

Studies of exotic-hadron candidates in high-energy reactions

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<https://indico3.cns.s.u-tokyo.ac.jp/event/315/>

May 29, 2024

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KEK-B
J-PARC
JLab/AMBER/EIC

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JLab/AMBER/EIC

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skip?

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JLab/EIC
Fermilab
NICA

6. Summary and prospects

References on our works

1. Exotic-hadron signature in fragmentation functions

M. Hirai, SK, M. Oka, and K. Sudoh, Phys. Rev. D77 (2008) 017504.

2. Constituent counting rule in perturbative QCD

H. Kawamura, SK, and T. Sekihara, Phys. Rev. D88 (2013) 034010;

W.-C. Chang, SK, and T. Sekihara, Phys. Rev. D93 (2016) 034006.

3. Internal structure of hadrons by GPDs

H. Kawamura and SK, Phys. Rev. D89 (2014) 054007;

SK, Qin-Tao Song, and O.V. Teryaev, Phys. Rev. D97 (2018) 014020;

SK, M. Strikman, and K. Sudoh, Phys. Rev. D80 (2009) 074003;

T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,

Phys. Rev. D93 (2016) 114034.

([Transition GPDs](#)) S. Diehl *et al.* (SK, 15th author), arXiv:2405.15386, Eur. Phys. J. A.

4. “Exotic” signatures in deuteron in DIS and Drell-Yan

W. Cosyn, Yu-Bing Dong, SK, and M. Sargsian, Phys. Rev. D95 (2017) 074036;

SK and Qin-Tao Song, Phys. Rev. D97 (2018) 014020; arXiv:1910.12523;

SK, J. Phys. Conf. Ser. 543 (2014) 012001; Phys. Rev. D94 (2016) 054022.

([Spin-1, short summary](#)) SK, to be submitted for Eur. Phys. J. A.

Introduction

Progress in exotic hadrons

$q\bar{q}$ Meson
 q^3 Baryon

$q^2\bar{q}^2$ Tetraquark
 $q^4\bar{q}$ Pentaquark
 q^6 Dibaryon

...
 $q^{10}\bar{q}$ e.g. Strange tribaryon

...
 gg Glueball
 ...

- $\Theta^+(1540)???:$ LEPS

$uudd\bar{s} ?$

Pentaquark?

- **Kaonic nuclei:** KEK-PS, ...
 Strange tribaryons, ...

$K^- pnn, K^- ppn ?$
 $K^- pp ?$

- **X (3872), Y(3940):** Belle
 Tetraquark, $D\bar{D}$ molecule

$c\bar{c}$
 $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
 $D^+(c\bar{d})D^-(\bar{c}d) ?$

- **$D_{sJ}(2317), D_{sJ}(2460):$** BaBar, CLEO, Belle
 Tetraquark, DK molecule

$c\bar{s}$
 $D^0(c\bar{u})K^+(u\bar{s})$
 $D^+(c\bar{d})K^0(d\bar{s}) ?$

- **Z (4430):** Belle
 Tetraquark, ...

$c\bar{c}u\bar{d}, D$ molecule?

- **$P_c(4380), T_{cc}(3875):$** LHCb

- ...
 $u\bar{c}udc : \bar{D}(u\bar{c})\Sigma_c^*(udc), \bar{D}^*(u\bar{c})\Sigma_c(udc)$ molecule?
 $cc\bar{u}\bar{d} : D^0(\bar{u}c)D^{*+}(d\bar{c})$ molecule?

Scalar mesons $J^P = 0^+$ at $M \sim 1$ GeV

based on my past experience

Naïve quark-model

$$\sigma = f_0(500) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$f_0(980) = s\bar{s} \rightarrow \text{denote } f_0 \text{ in this talk}$$

$$a_0(980) = u\bar{d}, \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

↕ contradiction

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

$a_1(1230)$

1.0 GeV

$a_0(980)$

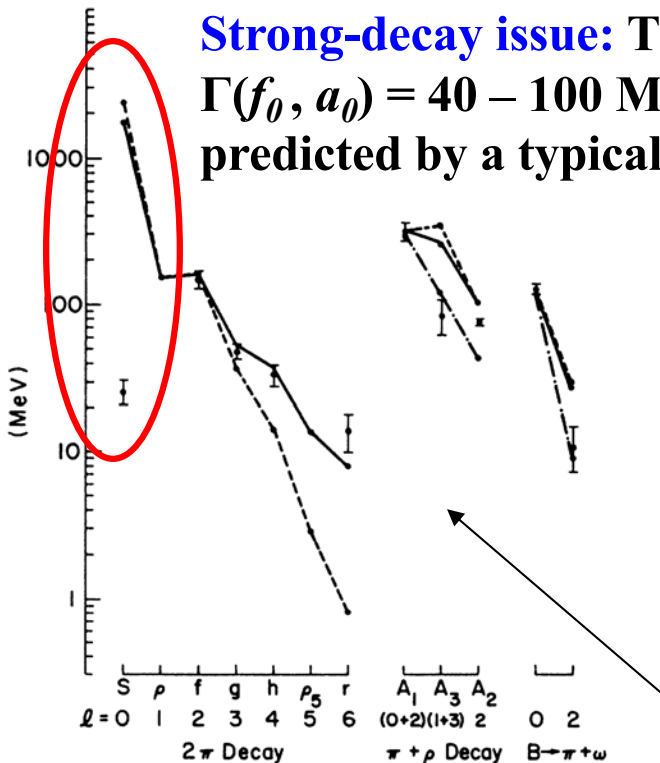
$f_0(980)$

$\rho(770)$

0.5 GeV

$f_0(500) = \sigma$

Strong-decay issue: The experimental widths $\Gamma(f_0, a_0) = 40 - 100$ MeV are too small to be predicted by a typical quark model.



These issues could be resolved

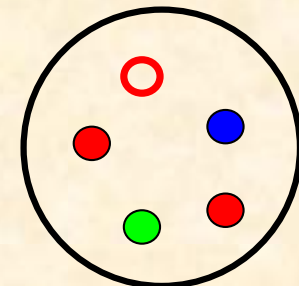
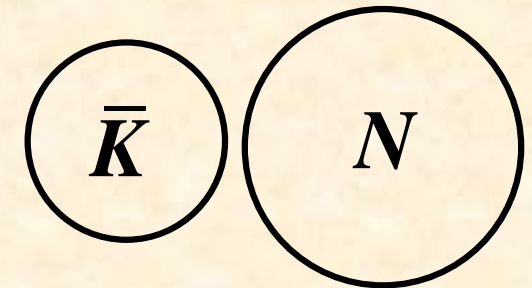
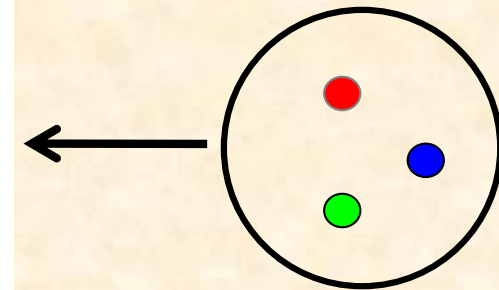
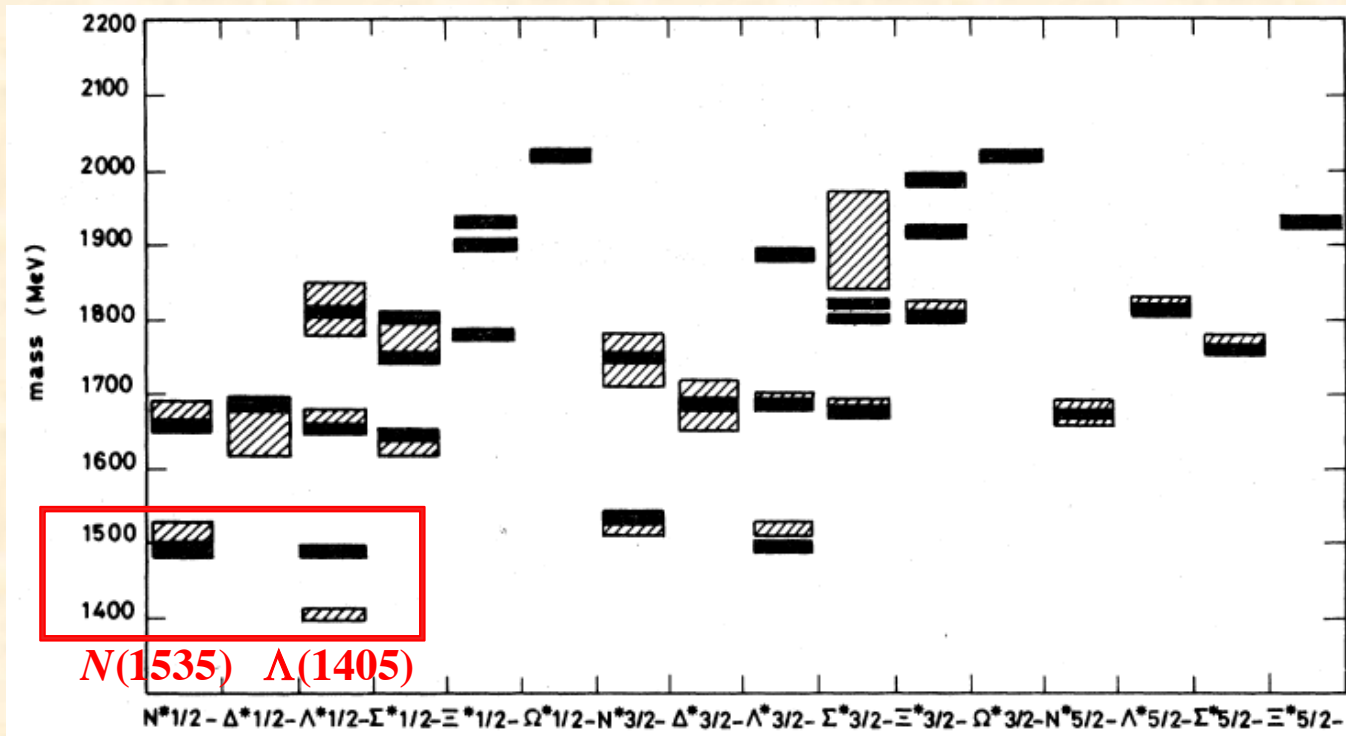
if f_0 is a tetraquark ($qq\bar{q}\bar{q}$) or a $K\bar{K}$ molecule, namely an "exotic" hadron.

Radiative decay: F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.

$\Lambda(1405)$: exotic hadron?

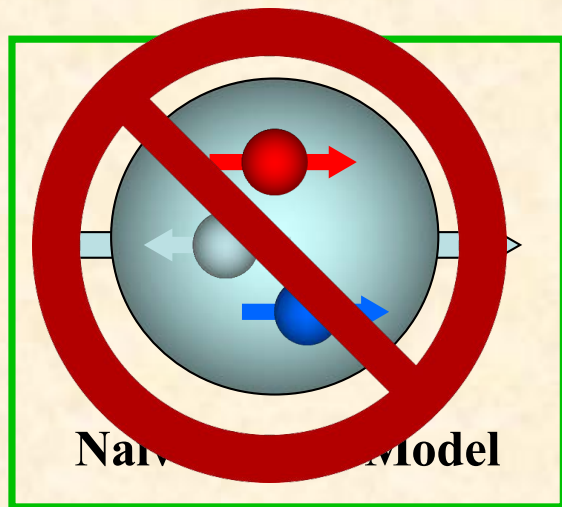
Negative-parity baryons
N. Isgur and G. Karl,
PRD 18 (1978) 4187.



Most spectra agree with the ones by a $3q$ -picture

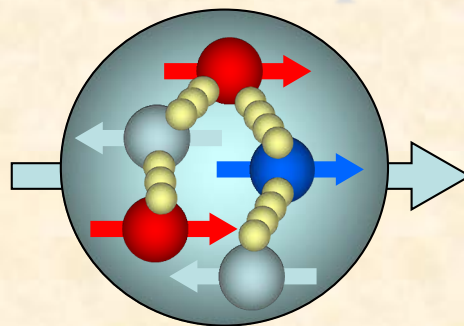
- Only $\Lambda(1405)$ deviates from the measurement.
- Difficult to understand the small mass of $\Lambda(1405)$ in comparison with $N(1535)$.
→ $\bar{K}N$ molecule or penta-quark ($qqqq\bar{q}$)?

Situation of tensor structure by b_1 for spin-1 deuteron

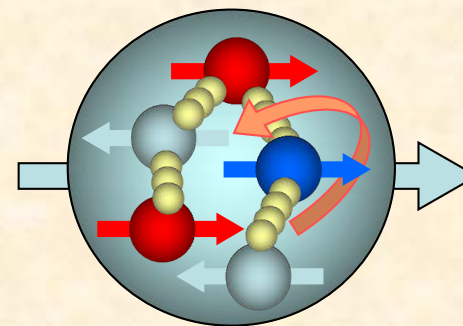


“old” standard model

Nucleon spin

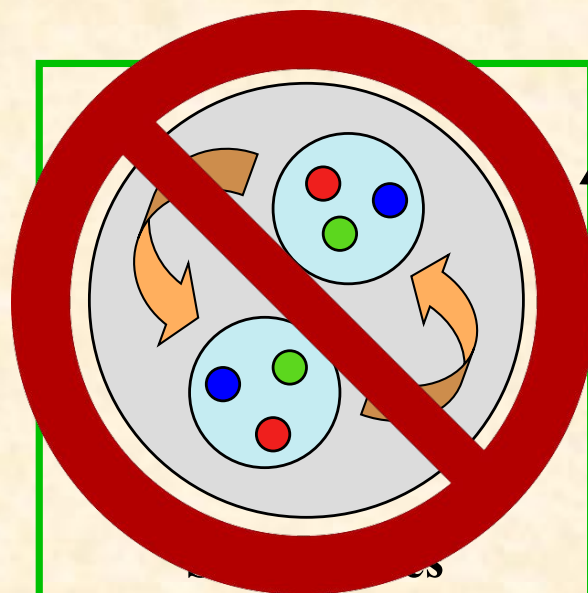
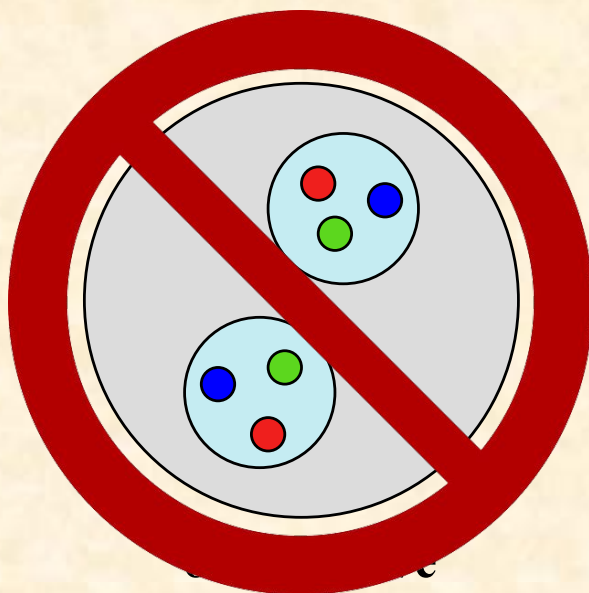


Nucleon spin crisis!?

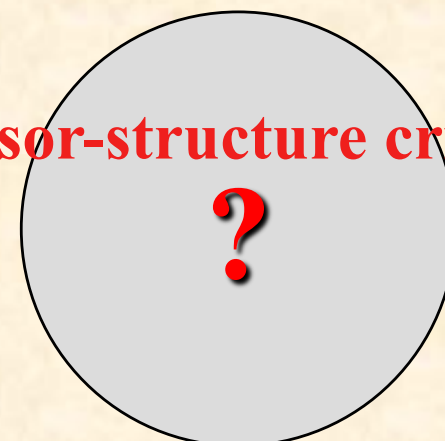


Tensor structure

We have shown in this work that the standard deuteron model does not work!?
→ new hadron physics?!



Tensor-structure crisis!?



b_1 experiment $\neq b_1$ “standard model”

Gluon transversity $\Delta_T g$

Note on our notations:

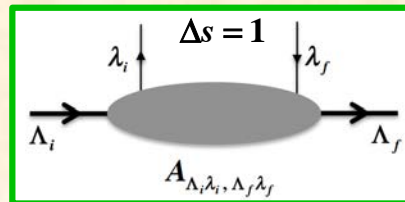
Tensor-polarized gluon distribution: $\delta_T g$

Gluon transversity: $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, +\frac{1}{2} + \frac{1}{2}\right) - A\left(+\frac{1}{2} - \frac{1}{2}, +\frac{1}{2} - \frac{1}{2}\right)$

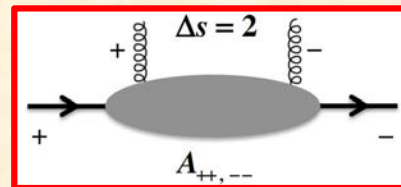
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right), \lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron:

$\Delta_T g(x) \sim A(+1+1, -1-1),$

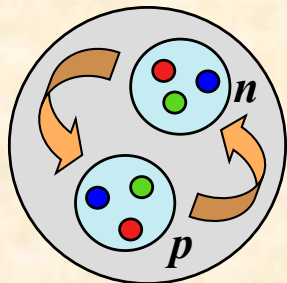
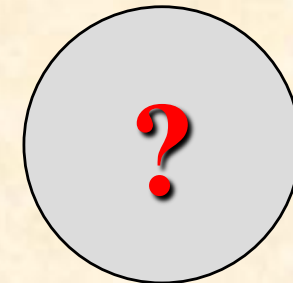
~~$A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$ not possible for nucleon~~



Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1 (\delta_T q, \delta_T g) \neq 0 \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?



S + D waves

Physics beyond “the standard model” in nuclear physics?
(Physics beyond the standard model in particle physics???)

