)}{9}-\frac{401}{324}\right)C_AC_F+\left(\frac{134}{27}-4\zeta(3)\right)C_A^2+\left(\frac{2407}{1458}-\frac{16\zeta(3)}{9}\right)C_F^2\right)+n_f^2\left(-\frac{697C_A}{729}-\frac{169C_F}{1458}\right)\right\}F^2\right]$$

$$\eta_{\mu\nu}T_g^{\mu\nu}=\frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F m\bar\psi\psi-\frac{11}{6}C_AF^2\right)+\left(\frac{\alpha_s}{4\pi}\right)^2\left[\left(C_F\left(\frac{812C_A}{27}-\frac{22n_f}{27}\right)+\frac{85C_F^2}{27}\right)m\bar\psi\psi+\left(\frac{28C_An_f}{27}-\frac{17C_A^2}{3}+\frac{5C_Fn_f}{54}\right)F^2\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9}-\frac{25229}{729}\right)C_F^2-\frac{2}{243}(4968\zeta(3)+1423)C_AC_F\right)+\left(\frac{32\zeta(3)}{3}-\frac{91753}{1458}\right)C_AC_F^2\right.\right.$$
$$\left.+\left(\frac{294929}{1458}-\frac{32\zeta(3)}{9}\right)C_A^2C_F-\frac{554}{243}C_Fn_f^2+\left(\frac{95041}{729}-\frac{64\zeta(3)}{9}\right)C_F^3\right\}m\bar\psi\psi$$
$$\left.+\left\{n_f\left(\left(\frac{1123}{162}-\frac{52\zeta(3)}{9}\right)C_AC_F+\left(4\zeta(3)+\frac{293}{36}\right)C_A^2+\frac{16}{729}(81\zeta(3)-98)C_F^2\right)+n_f^2\left(\frac{655C_A}{2916}-\frac{361C_F}{729}\right)-\frac{2857C_A^3}{108}\right\}F^2\right]$$

$$\eta_{\mu\nu}T^{\mu\nu}=\frac{\beta(g)}{2g}F^2+\left(1+\gamma_m(g)\right)m\bar\psi\psi \qquad C_A=N_c\,, \qquad C_F=\frac{N_c^2-1}{2N_c}$$

$$2M^2 = \langle N | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{\psi}\psi \right) | N \rangle \simeq \langle N | \frac{\beta(g)}{2g} F^2 | N \rangle$$

$$2M^2 = \eta_{\mu\nu} \langle N | T_q^{\mu\nu} | N \rangle + \eta_{\mu\nu} \langle N | T_g^{\mu\nu} | N \rangle$$

1-loop

$$\frac{\alpha_s}{4\pi} \frac{n_f}{3} F^2 \qquad \frac{\alpha_s}{4\pi} \left(-\frac{11C_A}{6} F^2 \right)$$

2-loop

$$\left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54} \right) F^2 \qquad \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54} \right) F^2$$

3-loop

$$\left(\frac{\alpha_s}{4\pi}\right)^3 \left[\left\{ n_f \left(\frac{52\zeta(3)}{9} - \frac{401}{324} \right) C_A C_F + \left(\frac{134}{27} - 4\zeta(3) \right) C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9} \right) C_F^2 \right\} + n_f^2 \left(-\frac{697C_A}{729} - \frac{169C_F}{1458} \right) \right] F^2$$

$$\left(\frac{\alpha_s}{4\pi}\right)^3 \left[n_f \left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9} \right) C_A C_F + \left(4\zeta(3) + \frac{293}{36} \right) C_A^2 + \frac{16}{729} (81\zeta(3) - 98) C_F^2 \right) + n_f^2 \left(\frac{655C_A}{2916} - \frac{361C_F}{729} \right) - \frac{2857C_A^3}{108} \right] F^2$$

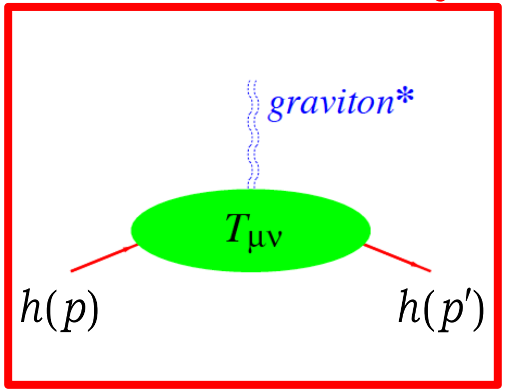
nucleon	**-1**	**:**	**6**
pion	**1**	**:**	**1**

$$2M_\pi^2 = \eta_{\mu\nu} \langle \pi | T_q^{\mu\nu} | \pi \rangle + \eta_{\mu\nu} \langle \pi | T_g^{\mu\nu} | \pi \rangle$$

