The 17th CNS International Summer School, Japan

MEAN-FIELD STUDY OF THE RADIATIVE CAPTURE $^{12}C(p,\gamma)^{13}N$ AND $^{13}C(p,\gamma)^{14}N$ REACTIONS

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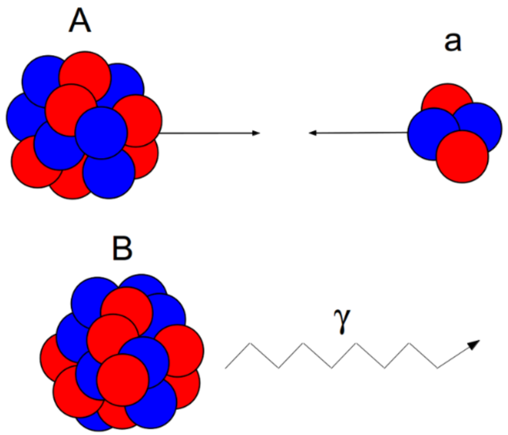
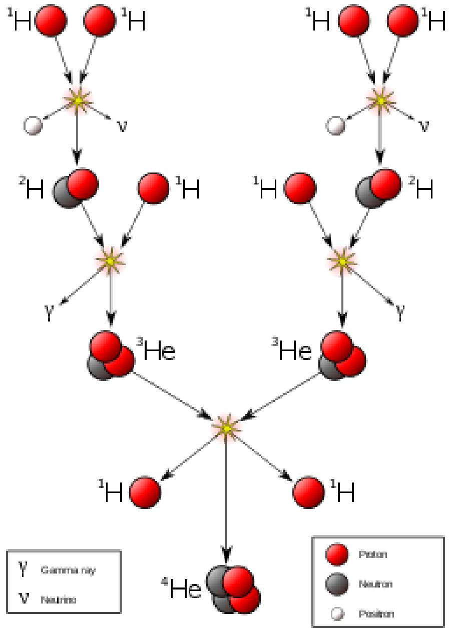
RADIATIVE CAPTURE

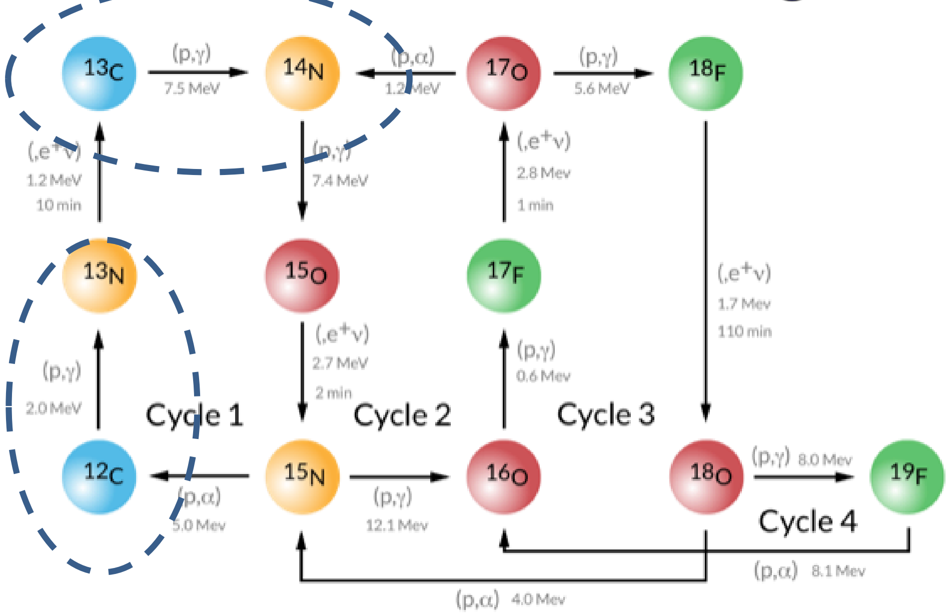
Radiative capture is an important process due to its astrophysical applications.