Exotic hadrons by GPDs

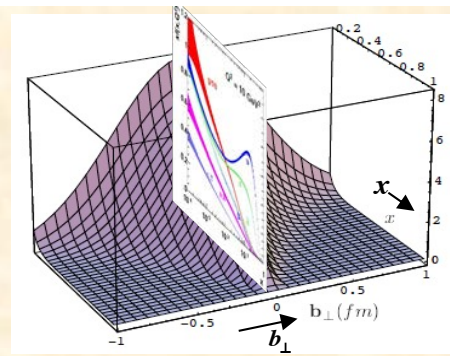
(Generalized Parton Distributions)

**H. Kawamura and SK,
Phys. Rev. D89 (2014) 054007.**

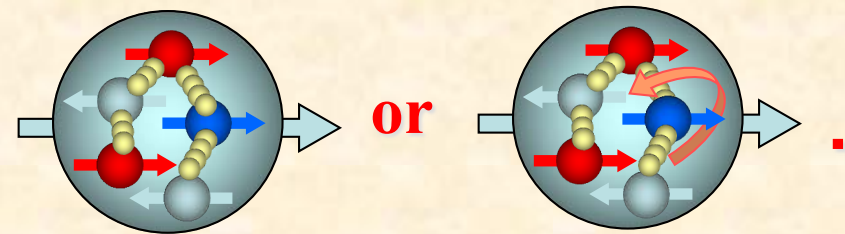
By hadron tomography



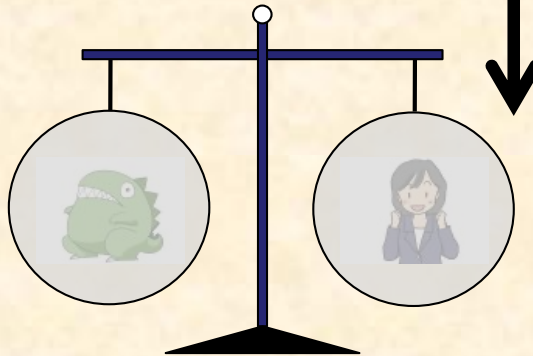
3D view
of hadrons



Origin of nucleon spin
By the tomography, we determine



Exotic hadrons



By tomography,
we determine

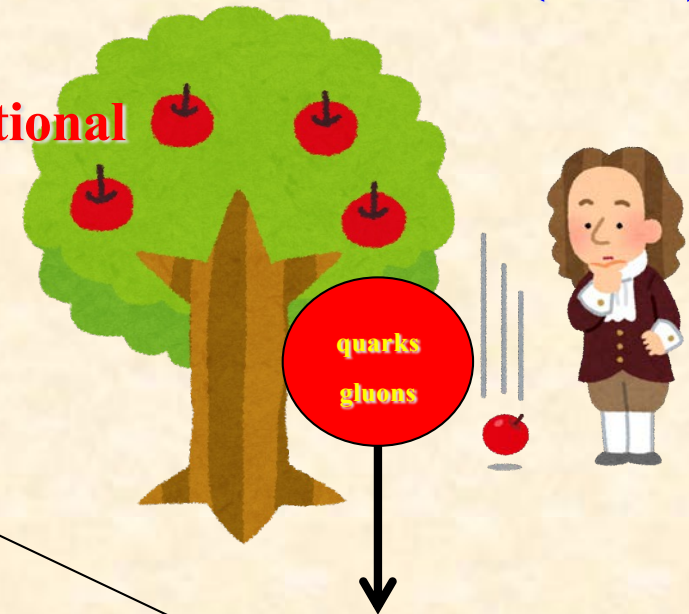


or

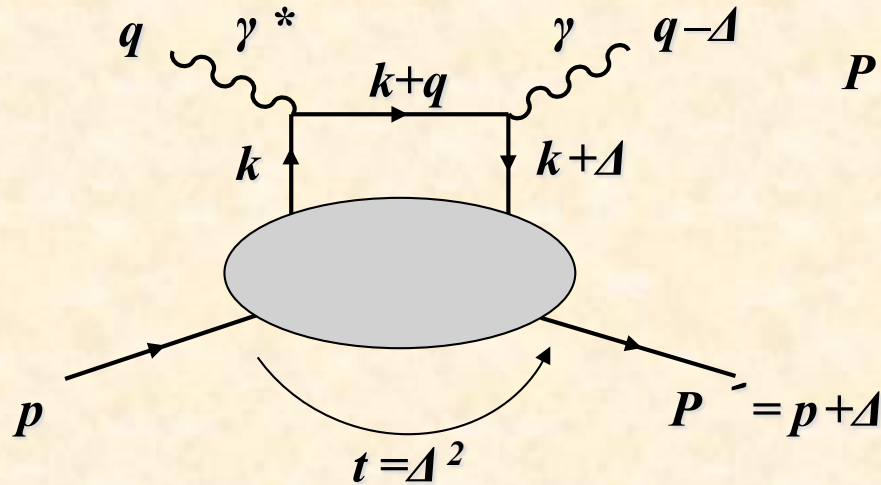


Origin of gravitational source (mass)

By tomography,
we determine gravitational
sources in terms of
quarks and gluons.



Generalized Parton Distributions (GPDs)



$$P = \frac{p^+ p'^+}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewness parameter $\xi = \frac{p'^+ - p^+}{p'^+ + p^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

\Rightarrow probe L_q , key quantity to solve the spin puzzle!

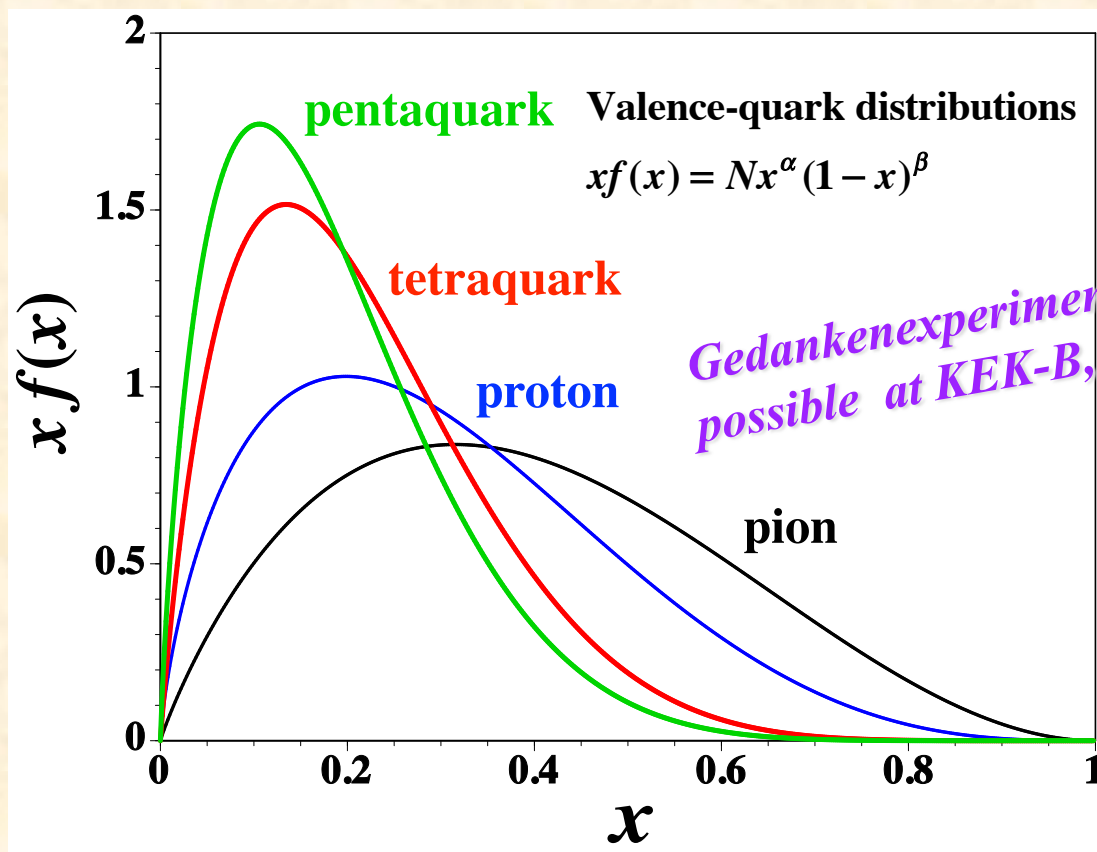
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

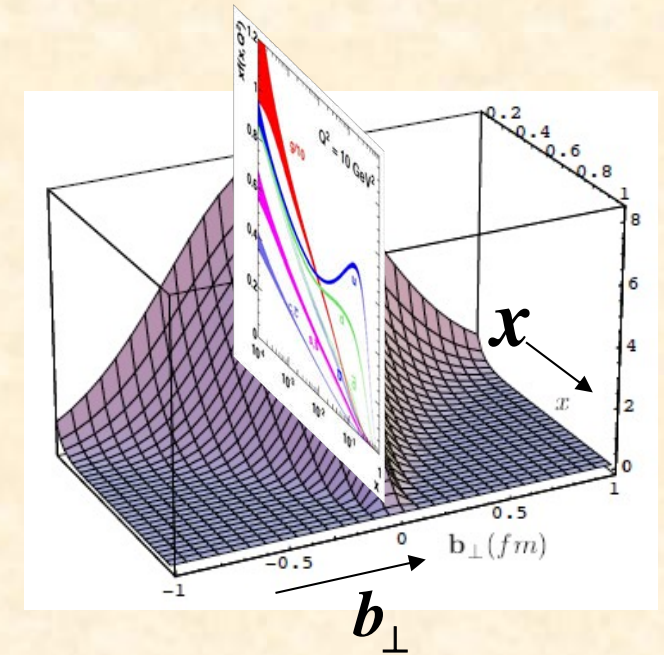
M. Guidal, M.V. Polyakov,
A.V. Radyushkin, M. Vanderhaeghen,
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent counting rule at $x \rightarrow 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx x f(x)$



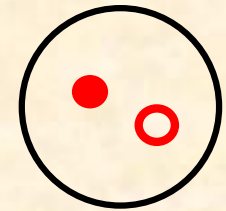
Gedankenexperiment, but possible at KEK-B, ILC, ...



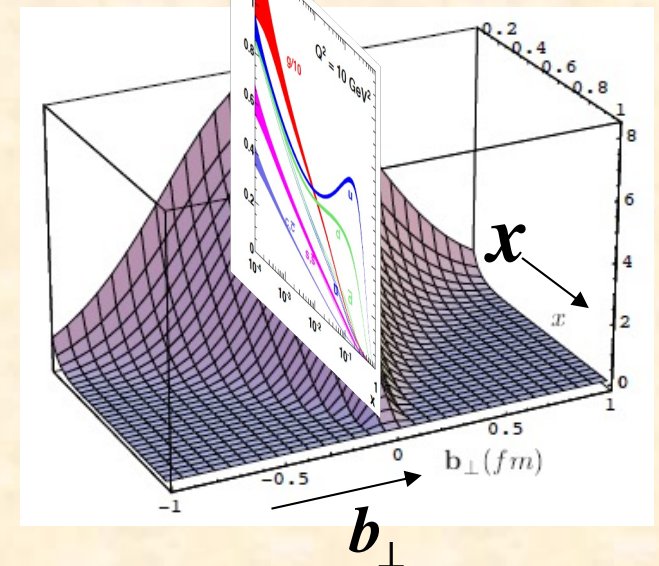
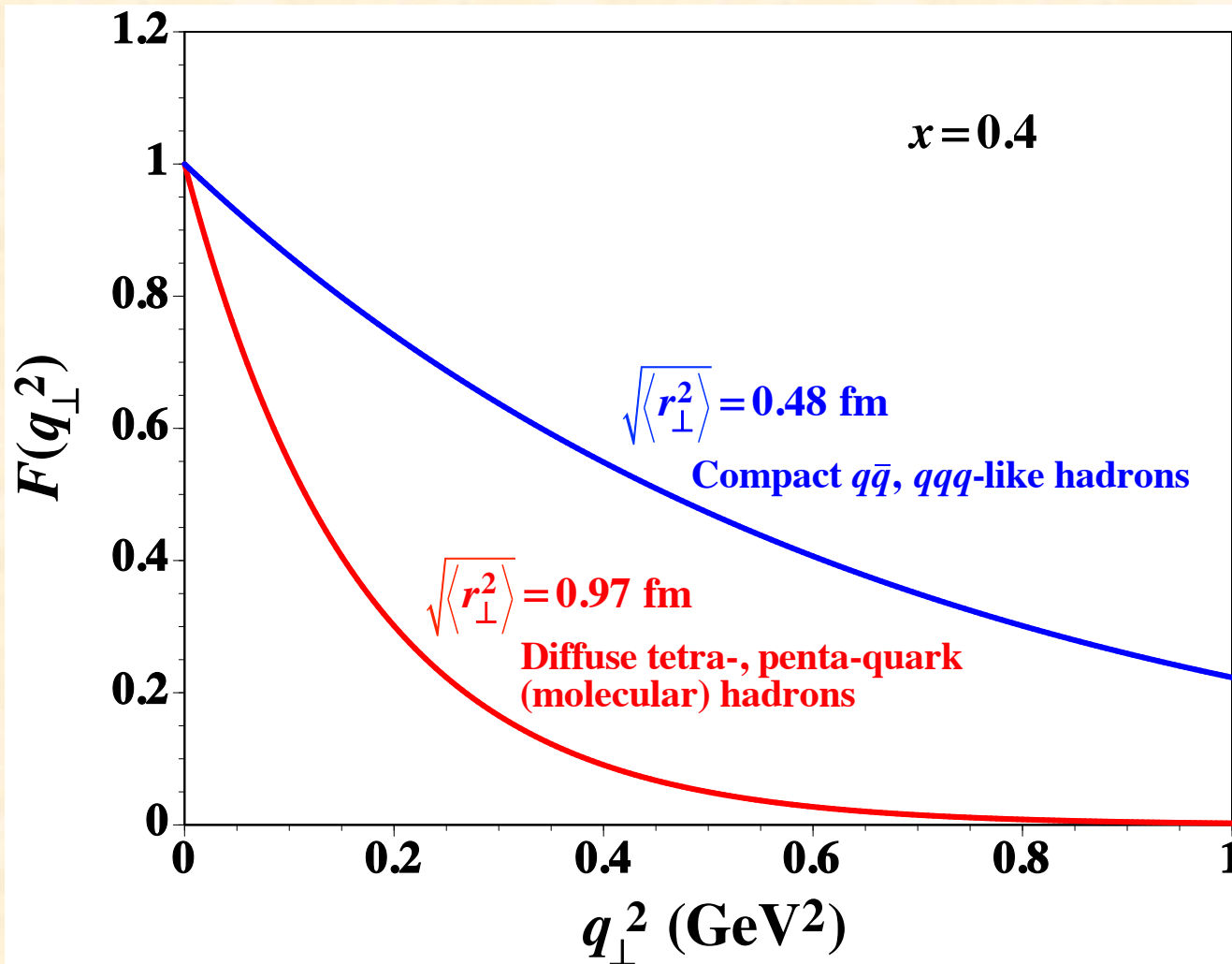
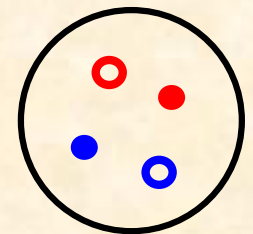
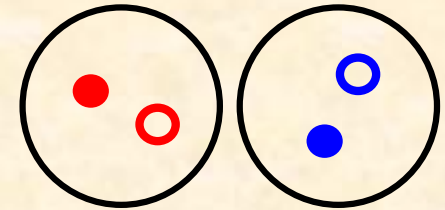
Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_{\perp}^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$

Ordinary $q\bar{q}$



Molecule $K\bar{K}$
or tetra-quark $qq\bar{q}\bar{q}$



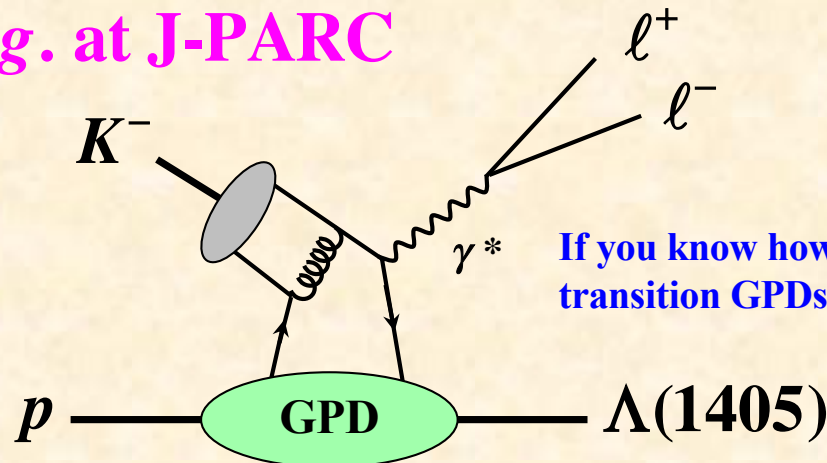
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed quantity = GDAs at KEKB, Linear Collider

e.g. at J-PARC

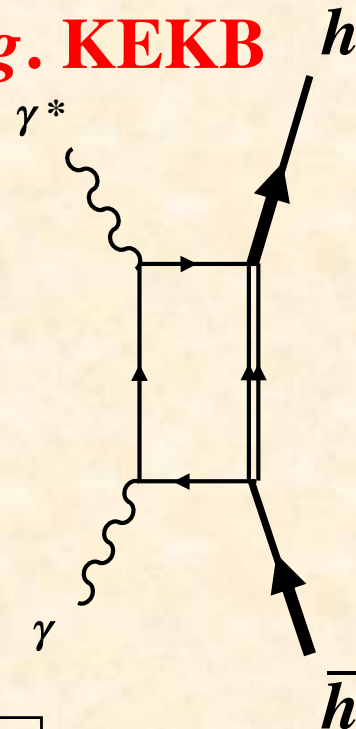


If you know how to handle this kind of transition GPDs $N \rightarrow \Lambda$, please inform me.

$$K^- (\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

e.g. KEKB



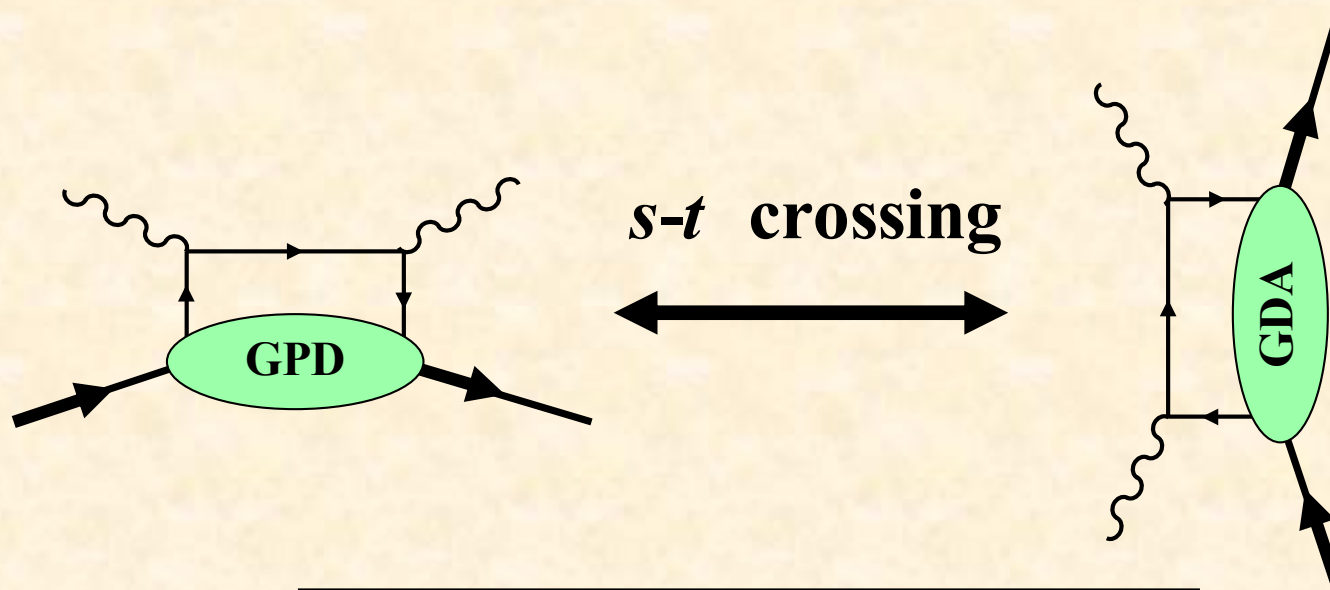
See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