KT, JHEP1901, 120

Summary

Gravitational form factors can be accessed @EIC



mass & energy distribution

spin distribution

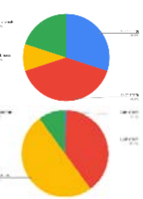
$$\langle N(p')|T^{\mu\nu}_{q,g}|N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}$$
$$+ D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

force & pressure distribution

mass & pressure distribution

$$\hat{H} = \hat{H}_q + \hat{H}_g = \int d^3x\,\psi^\dagger(-i\boldsymbol{D}\cdot\boldsymbol{\alpha} + m\beta)\psi \; + \cdots \; + \int d^3x\,\frac{1}{2}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right) \; + \cdots$$

$$M = M_q + M_g \qquad\qquad M_{q,g} = \left(A_{q,g}(0) + \bar{C}_{q,g}(0)\right)M$$



	M_q	M_g	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
nucleon	0.4	0.6	0.6, 0.4	−0.2, 0.2
pion	0.6	0.4	0.6, 0.4	−0.0, 0.0

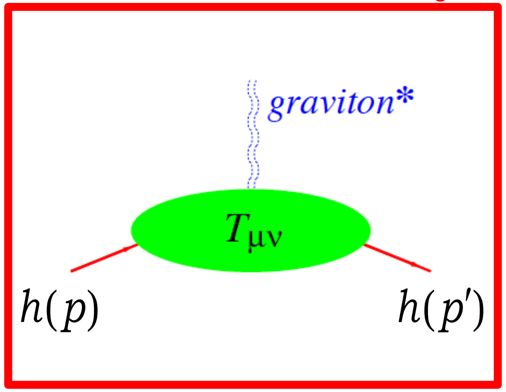
$$\eta_{\mu\nu}T^{\mu\nu} = \eta_{\mu\nu}T^{\mu\nu}_q + \eta_{\mu\nu}T^{\mu\nu}_g \qquad\qquad 2M^2 = \eta_{\mu\nu}\langle N|T^{\mu\nu}|N\rangle$$

$$M^2 = \tilde{M}^2_q + \tilde{M}^2_g \qquad\qquad \tilde{M}^2_{q,g} = \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)M^2$$

	$\tilde{M}^2_q$	$\tilde{M}^2_g$	$A_{q,g}(0)$	$\bar{C}_{q,g}(0)$
nucleon	−0.2	1.2	0.6, 0.4	−0.2, 0.2
pion	0.5	0.5	0.6, 0.4	−0.04, 0.04

Summary

Gravitational form factors can be accessed @EIC



mass & energy distribution; **spin** distribution

$$\langle N(p')\,|\,T^{\mu\nu}_{q,g}\,|\,N(p)\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}$$

$$+ D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)M\eta^{\mu\nu}\Big]u(p)$$

force & pressure distribution; **mass & pressure** distribution

$\bar{C}_q,\ \bar{C}_g$ trace anomaly for q/g part of energy-momentum tensor

$\bar{C}_q \sim \langle \bar{q}gq \rangle$ $\qquad\qquad$ $\bar{C}_g \sim \langle ggg \rangle$

$$\Delta^\mu\bar{u}(p')u(p)M\bar{C}_q(t) = \langle N(p')\,|\,\bar{\psi}igF^{\mu\nu}\gamma_\nu\psi\,|\,N(p)\rangle$$

$$\Delta^\mu\bar{u}(p',S')u(p,S)M\bar{C}_g(t) = \langle N(p')\,|\,F_a^{\mu\nu}iD^\rho_{ab}F^b_{\rho\nu}\,|\,N(p)\rangle$$

QCD EOMs $\quad (i\not{D} - m)\psi = 0, \quad D_\nu F^{\mu\nu} = g\bar{\psi}\gamma^\mu\psi$

$$\partial_\nu T^{\mu\nu}_q = -\bar{\psi}gF^{\mu\nu}\gamma_\nu\psi, \quad \partial_\nu T^{\mu\nu}_g = -F_a^{\mu\nu}D^\rho_{ab}F^b_{\rho\nu}$$

KT, PRD98, 034009 ('18)