BBN, stellar evolution, element synthesis, X-ray bursts, etc.





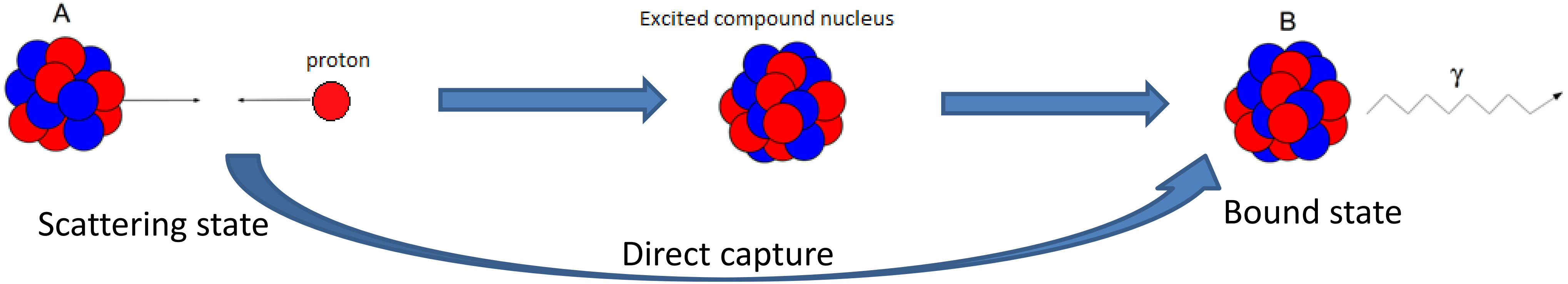
pp chain



CNO: $T_9 < 0.2$

CNO cycle

RADIATIVE CAPTURE



Scattering state

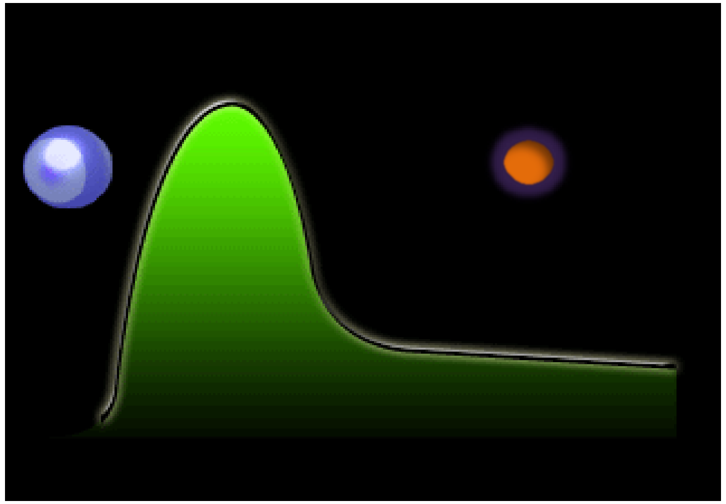
Direct capture

Bound state

The radial Schrödinger equation:

$$\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]u(r)+V(r)u(r)=Eu(r)\ \begin{cases}\text{Bound state}\\ \text{Scattering state}\end{cases}$$

Coulomb pot. Nuclear pot. Spin-orbit pot.