Timelike GPDs

= Generalized Distribution Amplitudes (GDAs)
and extraction of gravitational form factors
from KEKB data

Spacelike GPDs

GDA = Timelike GPDs



SK, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

GPD $H_q^h(x, \xi, t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z, \zeta, W^2)$

GPD: $H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$

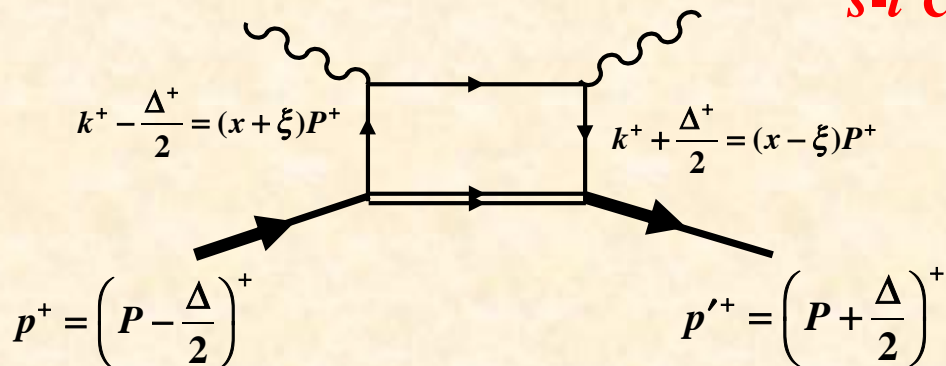
GDA: $\Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

$H_q^h(x, \xi, t)$

\longleftrightarrow
s-t crossing

$\Phi_q^{hh}(z, \zeta, W^2)$



$$z \Leftrightarrow \frac{1-x/\xi}{2}$$

$$\zeta \Leftrightarrow \frac{1-1/\xi}{2}$$

$$W^2 \Leftrightarrow t$$

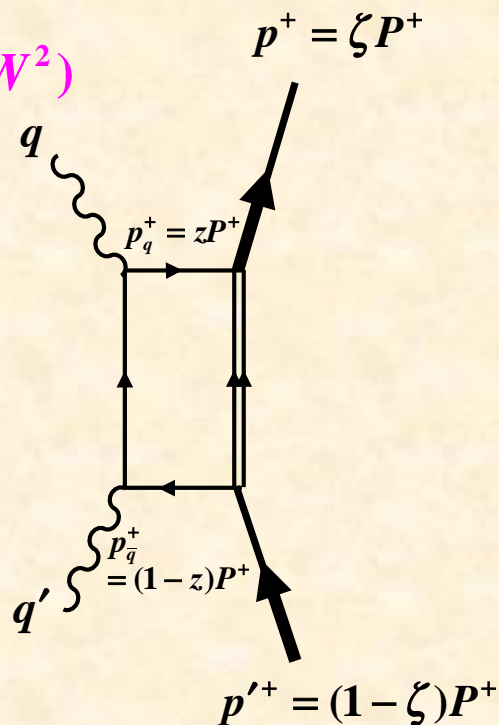
JLab / COMPASS

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$



KEKB

Bjorken variable for γ^* : $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1 + \beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i \int d^4\xi e^{-i\xi \cdot q} \langle \pi(p) \pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

$$\text{GDA (timelike GPD): } \Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) \pi(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle_{y^+=0, \vec{y}_\perp=0}$$

$$\frac{d\sigma}{d(\cos\theta)} \approx \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1) \zeta (1-\zeta) F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{[1 + (s - 4m_\pi^2) / \Lambda^2]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

- Resonances: There exist resonance contributions to the cross section.

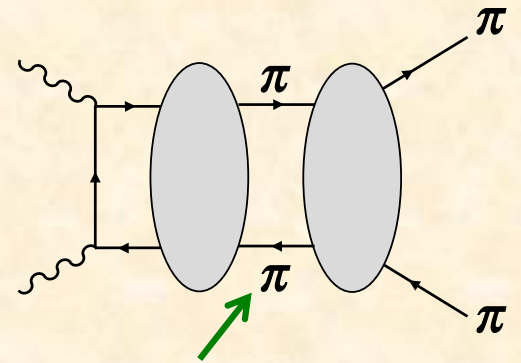
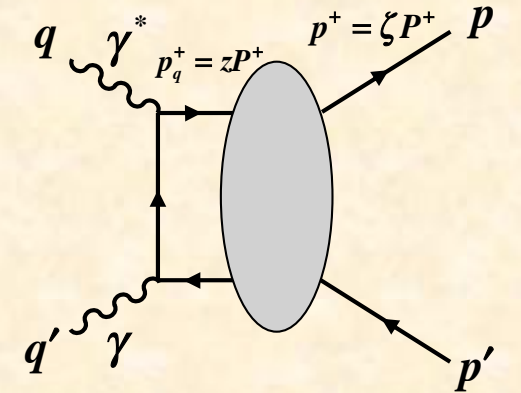
$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18N_f z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$\tilde{B}_{10}(W) = \text{resonance} [f_0(500), f_0(980)] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance} [f_2(1270)] + \text{continuum}$$

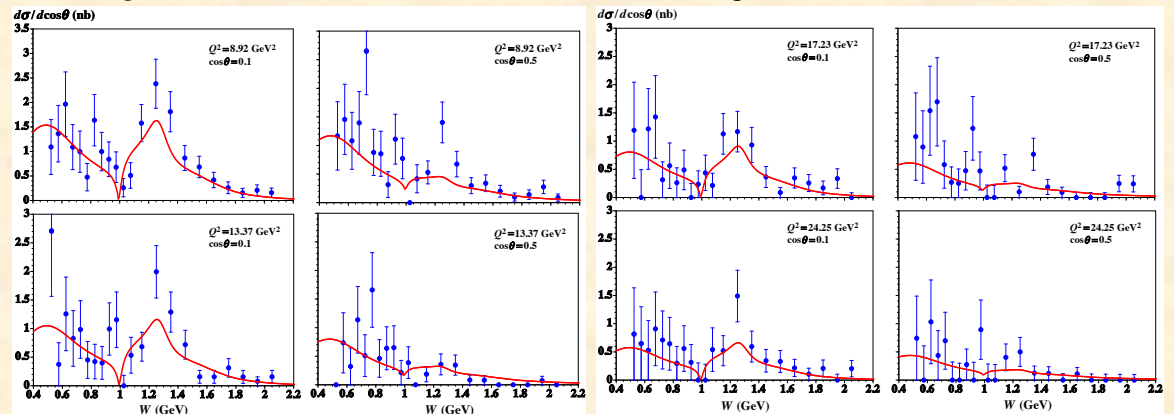
Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.



Including intermediate resonance contributions

$Q^2 = 8.92, 13.37 \text{ GeV}^2$

$Q^2 = 17.23, 24.25 \text{ GeV}^2$



Gravitational form factors and radii for pion

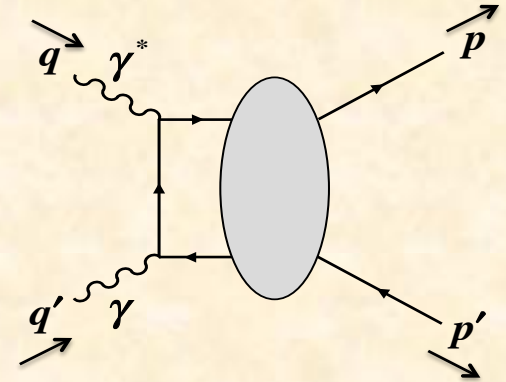
$$\int_0^1 dz (2z-1) \Phi_q^{\pi^0\pi^0}(z, \zeta, s) = \frac{2}{(P^+)^2} \langle \pi^0(p) \pi^0(p') | T_q^{++}(0) | 0 \rangle$$

$$\langle \pi^0(p) \pi^0(p') | T_q^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[(s g^{\mu\nu} - P^\mu P^\nu) \Theta_{1,q}(s) + \Delta^\mu \Delta^\nu \Theta_{2,q}(s) \right]$$

$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}, \Theta_{2,q}$: gravitational form factors for pion



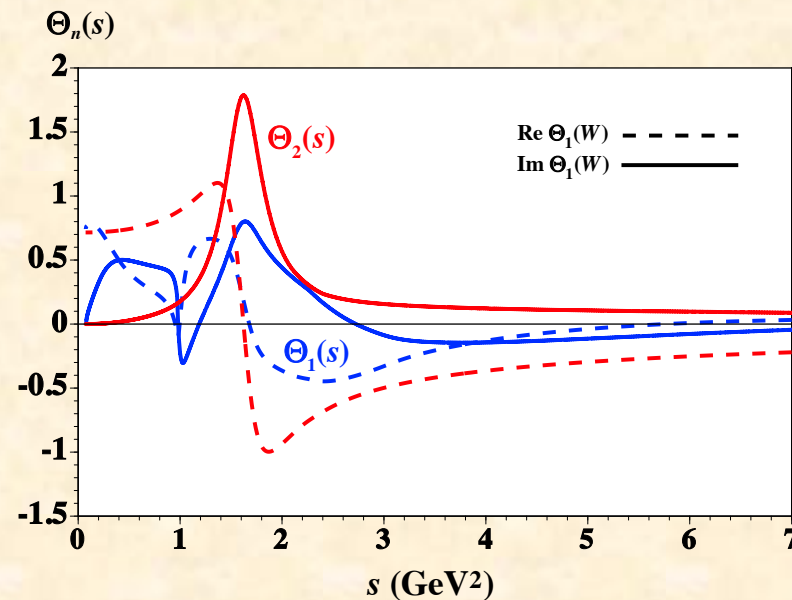
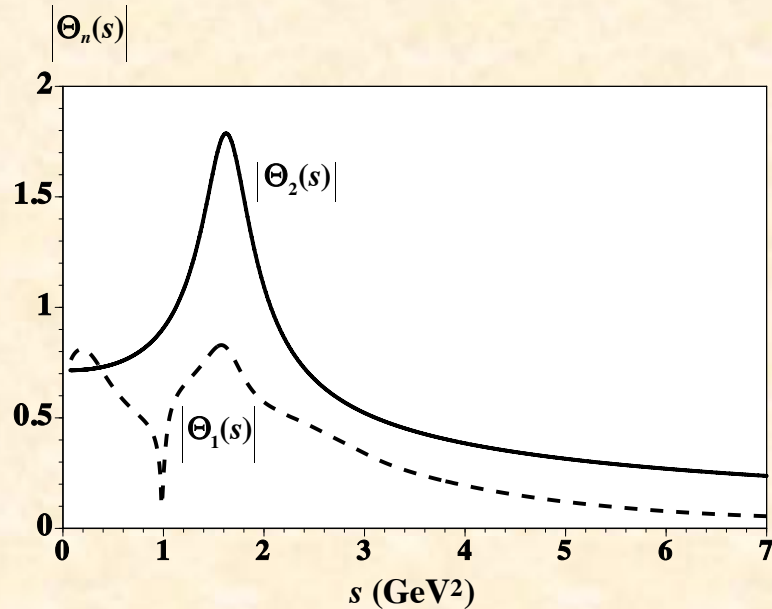
See also Hyeon-Dong Son,
Hyun-Chul Kim, PRD90 (2014) 111901.

Analysis of $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ cross section

- ⇒ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z, \zeta, s)$
- ⇒ Timelike gravitational form factors $\Theta_{1,q}(s), \Theta_{2,q}(s)$
- ⇒ Spacelike gravitational form factors $\Theta_{1,q}(t), \Theta_{2,q}(t)$
- ⇒ Gravitational radii of pion

Gravitational form factors:

- Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.
- Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009;
- Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;
- K. Tanaka, JHEP 01 (2019) 120.



Spacelike gravitational form factors and radii for pion

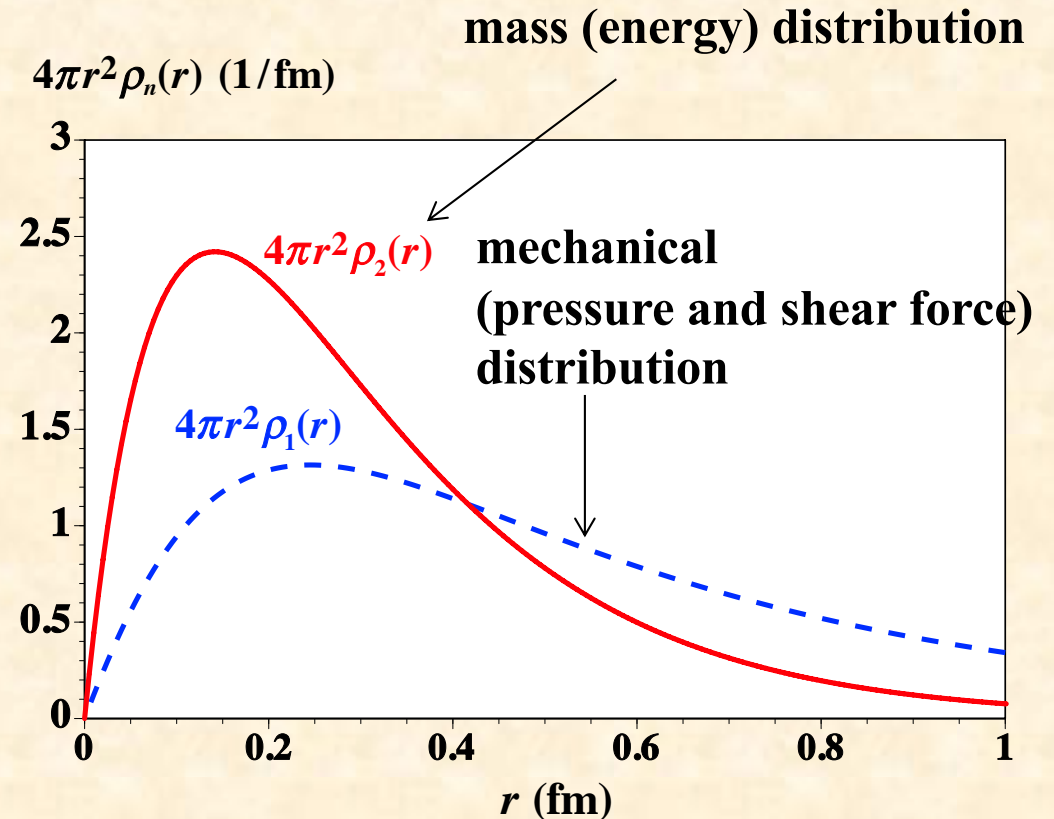
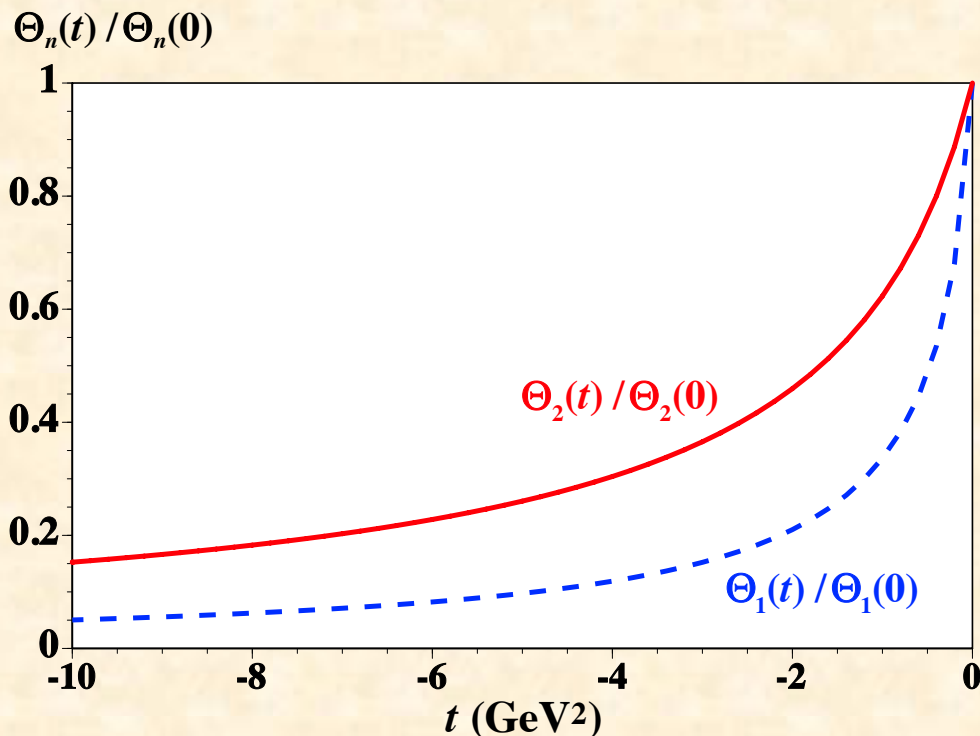
$$F(s) = \Theta_1(s), \Theta_1(s), \quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im}F(s)}{\pi(s-t-i\epsilon)}, \quad \rho(r) = \frac{1}{(2\pi)^3} \int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2} \frac{1}{r} \int_{4m_\pi^2}^{\infty} ds e^{-\sqrt{s}r} \text{Im}F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

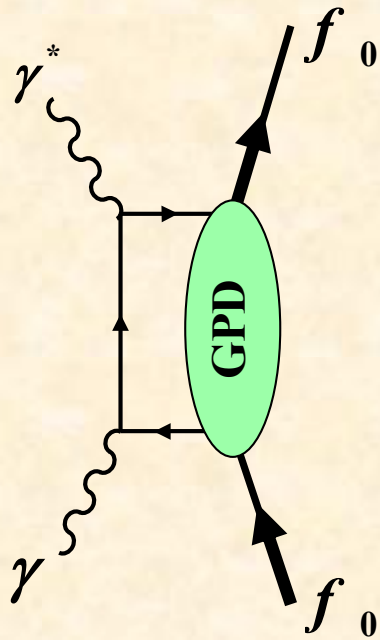
$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

First finding on gravitational radius from actual experimental measurements

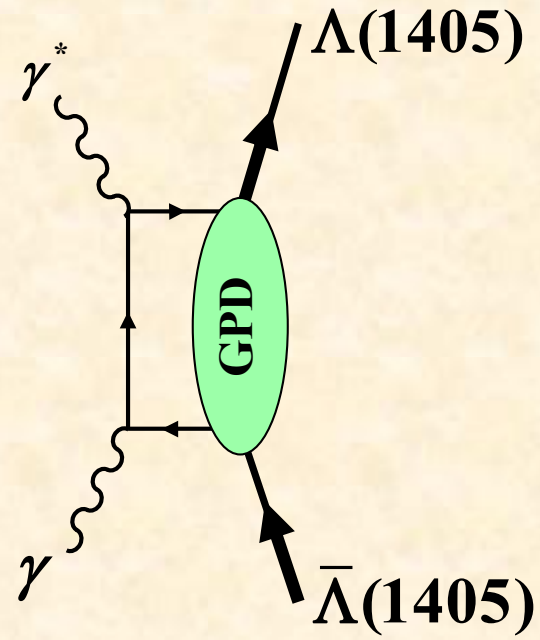
$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$



Timelike GPDs for exotic hadrons



Possible at super-KEKB?