Normalization

$$\begin{cases}\text{Bound state:} & u_J(r)\rightarrow C\exp(-k_B r)\\ \text{Scattering state:} & u_J(r)\rightarrow F_J(kr)\cos\delta_J+G_J(kr)\sin\delta_J\end{cases}$$

RADIATIVE CAPTURE

Using the balanced detail, the cross section for the radiative capture A(p;γ)B reaction is determined as

$$\sigma\left(J_f, E\right) \sim \sum_{\sigma\lambda} \underbrace{\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}}_{\text{Photon}} \underbrace{\left|\left\langle \Psi^{J_f} \middle\| M^{\sigma\lambda} \middle\| \Psi(E) \right\rangle\right|^2}_{\text{Nucleon}}$$

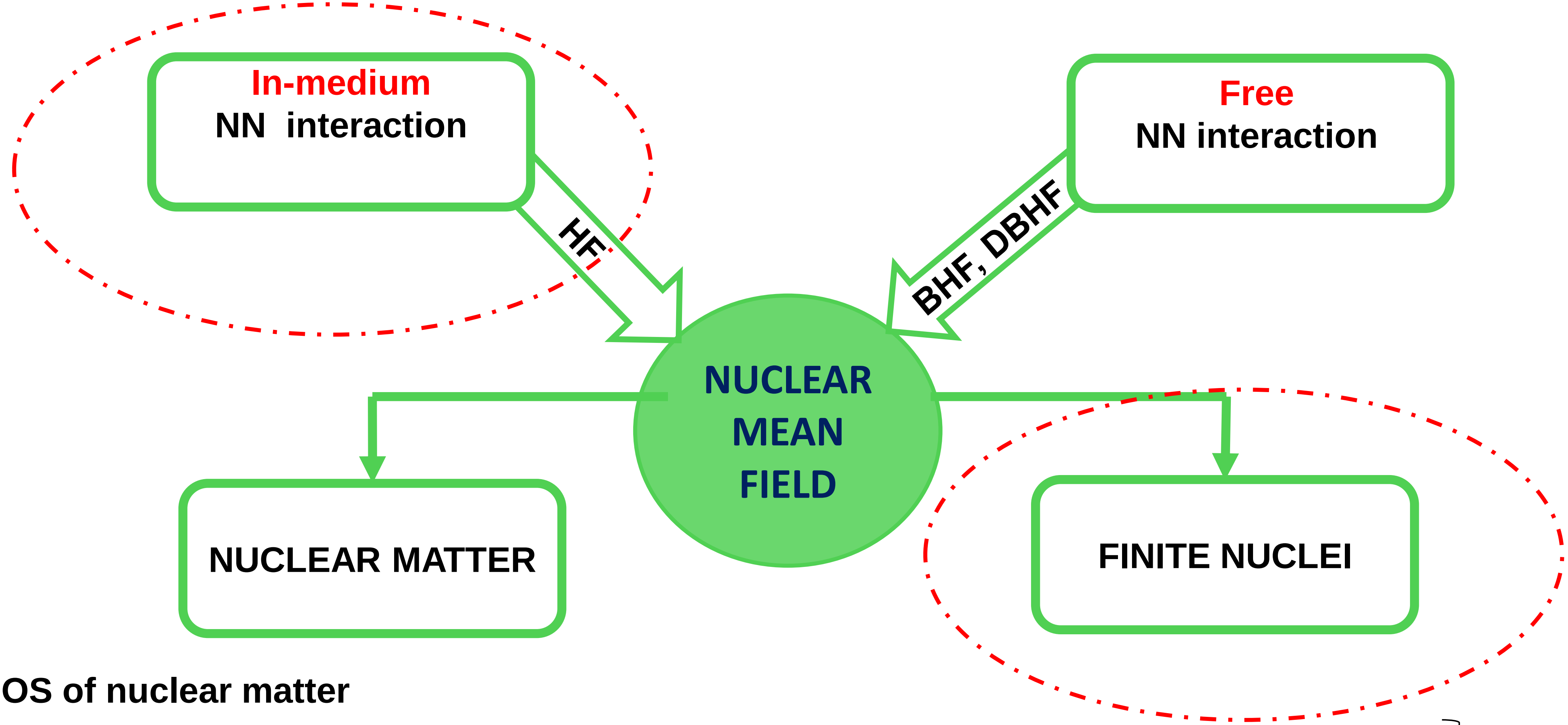
Matrix elements needed for electromagnetic transitions