Difficult even at super-KEKB?

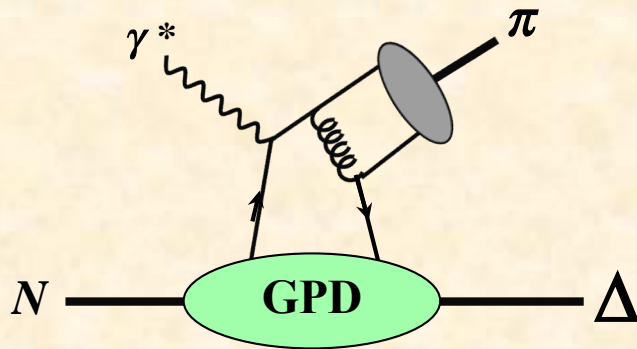
Transition GPDs for exotic hadrons

S. Diehl *et al.* (SK, 15th author),

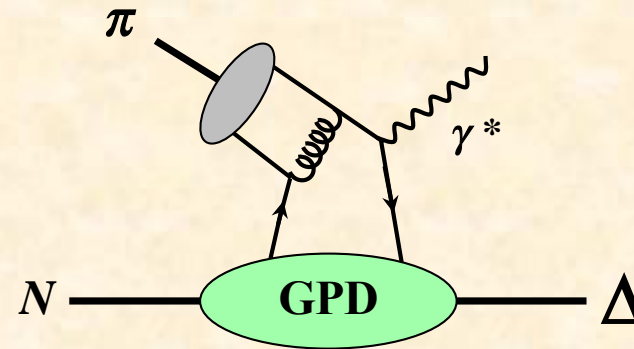
arXiv:2405.15386, submitted for Eur. Phys. J. A

Transition GPDs from N to Δ

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J-PARC

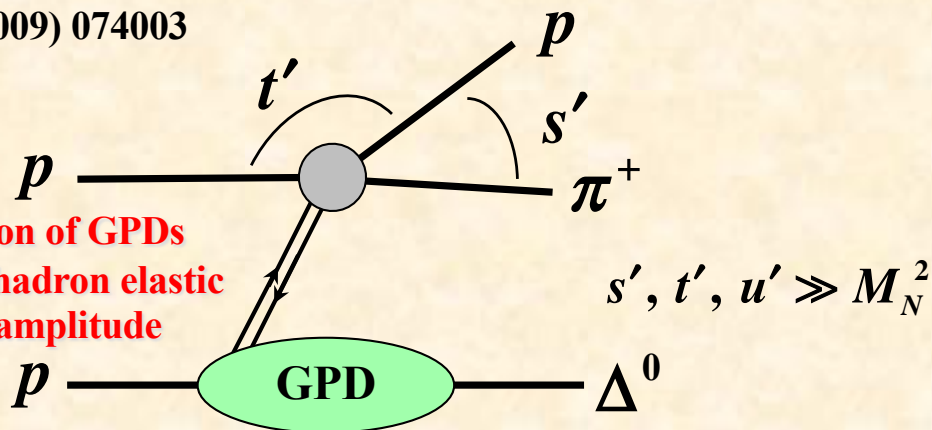


In future

$$K^- + p \rightarrow \Lambda_{1405} + \gamma^* ?$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003

Investigation of GPDs
with $2 \rightarrow 3$ hadron elastic
scattering amplitude



J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$$\pi + N \rightarrow \gamma + \gamma + N'$$

$$h + M_B \rightarrow h' + \gamma + M_D$$

$$h + M_B \rightarrow h' + M_C + M_D$$

$N \rightarrow \Delta$ transition GPDs

A. V. Belitsky, A. V. Radyushkin, Phys. Rept. 418 (2005) 1;

P. Kroll, K. Passek-Kumericki, Phys. Rev. D 107 (2023) 054009;

S. Diehl *et al.*, arXiv:2405.15386, submitted to Eur. Phys. J.

$$\begin{aligned} & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP \cdot z} \left\langle \Delta^{++}(p', \lambda') \left| \bar{u} \left(-\frac{1}{2}z \right) \gamma^+ d \left(\frac{1}{2}z \right) \right| p(p, \lambda) \right\rangle_{z^+=0, \bar{z}_\perp=0} \\ &= \frac{1}{2P^+} \bar{u}_\delta(p', \lambda') \left[\frac{\Delta^\delta n^\mu - \Delta^\mu n^\delta}{M_N} \left\{ \gamma_\mu G_1(x, \xi, t) + \frac{P_\mu}{M_N} G_2(x, \xi, t) + \frac{\Delta_\mu}{M_N} G_3(x, \xi, t) \right\} + \frac{\Delta^+ \Delta^\delta}{M_N^2} G_4(x, \xi, t) \right] \gamma_5 u(p, \lambda) \end{aligned}$$

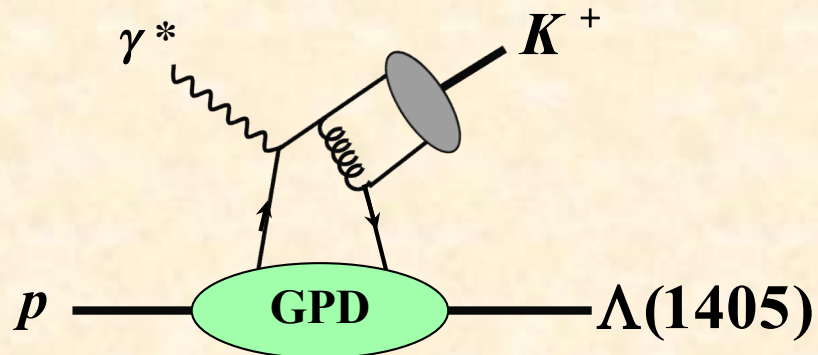
$$\begin{aligned} & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP \cdot z} \left\langle \Delta^{++}(p', \lambda') \left| \bar{u} \left(-\frac{1}{2}z \right) \gamma^+ \gamma^5 d \left(\frac{1}{2}z \right) \right| p(p, \lambda) \right\rangle_{z^+=0, \bar{z}_\perp=0} \\ &= \frac{1}{2P^+} \bar{u}_\delta(p', \lambda') \left[\frac{\Delta^\delta n^\mu - \Delta^\mu n^\delta}{M_N} \left\{ \gamma_\mu \tilde{G}_1(x, \xi, t) + \frac{P_\mu}{M_N} \tilde{G}_2(x, \xi, t) + \frac{\Delta_\mu}{M_N} \tilde{G}_3(x, \xi, t) \right\} + \frac{\Delta^+ \Delta^\delta}{M_N^2} \tilde{G}_4(x, \xi, t) \right] u(p, \lambda) \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP \cdot z} \left\langle \Delta^{++}(p', \lambda') \left| \bar{u} \left(-\frac{1}{2}z \right) i\sigma^{+j} d \left(\frac{1}{2}z \right) \right| p(p, \lambda) \right\rangle_{z^+=0, \bar{z}_\perp=0} \\ &= \frac{1}{2P^+} \bar{u}_\delta(p', \lambda') \left[\frac{P^\delta}{M_N} i\sigma^{+j} G_{1T}(x, \xi, t) + p^\delta \frac{P^+ \Delta^j - \Delta^+ P^j}{M_N^3} G_{2T}(x, \xi, t) + p^\delta \frac{\gamma^+ \Delta^j - \Delta^+ \gamma^j}{2M_N^2} G_{3T}(x, \xi, t) \right. \\ & \quad \left. + p^\delta \frac{\gamma^+ P^j - P^+ \gamma^j}{2M_N^2} G_{4T}(x, \xi, t) + (n^\delta \gamma^j - \gamma^\delta n^j) G_{5T}(x, \xi, t) + \frac{n^\delta \Delta^j - \Delta^\delta n^j}{M_N} G_{6T}(x, \xi, t) \right] \gamma_5 u(p, \lambda) \\ & \quad + \frac{1}{2P^+} \left[\left\{ \bar{u}^+(p', \lambda') \gamma^j - \bar{u}^j(p', \lambda') \gamma^+ \right\} G_{7T}(x, \xi, t) + \frac{\bar{u}^+(p', \lambda') \Delta^j - \bar{u}^j(p', \lambda') \Delta^+}{M_N} G_{8T}(x, \xi, t) \right] \gamma_5 u(p, \lambda) \end{aligned}$$

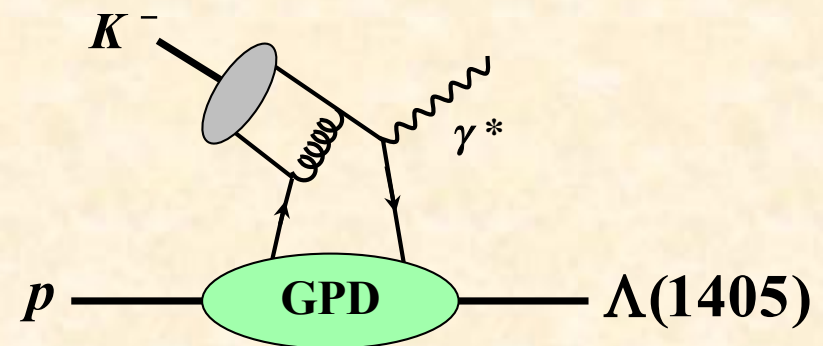
Rarita-Schwinger: $u^\alpha(p, \lambda) = \sum_{\rho, \sigma} \left\langle 1\rho; \frac{1}{2}\sigma \left| \frac{3}{2}\lambda \right\rangle \varepsilon^\alpha(p, \rho) u(p, \sigma)$

Transition GPDs for exotic hadrons

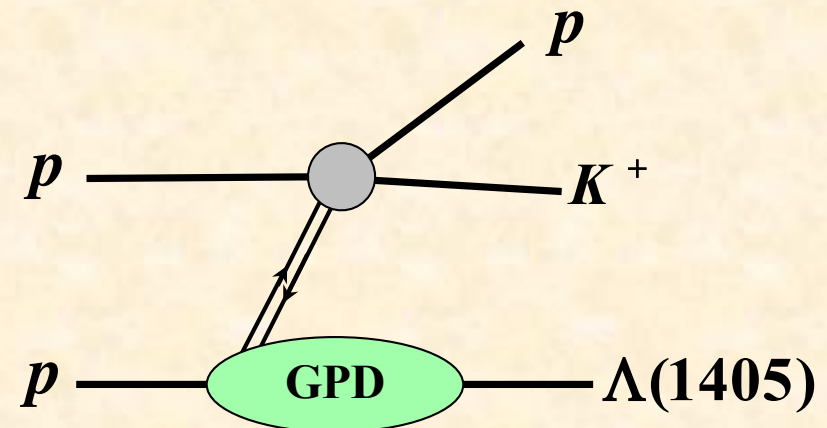
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However, there is no theoretical study on the $N \rightarrow \Lambda(1405)$ transition GPDs at this stage.



Constituent-counting rule for exotic hadrons

**H. Kawamura, S. Kumano, T. Sekihara,
Phys. Rev. 88 (2013) 034010.**

Research purposes

It is not easy to find undoubted evidence for exotic hadrons by global observables (mass, spin, parity, decay width) at low energies.

(1) Determination of internal structure of exotic hadrons by high energy processes, where quark-gluon degrees of freedom appear.

Constituent-counting rule could be used because it counts internal constituents.

(2) Investigation on transition from hadron degrees of freedom to quark-gluon degrees of freedom for exotic hadrons.

$$\frac{d\sigma_{a+b \rightarrow c+d}}{dt} \simeq \frac{1}{16\pi s^2} \sum_{pol} |M_{a+b \rightarrow c+d}|^2 \Rightarrow \frac{d\sigma_{a+b \rightarrow c+d}}{dt} = \frac{1}{s^{n-2}} f_{a+b \rightarrow c+d}(t/s) \quad \text{constituent-counting rule}$$
$$n = n_a + n_b + n_c + n_d$$

Constituent-counting rule in perturbative QCD: Form factor

Consider the magnetic form factor of the proton

$$\langle p' | J^\mu | p \rangle \approx \bar{u}(p') \gamma^\mu G_M(Q^2) u(p) \text{ at } Q^2 = -q^2 \gg m_N^2$$

$$G_M(Q^2) = \int d[x] d[y] \phi_p([y]) H_M([x], [y], Q^2) \phi_p([x])$$

ϕ_p = proton distribution amplitude, H_M = hard amplitude (calculated in pQCD)

In the Breit frame with $q = (0, \vec{q})$, $|\vec{p}| = |\vec{p}'| \equiv P \sim O(Q)$.

$u^\dagger u = 2E \Rightarrow$ External quark line: $u \sim \sqrt{P} \sim \sqrt{Q}$

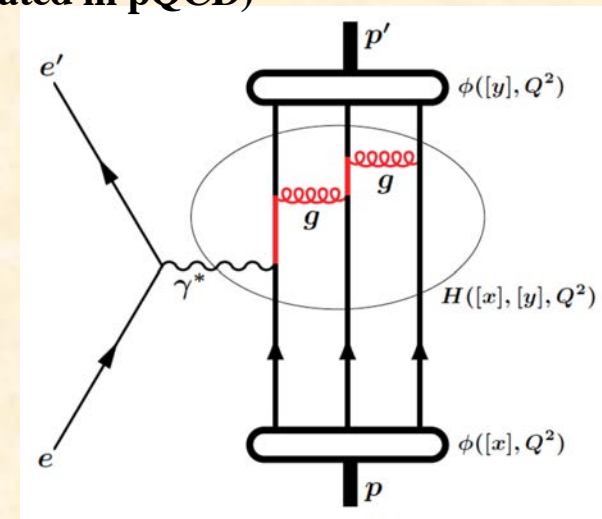
$$\langle p' | J^\mu | p \rangle \approx \bar{u}(p') \gamma^\mu G_M(Q^2) u(p) \sim (\sqrt{Q})^2 G_M(Q^2)$$

- Two quark propagators: $\frac{1}{Q^2}$
- Two gluon propagators: $\frac{1}{(Q^2)^2}$
- Six external quark lines: $(\sqrt{Q})^6$

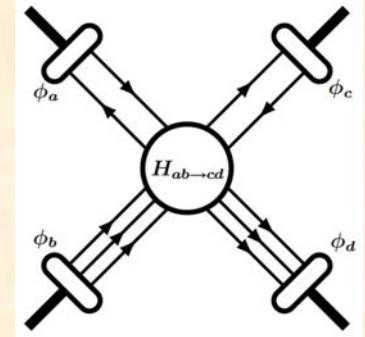
$$\langle p' | J^\mu | p \rangle \sim \frac{1}{Q^2} \frac{\alpha_s(Q^2)}{(Q^2)^2} (\sqrt{Q})^6 = \frac{\alpha_s(Q^2)}{(Q^2)^{3/2}}$$

$$\Rightarrow G_M(Q^2) \sim \frac{1}{(\sqrt{Q})^2} \langle p' | J^\mu | p \rangle \sim \frac{1}{(Q^2)^{1/2}} \frac{\alpha_s(Q^2)}{(Q^2)^{3/2}} = \frac{\alpha_s(Q^2)}{(Q^2)^2} \sim \frac{1}{t^{n_N-1}}, \quad n_N = 3, \quad -t = Q^2$$

Counting rule for the form factor: $G_M(Q^2) \sim \frac{1}{t^{n_N-1}}, \quad n_N = 3$



Constituent-counting rule in perturbative QCD: Hard exclusive processes $a + b \rightarrow c + d$



Consider the hard exclusive hadron reaction $a + b \rightarrow c + d$

$$M_{ab \rightarrow cd} = \int d[x_a] d[x_b] d[x_c] d[x_d] \phi_c([x_c]) \phi_d([x_d]) H_M([x_a], [x_b], [x_c], [x_d], Q^2) \phi_a([x_a]) \phi_b([x_b])$$

ϕ_p = proton distribution amplitude, H_M = hard amplitude (calculated in pQCD)

Rule for estimating $M_{ab \rightarrow cd}$

- (1) Feynman diagram: Draw leading and connected Feynman diagram by connecting $n/2$ quark lines by gluons.
- (2) Gluon propagators: The factor $1/P^2$ is assigned for each gluon propagator.

There are $n/2 - 1$ gluon propagators $\sim 1/(P^2)^{n/2-1}$.

- (3) Quark propagators: The factor $1/P$ is assigned for each quark propagator.

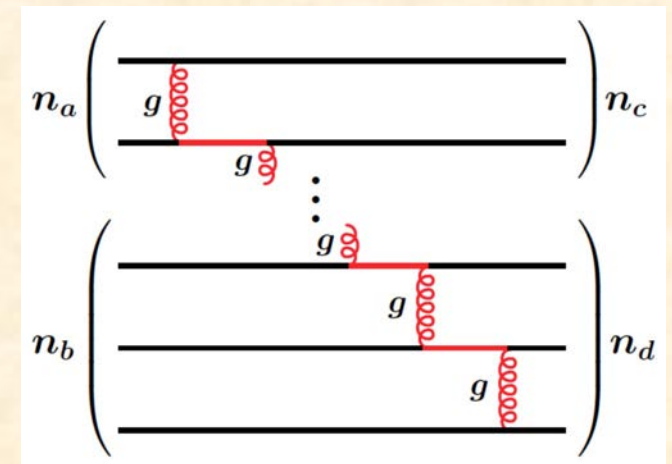
There are $n/2 - 2$ quark propagators $\sim 1/(P)^{n/2-2}$.