$$\left\langle \Psi^{J_f} \middle\| M^{\sigma\lambda} \middle\| \Psi(E) \right\rangle = \underbrace{eZ_{eff} \left\langle J_f \lambda M_f \mu \middle| J_i M_i \right\rangle \sqrt{\frac{(2J_i+1)(2\lambda+1)}{4\pi\left(2J_f+1\right)}}}_{\text{Coupling coefficient}} \underbrace{\int_0^\infty \overset{\substack{\text{Scattering state} \\ \uparrow}}{u_i(E,r)} r^\lambda \overset{\substack{\text{Bound state} \\ \uparrow}}{u_f(r)}\, dr}_{\boxed{\text{Overlap integral}}}$$

Long wavelength approximation

$$\sigma\lambda = \boxed{E_1} \gg E_2 \approx M_1 \gg E_3 \approx M_2, \ldots$$

NUCLEAR MEAN-FIELD POTENTIAL



In-medium
NN interaction

HF

Free
NN interaction

BHF, DBHF

NUCLEAR MEAN FIELD

NUCLEAR MATTER

FINITE NUCLEI

- ✓ **EOS of nuclear matter**
- ✓ **Single particle potential**
- ✓ **Symmetry energy**
- ✓ **Neutron star**

Ngo Hai Tan *et al, Phys. Rev. C 93, 035806*

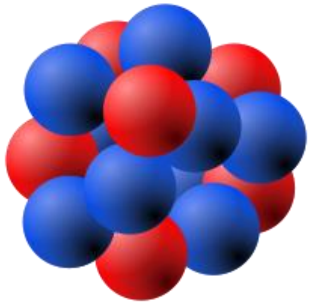
Doan Thi Loan *et al, Phys. Rev. C 92, 034304*

Dao T. Khoa *et al, Phys. Rev. C 94, 034612*

- ✓ **Bound problems**
- ✓ **Scattering problems**

} ***Radiative capture***

NUCLEAR MEAN-FIELD POTENTIAL



Fermions

Antisymmetry of wave functions

(Slater's determinant)

HF Approximation

$$v = v^{D} + v^{EX}$$

M3Y

In-medium (density dependent) NN interaction

$$v^{D(EX)}(\rho, s) = F_0(\rho) v_{00}^{D(EX)}(s) + F_1(\rho) v_{01}^{D(EX)}(s) \overrightarrow{\tau_1} \cdot \overrightarrow{\tau_2} \quad \text{with} \quad s = |\vec{r}_1 - \vec{r}_2|$$

CDM3Yn density dependence

D.T. Khoa, G.R. Satchler and W. von Oertzen, *Phys. Rev.* C 56, 954 (1997);
D.T. Loan, B.M. Loc, and D.T. Khoa, *Phys. Rev.* C 92, 034304 (2015).