- (4) External quarks: The factor \sqrt{P} is assigned for each external quark.

There are n external quarks $\sim (\sqrt{P})^n$.

$$M_{ab \rightarrow cd} \sim \frac{1}{(P^2)^{n/2-1}} \frac{1}{(P)^{n/2-2}} (\sqrt{P})^n = \frac{(P)^{n/2}}{(P)^{n-2} (P)^{n/2-2}} = \frac{1}{(P)^{n-4}} \sim \frac{1}{s^{n/2-2}}$$

Cross section: $\frac{d\sigma_{ab \rightarrow cd}}{dt} \simeq \frac{1}{16\pi^2} \sum_{\text{spol}} |M_{ab \rightarrow cd}|^2 \sim \frac{1}{s^{n-2}}$



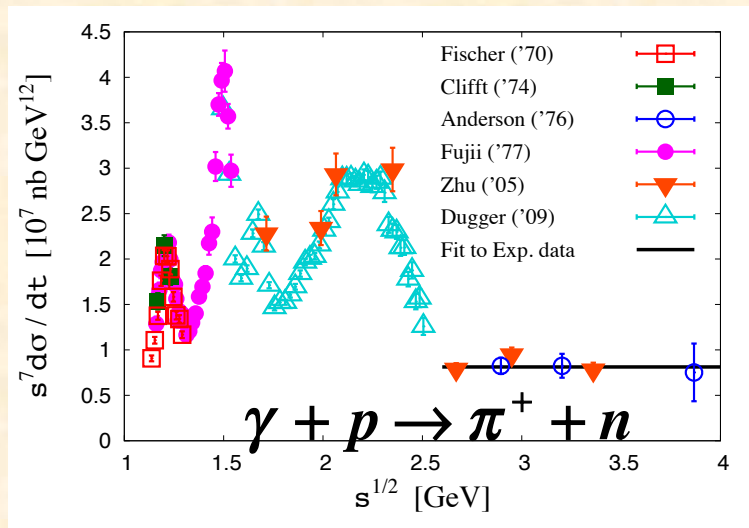
Constituent-counting rule, Transition from hadron degrees of freedom to quark-gluon ones

Typical current situation

- Transition from hadron d.o.f to quark d.o.f.
- (Looks like) Constituent-counting scaling

BNL experiment

C. White et al., PRD 49 (1994) 58.



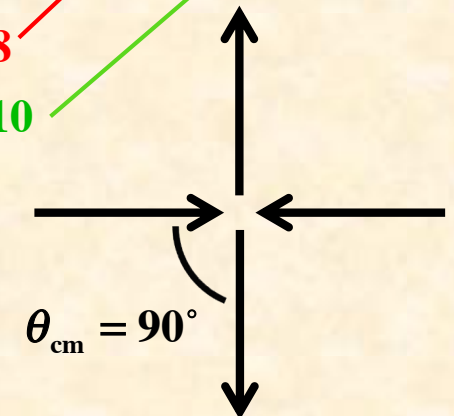
No.	Interaction	Cross section		$n-2$ ($\frac{d\sigma}{dt} \sim 1/s^{n-2}$)
		E838	E755	
1	$\pi^+ p \rightarrow p\pi^+$	132 ± 10	4.6 ± 0.3	6.7 ± 0.2
2	$\pi^- p \rightarrow p\pi^-$	73 ± 5	1.7 ± 0.2	7.5 ± 0.3
3	$K^+ p \rightarrow pK^+$	219 ± 30	3.4 ± 1.4	$8.3^{+0.6}_{-1.0}$
4	$K^- p \rightarrow pK^-$	18 ± 6	0.9 ± 0.9	≥ 3.9
5	$\pi^+ p \rightarrow p\rho^+$	214 ± 30	3.4 ± 0.7	8.3 ± 0.5
6	$\pi^- p \rightarrow p\rho^-$	99 ± 13	1.3 ± 0.6	8.7 ± 1.0
13	$\pi^+ p \rightarrow \pi^+ \Delta^+$	45 ± 10	2.0 ± 0.6	6.2 ± 0.8
15	$\pi^- p \rightarrow \pi^- \Delta^-$	24 ± 5	≤ 0.12	≥ 10.1
17	$pp \rightarrow pp$	3300 ± 40	48 ± 5	9.1 ± 0.2
18	$\bar{p}p \rightarrow \bar{p}p$	75 ± 8	≤ 2.1	≥ 7.5

JLab: L.Y. Zhu *et al.*, PRL 91, 022003 (2003);
 PRC 71, 044603 (2005);
 W. Chen *et al.*, PRL 103, 012301 (2009).

$$n-2: (2+3+2+3)-2=8$$

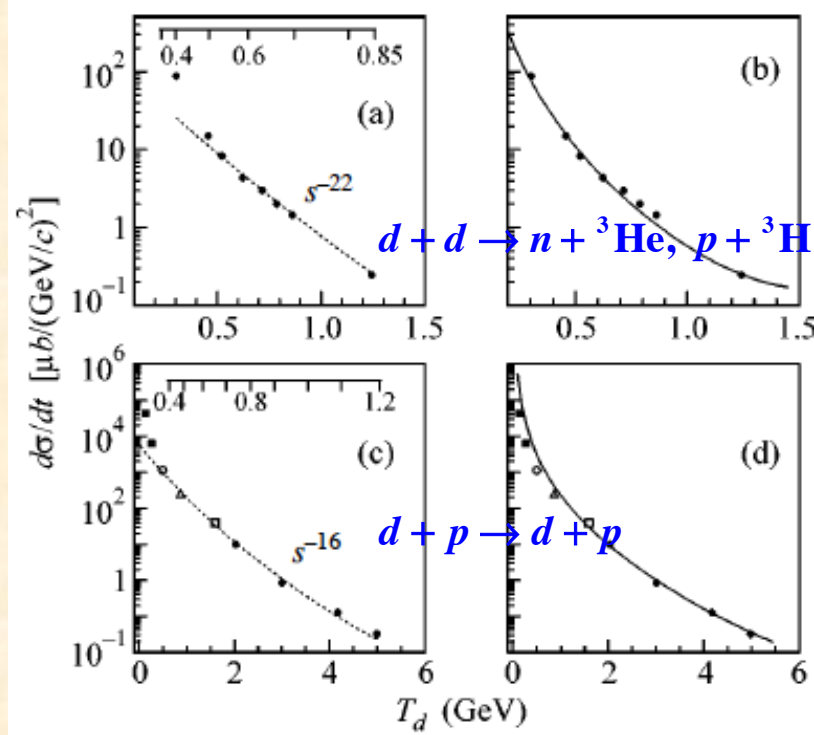
$$(3+3+3+3)-2=10$$

see R. A. Schumacher and M. M. Sargsian,
 PRC 83 (2011) 025207 for hyperon production

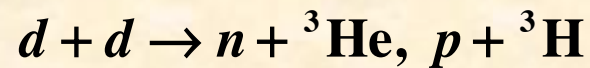
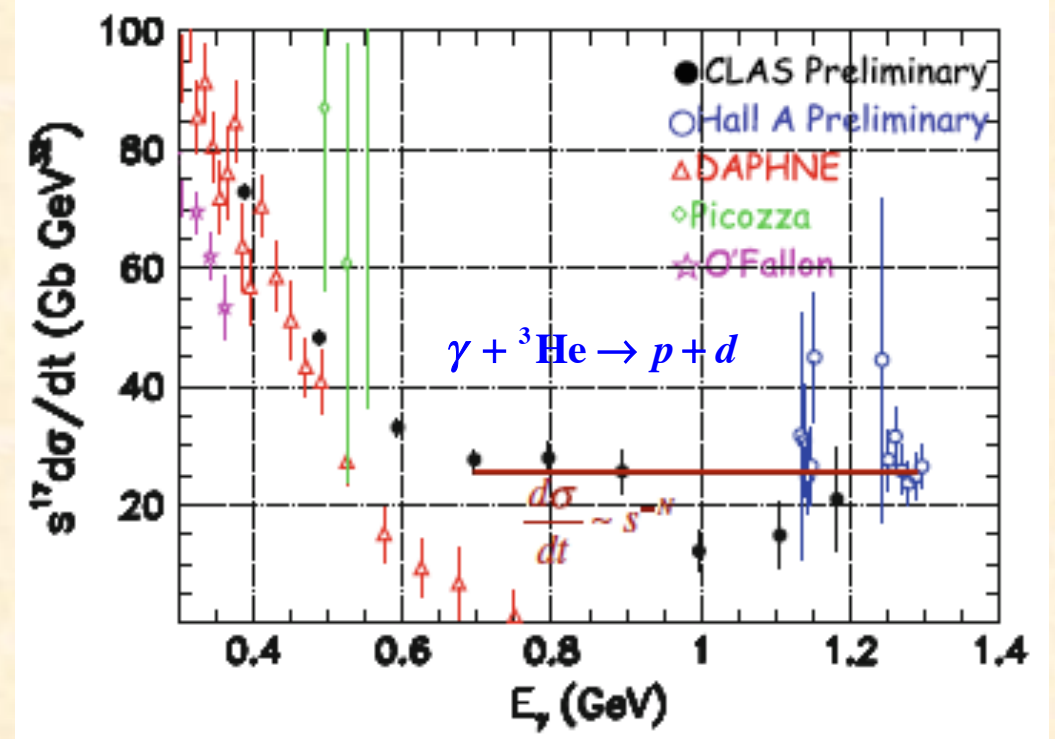


Constituent-counting rule for “molecular” systems

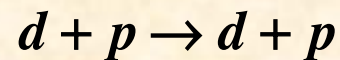
Y. N. Uzikov, JETP Lett. 81, 303 (2005).



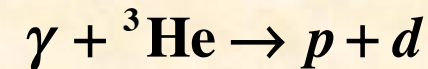
Y. Ilieva, Few Body Syst. 54, 989 (2013).



$$n - 2 = (6 + 6 + 3 + 9) - 2 = 22$$



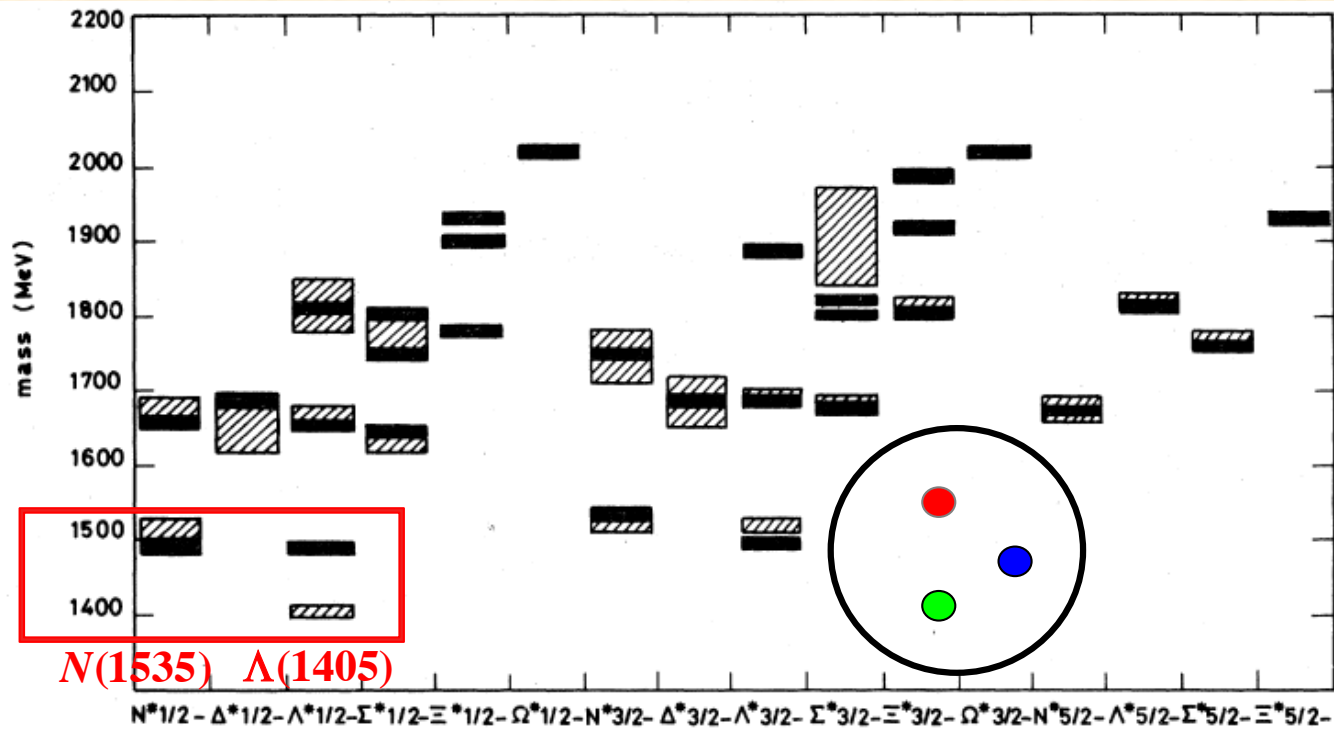
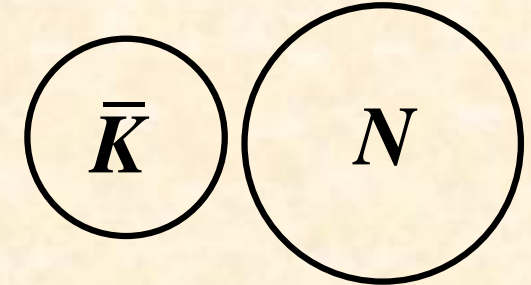
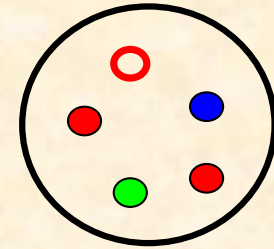
$$n - 2 = (6 + 3 + 6 + 3) - 2 = 16$$



$$n - 2 = (1 + 9 + 3 + 6) - 2 = 17$$

$\Lambda(1405)$: exotic hadron?

Negative-parity baryons
N. Isgur and G. Karl,
PRD 18 (1978) 4187.



Our proposal:

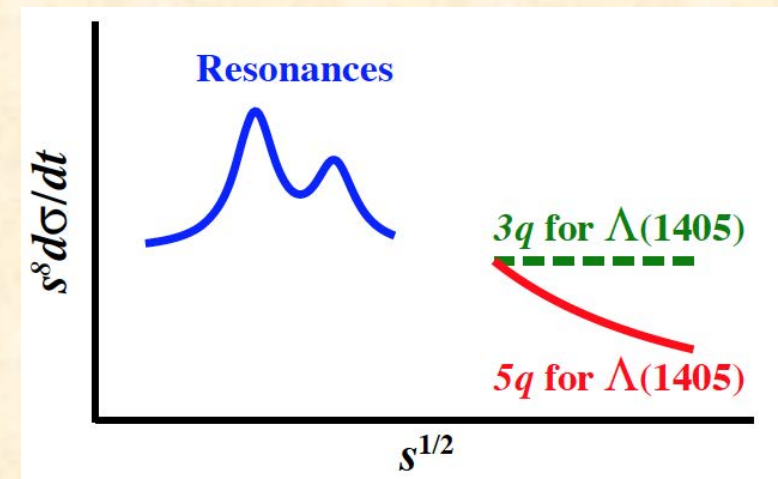
Exotic hadron production

$\pi^- + p \rightarrow K^0 + \Lambda(1405)$: J-PARC, COMPASS?

$\gamma + p \rightarrow K^+ + \Lambda(1405)$: JLab

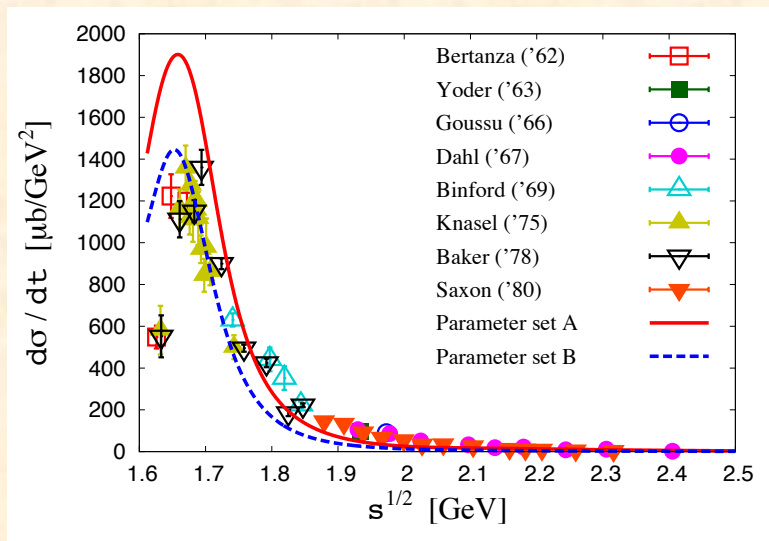
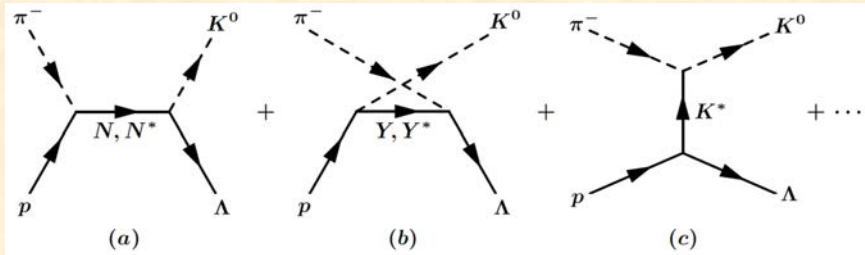
Most spectra agree with the ones by a 3q-picture

- Only $\Lambda(1405)$ deviates from the measurement.
- Difficult to understand the small mass of $\Lambda(1405)$ in comparison with $N(1535)$.
→ $\bar{K}N$ molecule or penta-quark ($qqqq\bar{q}$)?

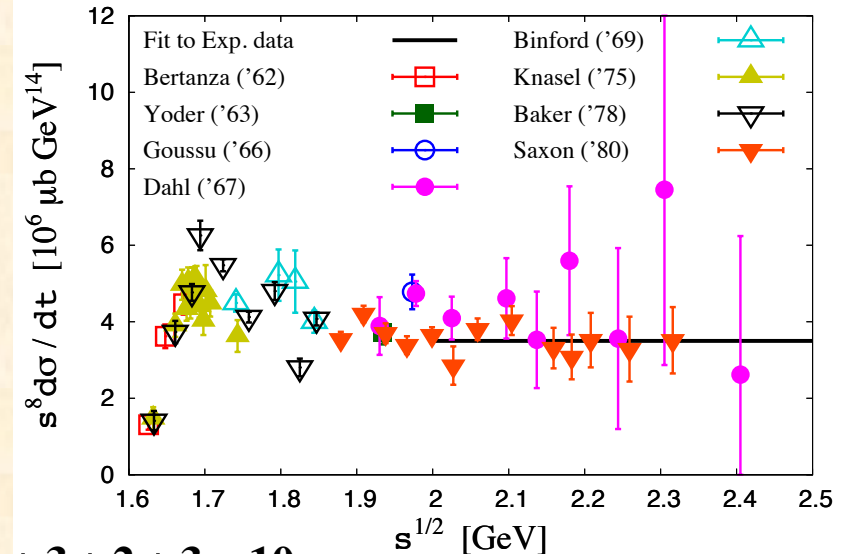


Ordinary-hadron production $\pi^- + p \rightarrow K^0 + \Lambda$ as a reference

At low energies



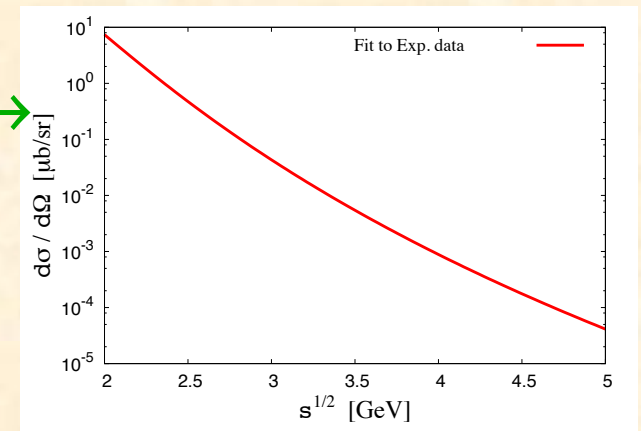
From low to higher energies



$$n = 2 + 3 + 2 + 3 = 10$$

$$\frac{d\sigma_{ab \rightarrow cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10.1 \pm 0.6, \text{ encouraging!}$$

Our prediction at high energies →



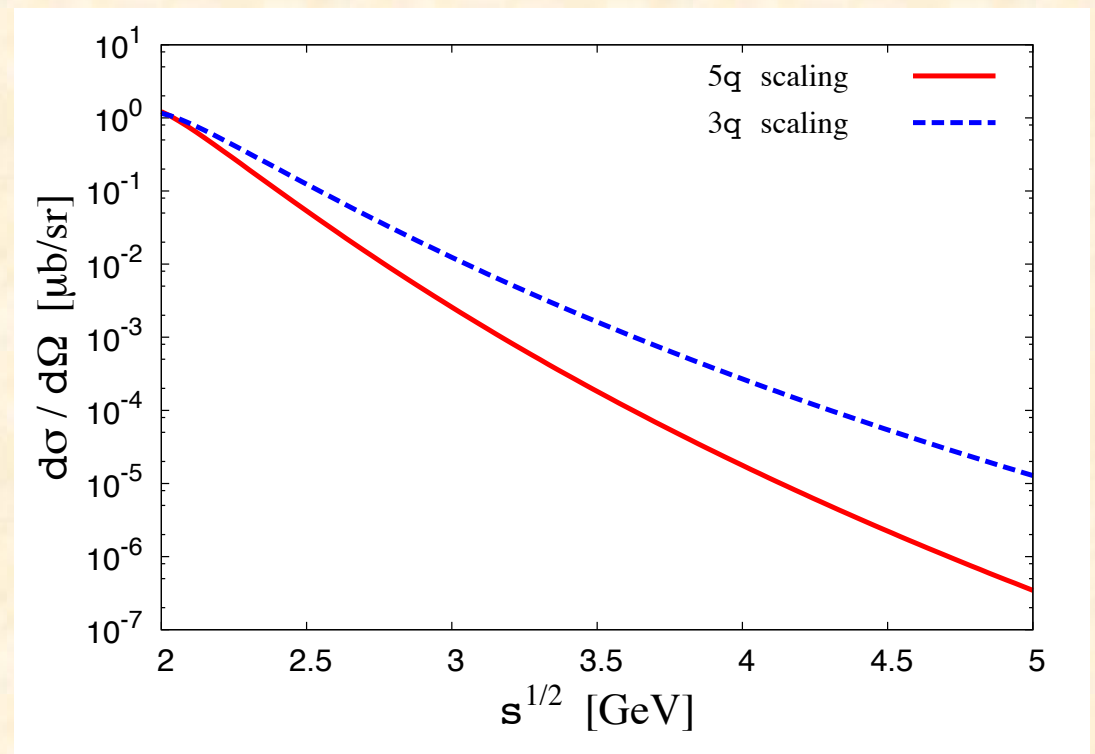
Exotic-hadron production $\pi^- + p \rightarrow K^0 + \Lambda(1405)$

Theoretical and experimental situation
is no as good as the one for the ground Λ .

$$\begin{aligned} n &= 2 + 3 + 2 + 3 = 10 \text{ if } \Lambda(1405) = \text{three-quark state} \\ &= 2 + 3 + 2 + 5 = 12 \text{ if } \Lambda(1405) = \text{five-quark state} \\ &\quad \text{(including } \bar{K}N \text{ molecule)} \end{aligned}$$

$$\frac{d\sigma_{ab \rightarrow cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10 \text{ or } 12$$

Our prediction at high energies

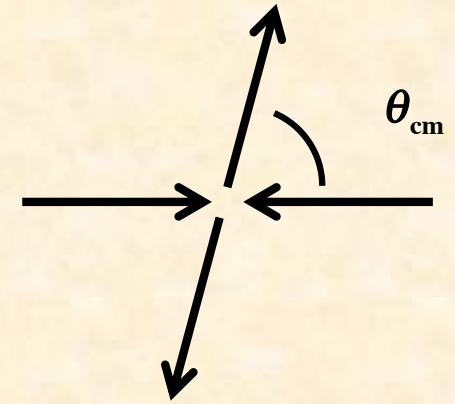
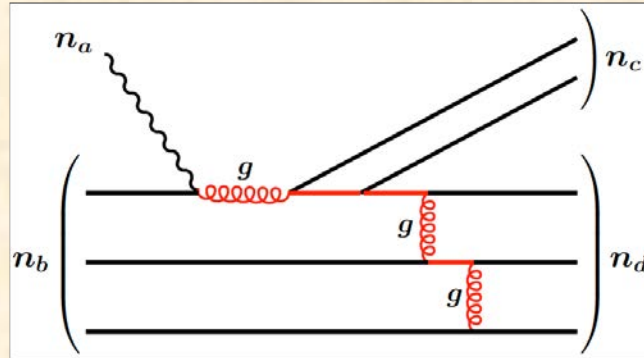
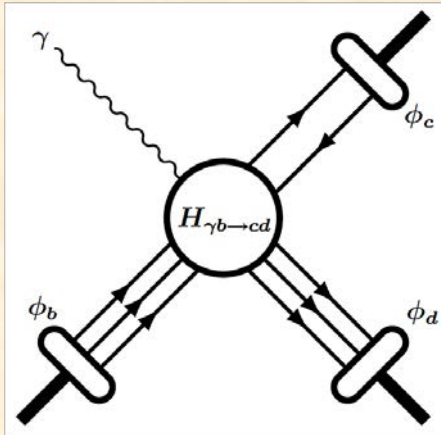


Hard production of hyperons

W.-C. Chang, S. Kumano, and T. Sekihara

Phys. Rev. D 93 (2016) 034006 (arXiv:1512.06647).

JLab hyperon productions



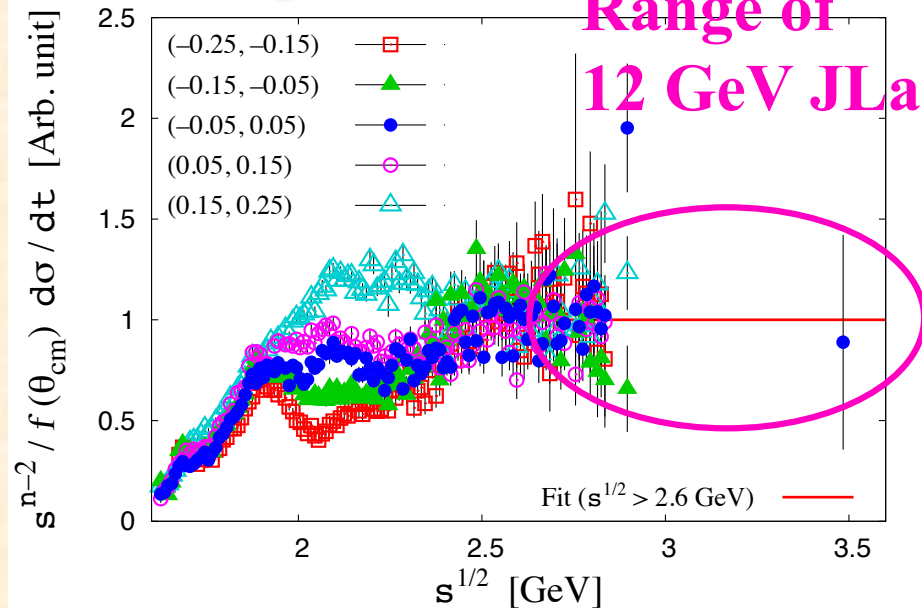
5 bins: $-0.25 < \cos \theta_{cm} < -0.15$, ..., $0.15 < \cos \theta_{cm} < 0.25$

4 bins: $-0.20 < \cos \theta_{cm} < -0.10$, ..., $0.10 < \cos \theta_{cm} < 0.20$

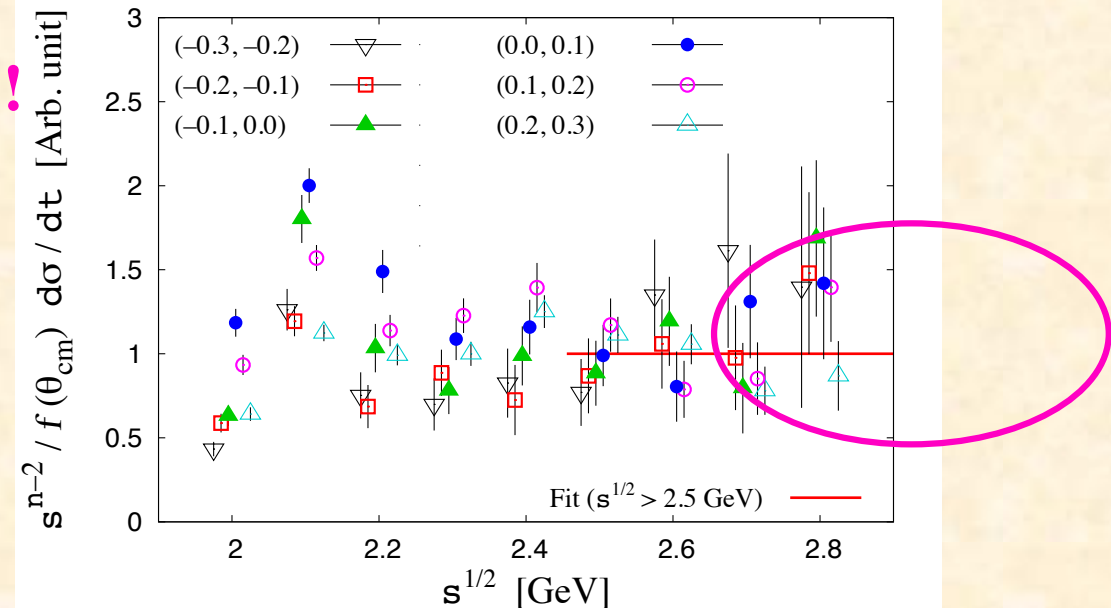
...

1 bin: $-0.05 < \cos \theta_{cm} < +0.05$

ground Λ

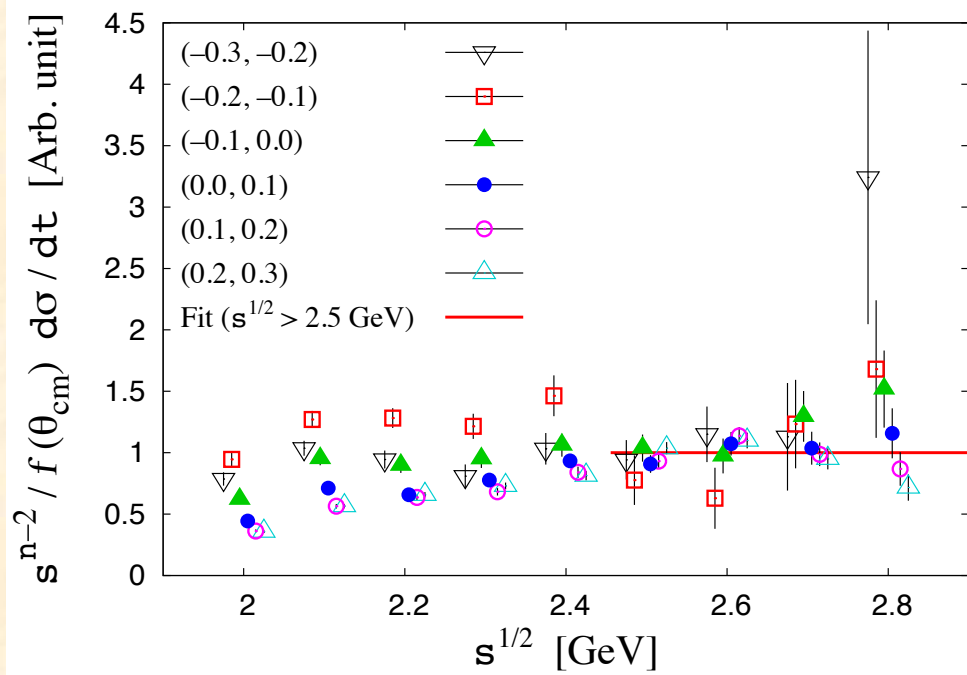


$\Lambda(1405)$

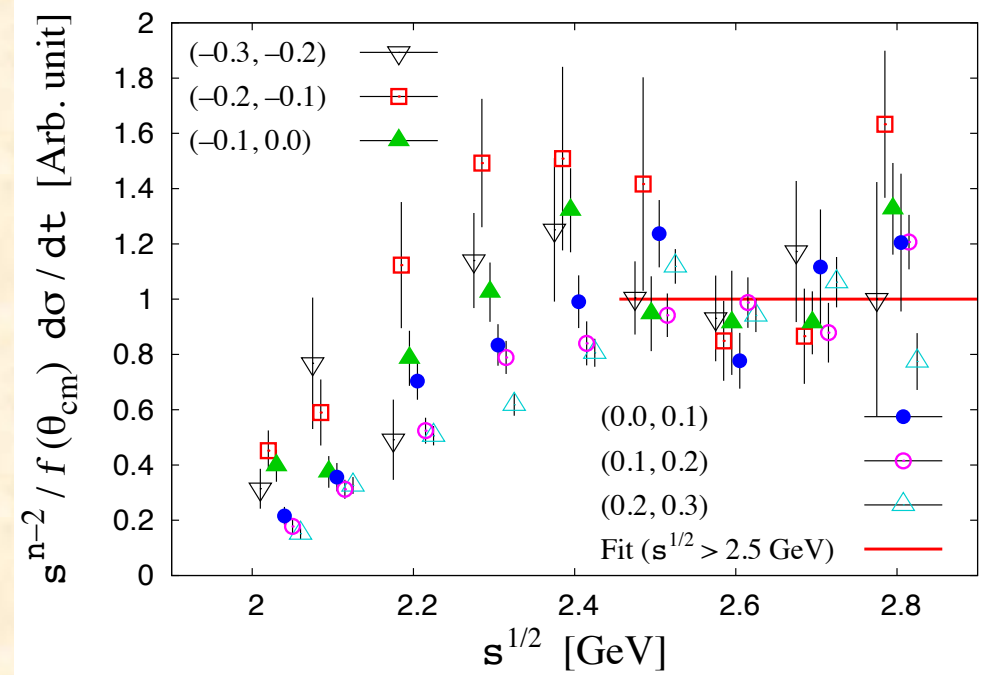


Hyperon productions

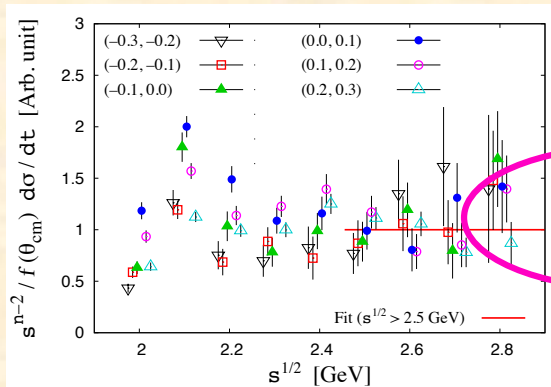
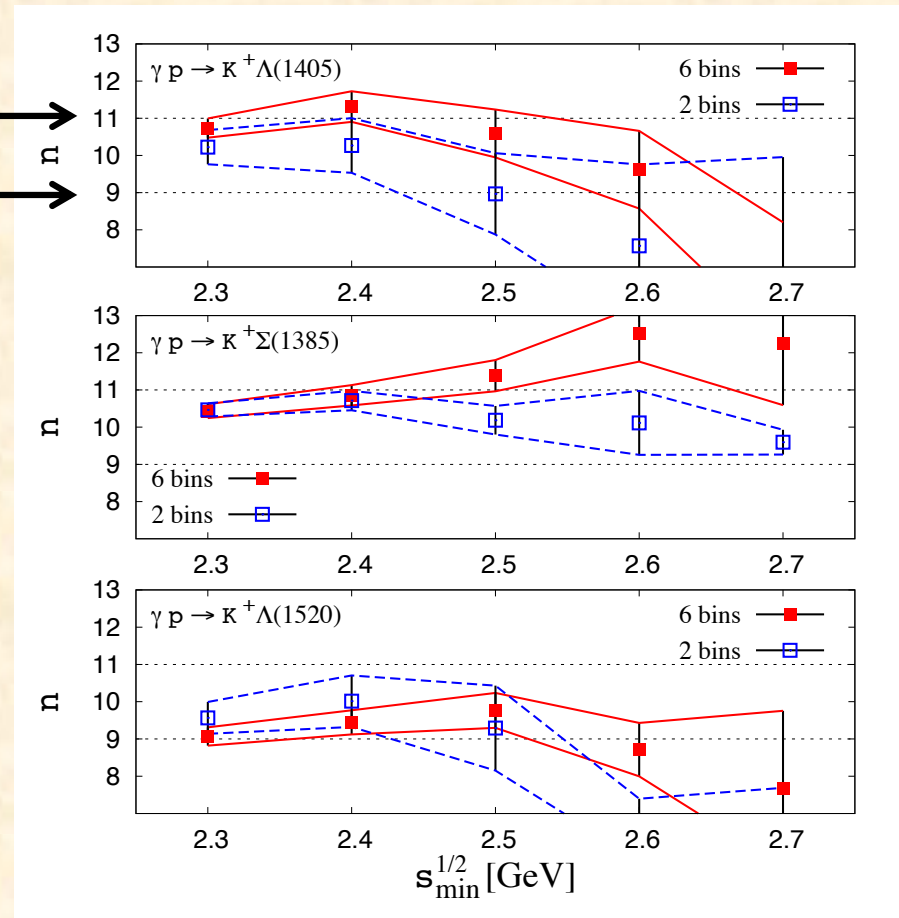
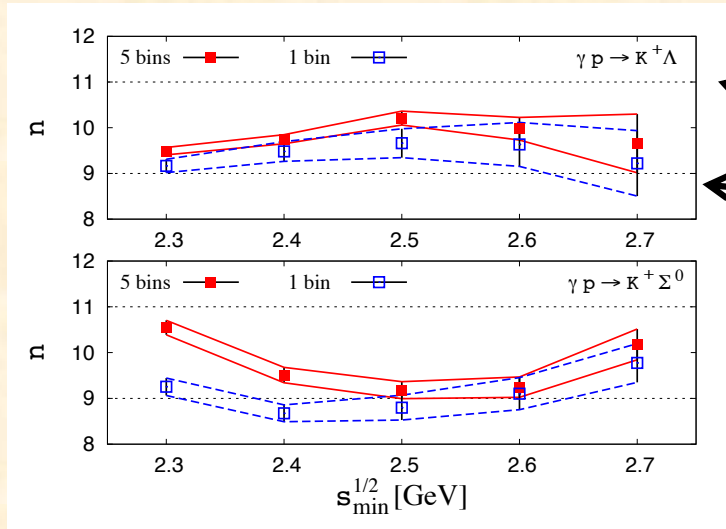
$\Sigma^0(1385)$



$\Lambda(1520)$



JLab hyperon productions including $\Lambda(1405)$



Range of
12 GeV JLab!