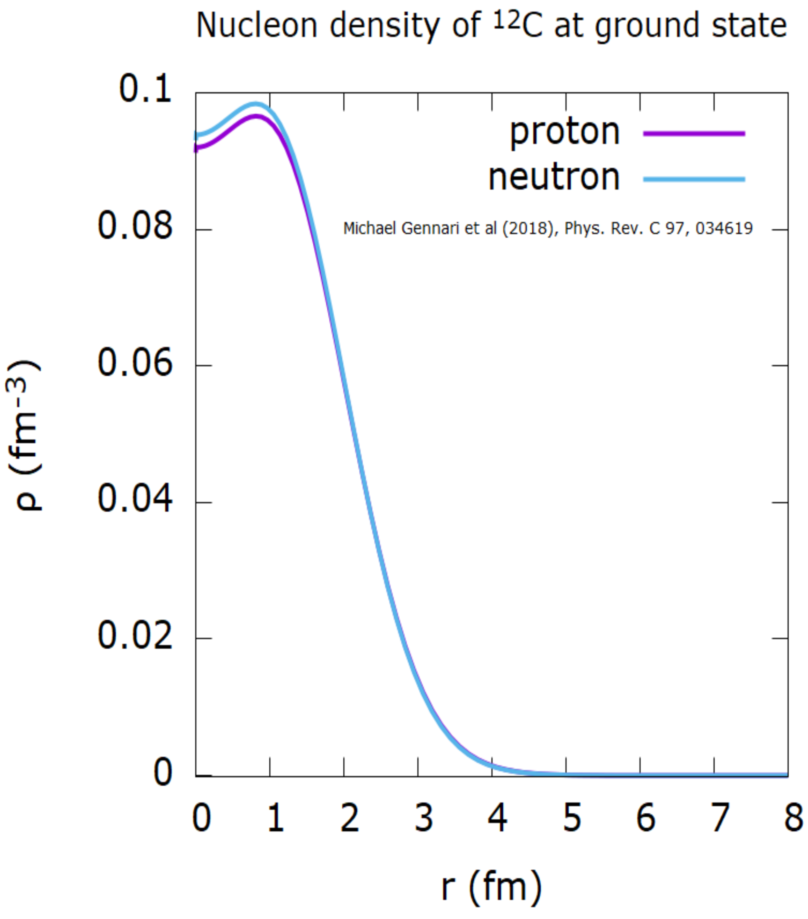
G-matrix based on M3Y interaction

N. Anantaraman, H. Toki, G.F. Bertsch, *Nucl. Phys*. A 398, 269 (1983).

HF calculation $\xrightarrow{\text{HvH theorem}}$ **Extended HF calculation**

Doan Thi Loan, Bui Minh Loc, and Dao T. Khoa, *Phys. Rev.* C 92, 034304 (2015),

NUCLEAR MEAN-FIELD POTENTIAL


Nucleon density of ^{12}C at ground state

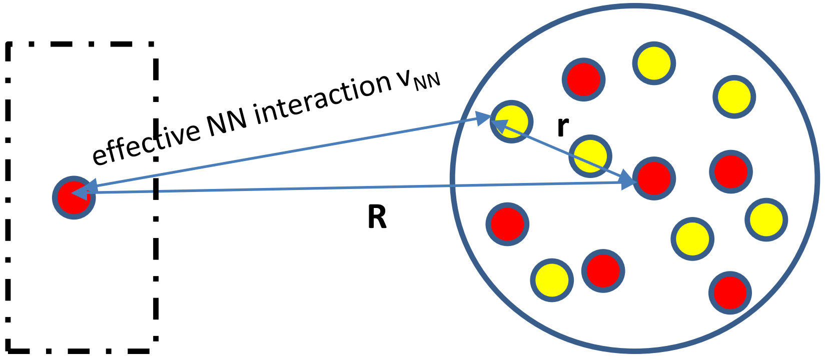
Michael Gennari et al (2018), Phys. Rev. C 97, 034619