- Λ , $\Lambda(1520)$ and Σ seem to be consistent with ordinary baryons with $n = 3$.
- $\Lambda(1405)$ looks penta-quark at low energies but $n \sim 3$ at high energies???
- $\Sigma(1385)$: $n = 5$???

→ In order to clarify the nature of $\Lambda(1405)$ [$qqq, \bar{K}N, qq\bar{q}\bar{q}$], the JLab 12-GeV and EIC experiment plays an important role!

W.-C. Chang, SK, T. Sekihara,
PRD 93 (2016) 034006.

Summary on exotic hadron structure by hard exclusive processes

- We propose to use hard exclusive production of exotic hadrons for probing internal quark-gluon structure

by the constituent counting rule, $\frac{d\sigma}{dt} = \frac{\text{const}}{s^{n-2}}$.

- As an example, $\pi^- + p \rightarrow K^0 + \Lambda(1405)$ is studied together with $\pi^- + p \rightarrow K^0 + \Lambda$ as a reference of an ordinary hadron.

- Exclusive processes of exotic hadrons can be investigated at many facilities in the world.

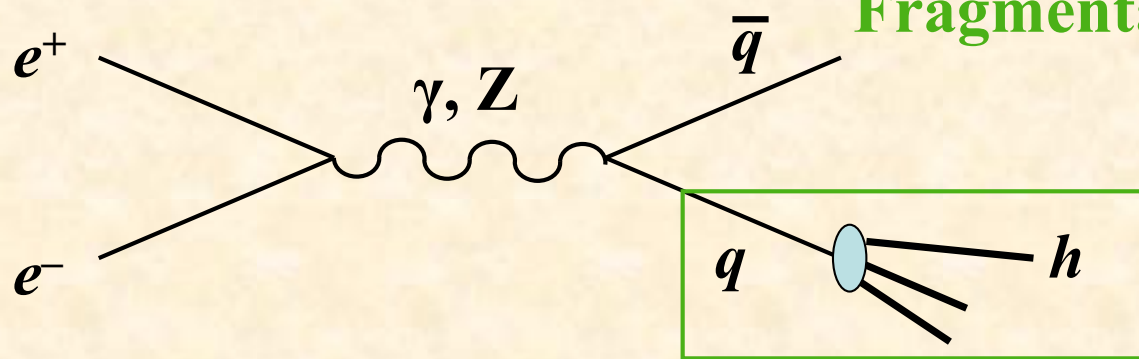
For example, J-PARC, KEK-B, JLab, AMBER, EIC, ...

in general any hadron facilities like GSI, Fermilab, RHIC, LHC, ...

Exotic hadrons by fragmentation functions

**M. Hirai, S. Kumano, M. Oka, K. Sudoh,
Phys. Rev. D77 (2008) 017504 (arXiv:0708.1816).**

Fragmentation Functions



Fragmentation: hadron production from a quark, antiquark, or gluon

Total fragmentation function is defined by

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

$$F^h(z, Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz}$$

σ_{tot} = total hadronic cross section

Variable z

- Hadron energy / Beam energy
- Hadron energy / Primary quark energy

A fragmentation process occurs from quarks, antiquarks, and gluons, so that F^h is expressed by their individual contributions:

$$F^h(z, Q^2)_{LO} = \sum_i D_i^h(y, Q^2)$$

$D_i^h(z, Q^2)$ = fragmentation function of hadron h from a parton i

Momentum (energy) sum rule

$D_i^h(z, Q^2)$ = probability to find the hadron h from a parton i
with the energy fraction z

Energy conservation:
$$\sum_h \int_0^1 dz z D_i^h(z, Q^2) = 1$$

$$h = \pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, p, \bar{p}, n, \bar{n}, \dots$$

Favored and disfavored fragmentation functions

Simple quark model: $\pi^+(u\bar{d})$, $K^+(u\bar{s})$, $p(uud)$, ...

Differences between them
could be used
for exotic hadron studies.

Favored fragmentation: $D_u^{\pi^+}$, $D_{\bar{d}}^{\pi^+}$, ...

(from a quark which exists in a naive quark model)

Disfavored fragmentation: $D_d^{\pi^+}$, $D_{\bar{u}}^{\pi^+}$, $D_s^{\pi^+}$, ...

(from a quark which does not exist in a naive quark model)

Exotic hadrons by fragmentation functions

“Favored” and “disfavored” (unfavored) fragmentation functions

Hirai, SK, Oka, Sudoh,
PRD 77 (2008) 017504.

Possibility of finding exotic hadrons in high-energy processes

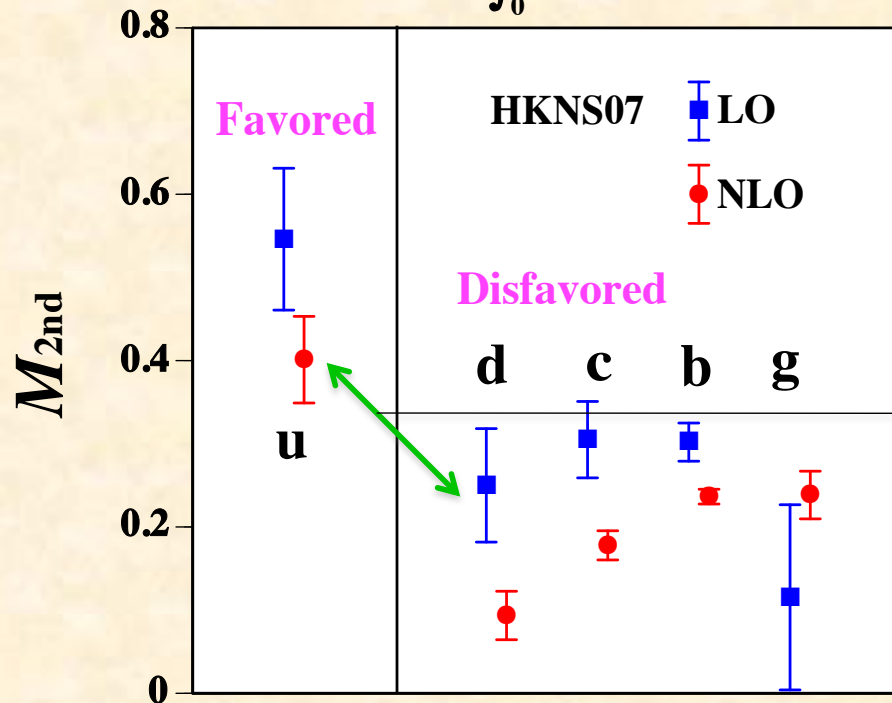
Possibilities for $f_0(980)$: $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $s\bar{s}$, $\frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$, $K\bar{K}$, or gg

e.g. if $f_0(980) = s\bar{s}$: **favored** $s, \bar{s} \rightarrow f_0$; **disfavored** $u, d, \bar{u}, \bar{d} \rightarrow f_0, \dots$

*$f_0(980)$: Belle analysis
is possible in principle.*

Pion case

$$M_{2nd} = \int_0^1 dz z D_i^{\pi^+}(z)$$



2nd moments of
M. Hirai, SK, T.-H. Nagai, K. Sudoh,
PRD 75 (2007) 094009.

There are distinct differences between
the favored and disfavored 2nd moments.
→ It could be used for exotic-hadron studies.

Criteria for determining f_0 structure by its fragmentation functions

Possible configurations of f_0 (980)

(1) ordinary u, d - meson $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

(2) strange meson, $s\bar{s}$

(3) tetraquark ($K\bar{K}$), $\frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$

(4) glueball gg

Contradicts with experimental widths

$$\Gamma_{\text{theo}}(f_0 \rightarrow \pi\pi) = 500 - 1000 \text{ MeV} \\ \gg \Gamma_{\text{exp}} = 40 - 100 \text{ MeV}$$

$$\Gamma_{\text{theo}}(f_0 \rightarrow \gamma\gamma) = 1.3 - 1.8 \text{ keV} \\ \gg \Gamma_{\text{exp}} = 0.205 \text{ keV}$$

Contradicts with lattice-QCD estimate

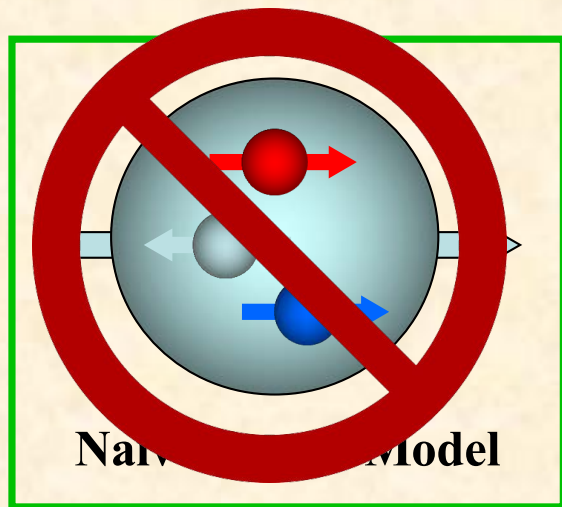
$$m_{\text{lattice}}(f_0) = 1600 \text{ MeV} \\ \gg m_{\text{exp}} = 980 \text{ MeV}$$

There could be a difference in fragmentation functions for f_0 depending on its internal structure.

- Favored and disfavored fragmentation functions
- 2nd moments and functional forms

Exotic signatures in deuteron

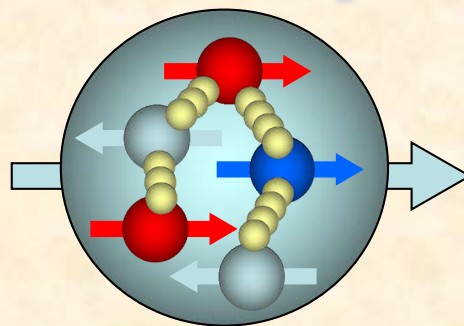
Situation of tensor structure by b_1 for spin-1 deuteron



Naive Model

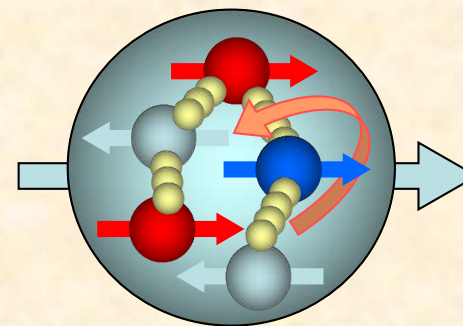
“old” standard model

Nucleon spin



Sea-quarks and gluons?

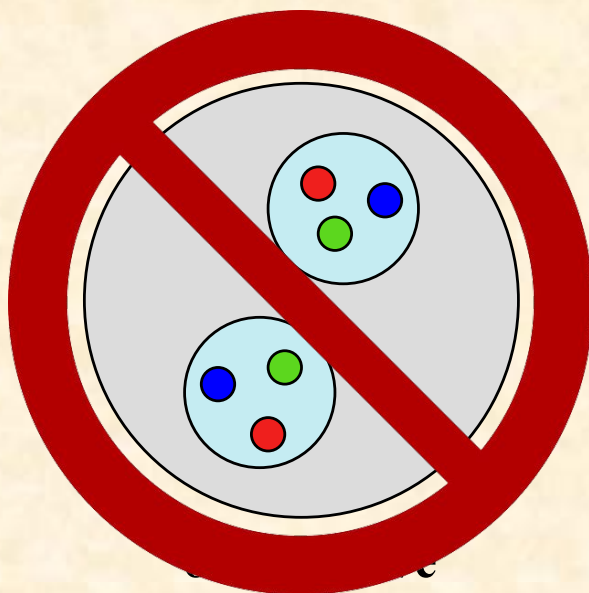
Nucleon spin crisis!?



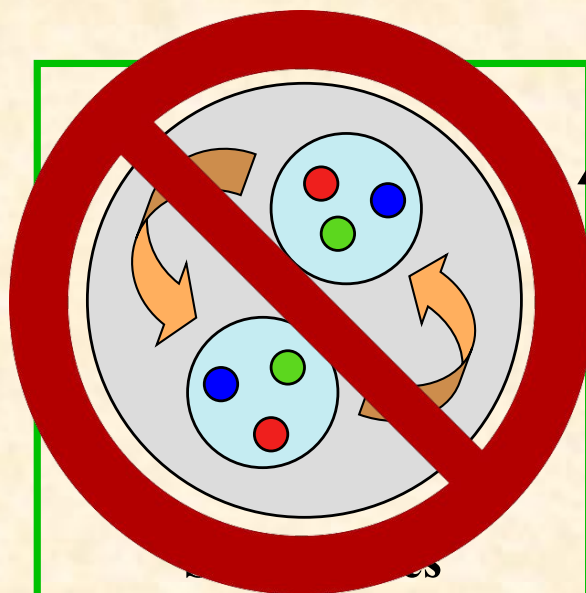
Orbital angular momenta ?

Tensor structure

We have shown in this work that the standard deuteron model does not work!?
→ new hadron physics?!

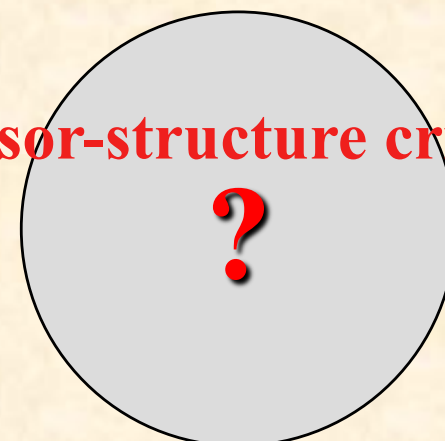


$b_1 = 0$



standard model $b_1 \neq 0$

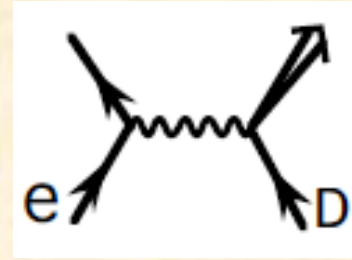
Tensor-structure crisis!?



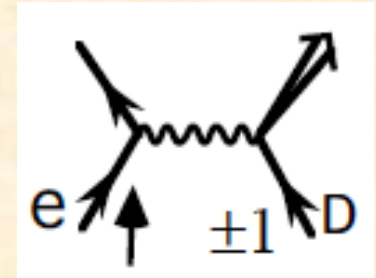
b_1 experiment $\neq b_1$ “standard model”

Structure Functions

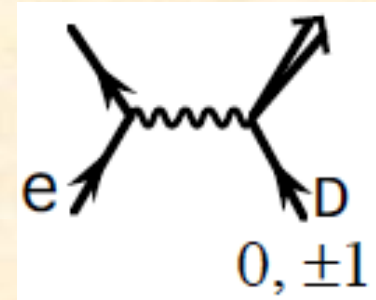
$$F_1 \propto \langle d\sigma \rangle$$



$$g_1 \propto d\sigma(\uparrow, +1) - d\sigma(\uparrow, -1)$$



$$b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2}$$



note: $\sigma(0) - \frac{\sigma(+1) + \sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2}[\sigma(+1) + \sigma(-1)]$

Parton Model

$$F_1 = \frac{1}{2} \sum_i e_i^2 (q_i + \bar{q}_i)$$

$$q_i = \frac{1}{3} (q_i^{+1} + q_i^0 + q_i^{-1})$$

$$g_1 = \frac{1}{2} \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$$

$$\Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1}$$

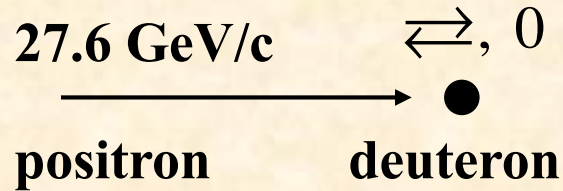
$$[q_{\uparrow}^H(x, Q^2)]$$

$$b_1 = \frac{1}{2} \sum_i e_i^2 (\delta_T q_i + \delta_T \bar{q}_i)$$

$$\delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2}$$

HERMES results on b_1

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.



b_1 measurement in the kinematical region

$$0.01 < x < 0.45, \quad 0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$$

b_1 sum rule

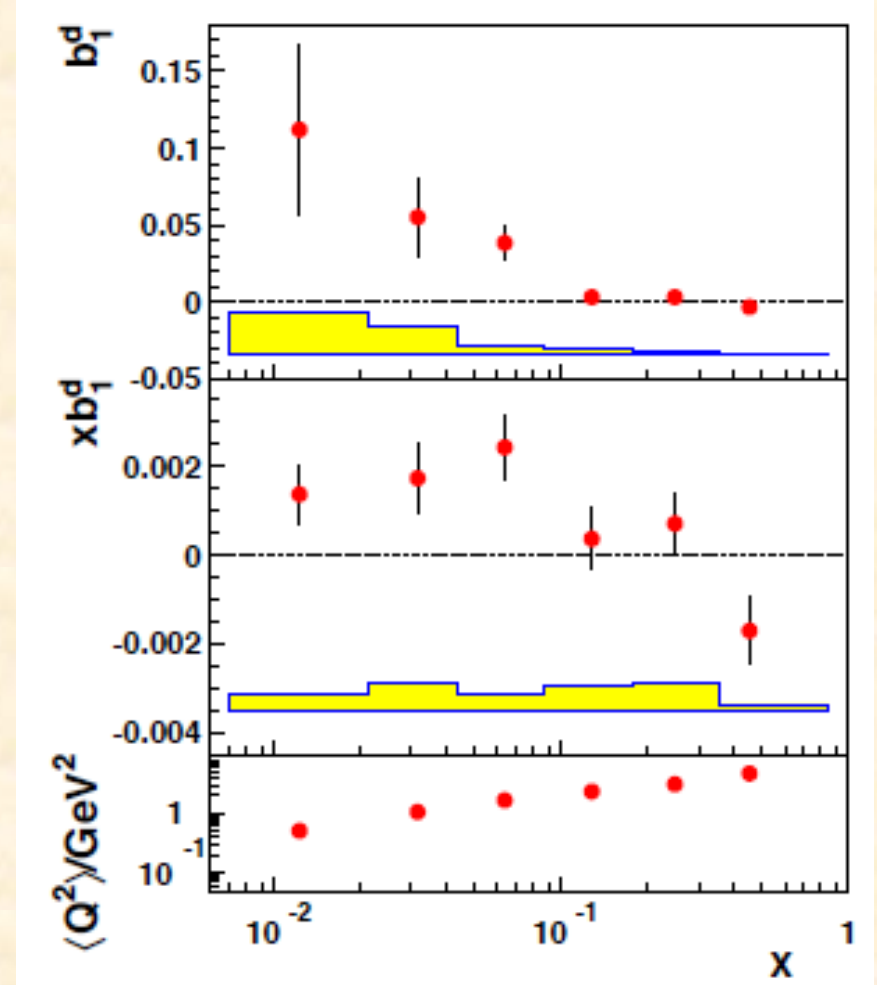
$$\int_{0.002}^{0.85} dx b_1(x) = [1.05 \pm 0.34(\text{stat}) \pm 0.35(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5 \text{ GeV}^2$

In the restricted Q^2 range $Q^2 > 1 \text{ GeV}^2$

$$\int_{0.02}^{0.85} dx b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

at $Q^2 = 5 \text{ GeV}^2$



$$\int dx b_1^D(x) = \lim_{t \rightarrow 0} -\frac{5}{12} \frac{t}{M^2} F_Q(t) + \frac{1}{9} (\delta Q + \delta \bar{Q})_{\text{sea}} = 0 ?$$

$$\int \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} \int dx [u_v - d_v] + \frac{2}{3} \int dx [\bar{u} - \bar{d}] \neq 1/3$$

b_1 sum rule: F. E. Close and SK, PRD 42 (1990) 2377.

Drell-Yan experiments probe these antiquark distributions.

Basic convolution approach

Convolution model: $A_{hH, hH}(x, Q^2) = \int \frac{dy}{y} \sum_s f_s^H(y) \hat{A}_{hs, hs}(x/y, Q^2) \equiv \sum_s f_s^H(y) \otimes \hat{A}_{hs, hs}(y, Q^2)$

$$A_{hH, h'H'} = \varepsilon_{h'}^{*\mu} W_{\mu\nu}^{H'H} \varepsilon_h^\nu, \quad b_1 = A_{+,0,+0} - \frac{A_{++,++} + A_{+,-,+}}{2}$$

$$\hat{A}_{+\uparrow,+\uparrow} = F_1 - g_1, \quad \hat{A}_{+\downarrow,+\downarrow} = F_1 + g_1$$

Momentum distribution: $f^H(y) = \int d^3 p y |\phi^H(\vec{p})|^2 \delta\left(y - \frac{E - p_z}{M_N}\right)$

$$y = \frac{M p \cdot q}{M_N P \cdot q} \approx \frac{2p^-}{P^-}, \quad f^H(y) \equiv f_{\uparrow}^H(y) + f_{\downarrow}^H(y)$$

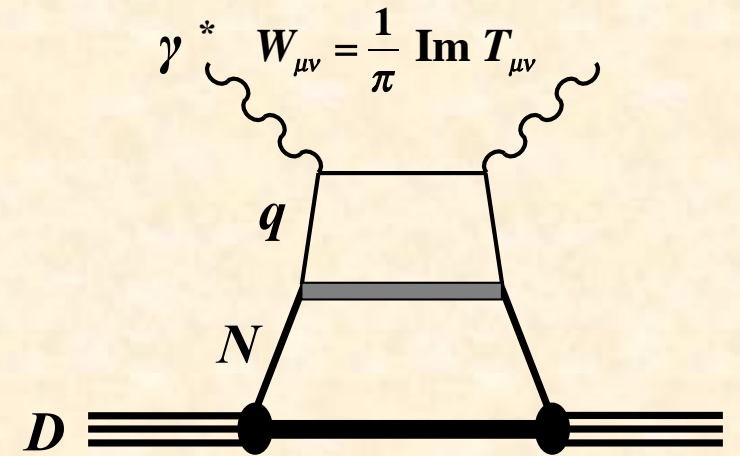
D-state admixture: $\phi^H(\vec{p}) = \phi_{\ell=0}^H(\vec{p}) + \phi_{\ell=2}^H(\vec{p})$

⇓

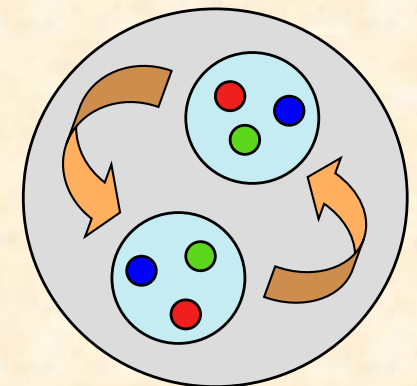
$$b_1(x) = \int \frac{dy}{y} \delta_T f(y) F_1^N(x/y, Q^2)$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y) + f^-(y)}{2}$$

$$= \int d^3 p y \left[-\frac{3}{4\sqrt{2}\pi} \phi_0(p) \phi_2(p) + \frac{3}{16\pi} |\phi_2(p)|^2 \right] (3 \cos^2 \theta - 1) \delta\left(y - \frac{p \cdot q}{M_N v}\right)$$

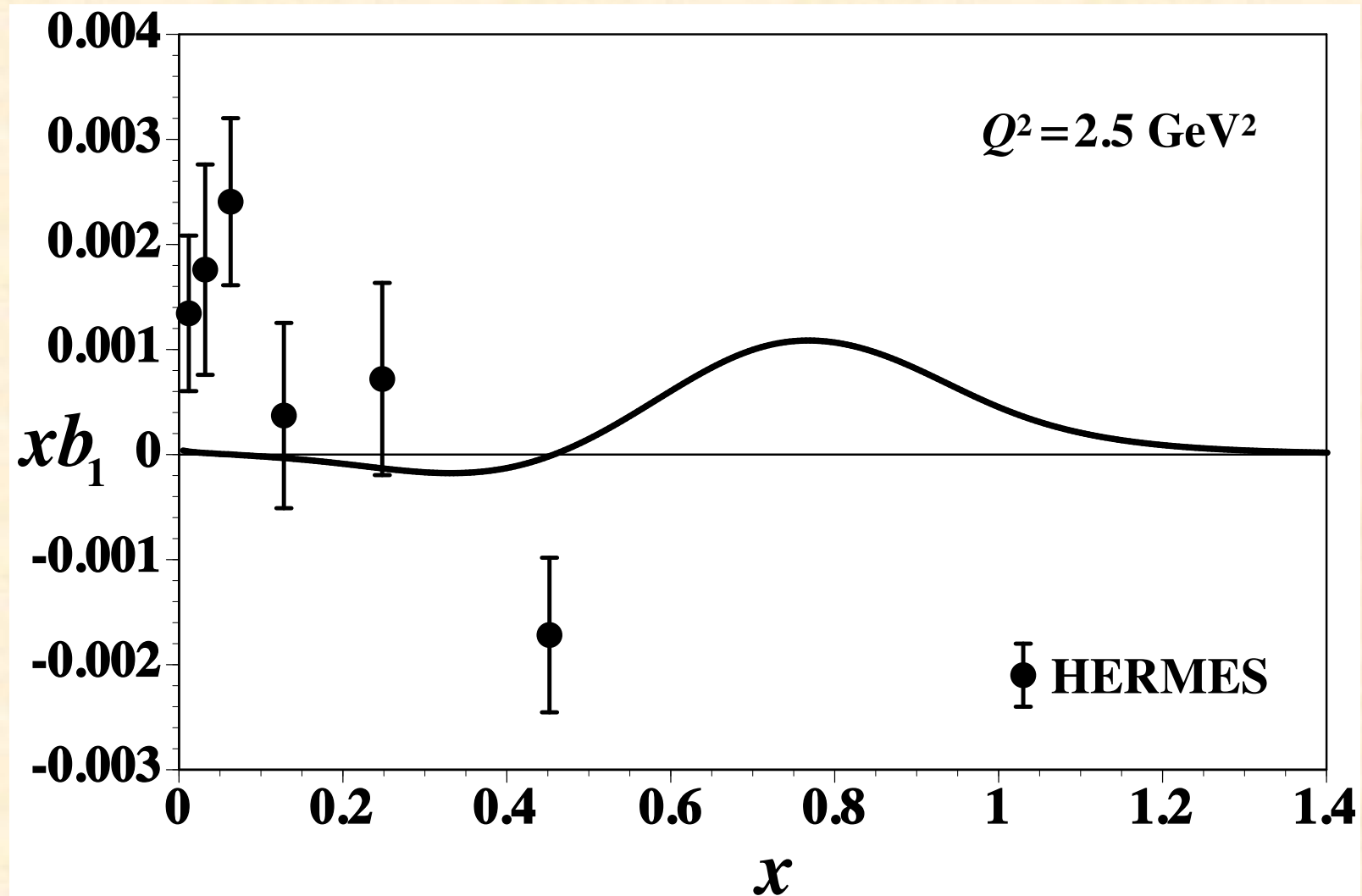


Standard model of the deuteron



S + D waves

Comparison with HERMES measurements



$$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$$

Standard convolution model does not work for the deuteron tensor structure!?

JLab PAC-38 (Aug. 22-26, 2011) proposal, PR12-11-110

(2023 update PR12-13-011)

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),
K. Allada, A. Camsonne, A. Deur, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

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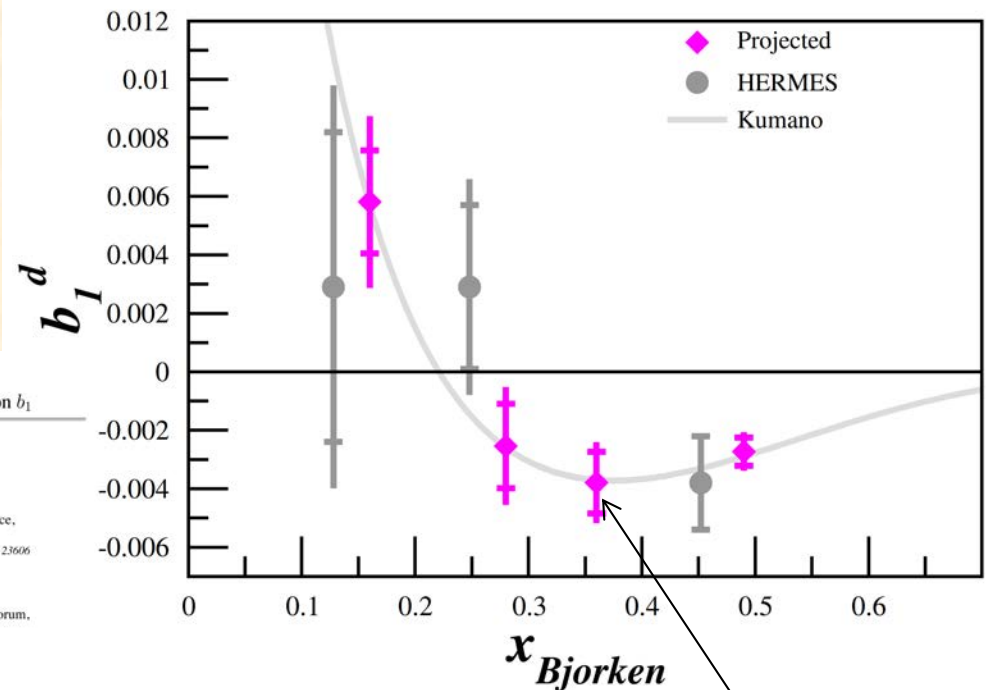
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Seonho Choi, Hoyoung Kang, Hyekoo Kang, Yoomin Oh
Seoul National University, Seoul 151-747 Korea

Review paper "Tensor Spin Observables"
under preparation for European Physical Journal A.



PR12-13-011

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

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[†]Spokesperson
[‡]Contact

Approved!
(Fully approved in 2023.)

Expected errors
by JLab

Gluon transversity $\Delta_T g$

Note on our notations:

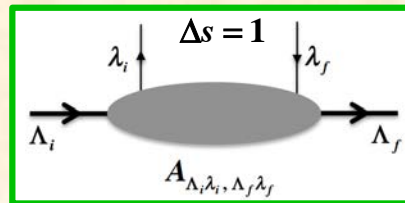
Tensor-polarized gluon distribution: $\delta_T g$

Gluon transversity: $\Delta_T g$

Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, +\frac{1}{2} + \frac{1}{2}\right) - A\left(+\frac{1}{2} - \frac{1}{2}, +\frac{1}{2} - \frac{1}{2}\right)$

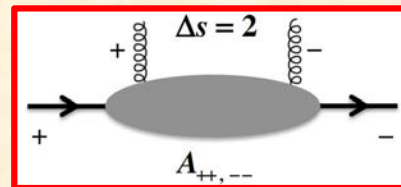
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2} + \frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \rightarrow \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron:

$\Delta_T g(x) \sim A(+1+1, -1-1)$,

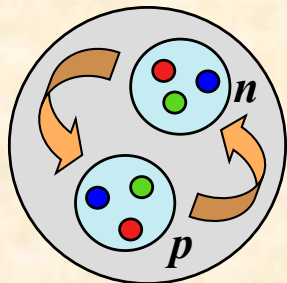
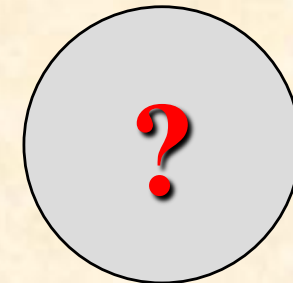
~~$A\left(+\frac{1}{2} + 1, -\frac{1}{2} - 1\right)$ not possible for nucleon~~



Note: Gluon transversity does not exist for spin-1/2 nucleons.

$b_1(\delta_T q, \delta_T g) \neq 0 \Leftrightarrow$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?



S + D waves

Physics beyond “the standard model” in nuclear physics?
(Physics beyond the standard model in particle physics???)

Proposed JLab experiment



LoI, arXiv:1803.11206

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016
 Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

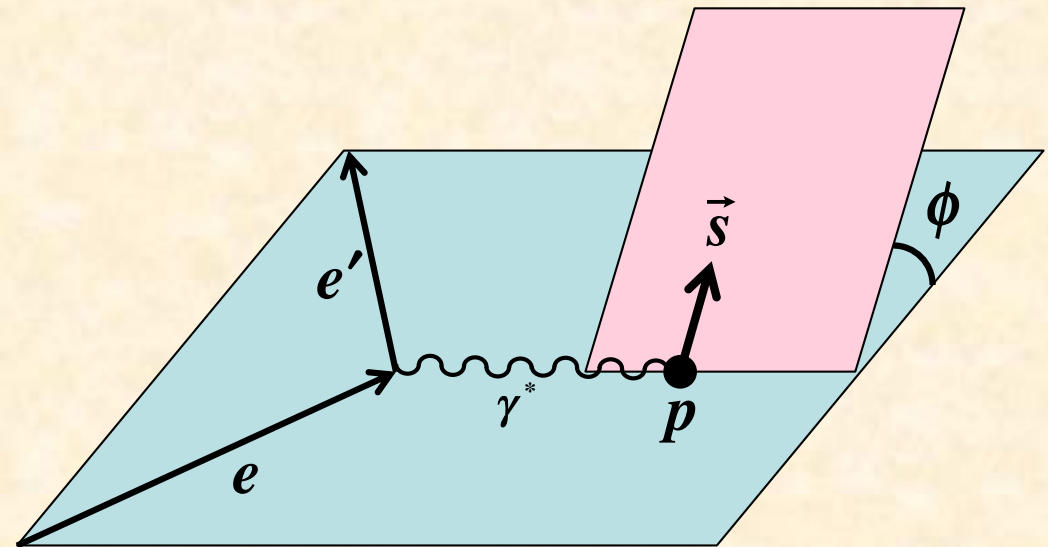
Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831



Electron scattering with polarized-proton target

$$\left. \frac{d\sigma}{dx dy d\phi} \right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4} \left[xy^2 F_1(x, Q^2) + (1-y)F_2(x, Q^2) - \frac{1}{2}x(1-y)\Delta(x, Q^2) \cos(2\phi) \right]$$

$$\Delta(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3} \Delta_T g(y, Q^2)$$

By looking at the proton polarization angle ϕ , the quark transversity $\Delta_T g$ can be measured.

Summary

In general, it is not easy to find internal structure of exotic-hadron candidates by global observables, such as mass, spin, ...

High energies = Quark and gluon degrees of freedom.

It could be appropriate to use high-energy processes for determination of internal configurations for exotic-hadron candidates.

- 3D tomography by GPDs**
- Constituent-counting rule**
- Fragmentation functions**
- (• New observables in spin-1 hadrons)**

Experimental projects

JLab, J-PARC, KEK-B, AMBER, EIC, ...

The End

The End