Single folding model

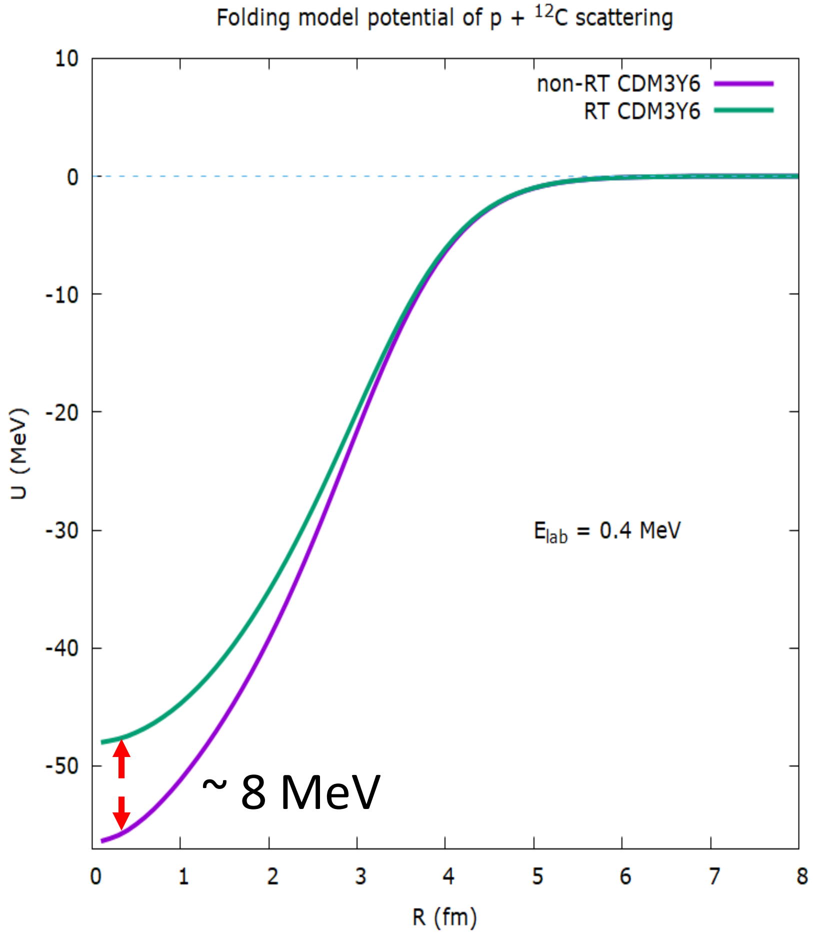
$$U = \sum_{j \in A} \langle \vec{k}, j | v_c^D | \vec{k}, j \rangle + \langle \vec{k}, j | v_c^{EX} | j, \vec{k} \rangle$$

$$U(\vec{R}) = \int d r \rho_A(\vec{r}) v_{NN}(\vec{r} - \vec{R}, \rho)$$

Two-body problem



mean-field

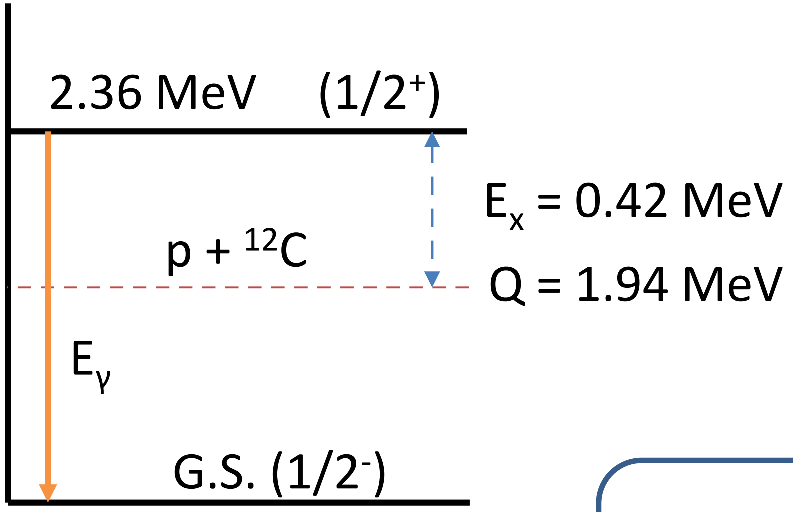

Folding model potential of p + ^{12}C scattering

E_{lab} = 0.4 MeV

~ 8 MeV

MEAN-FIELD DESCRIPTION OF THE $^{12}C(p,\gamma)^{13}N$ AND $^{13}C(p,\gamma)^{14}N$ REACTIONS

$^{12}C(p,\gamma)^{13}N$


2.36 MeV (1/2⁺)

p + ^{12}C

E_x = 0.42 MeV

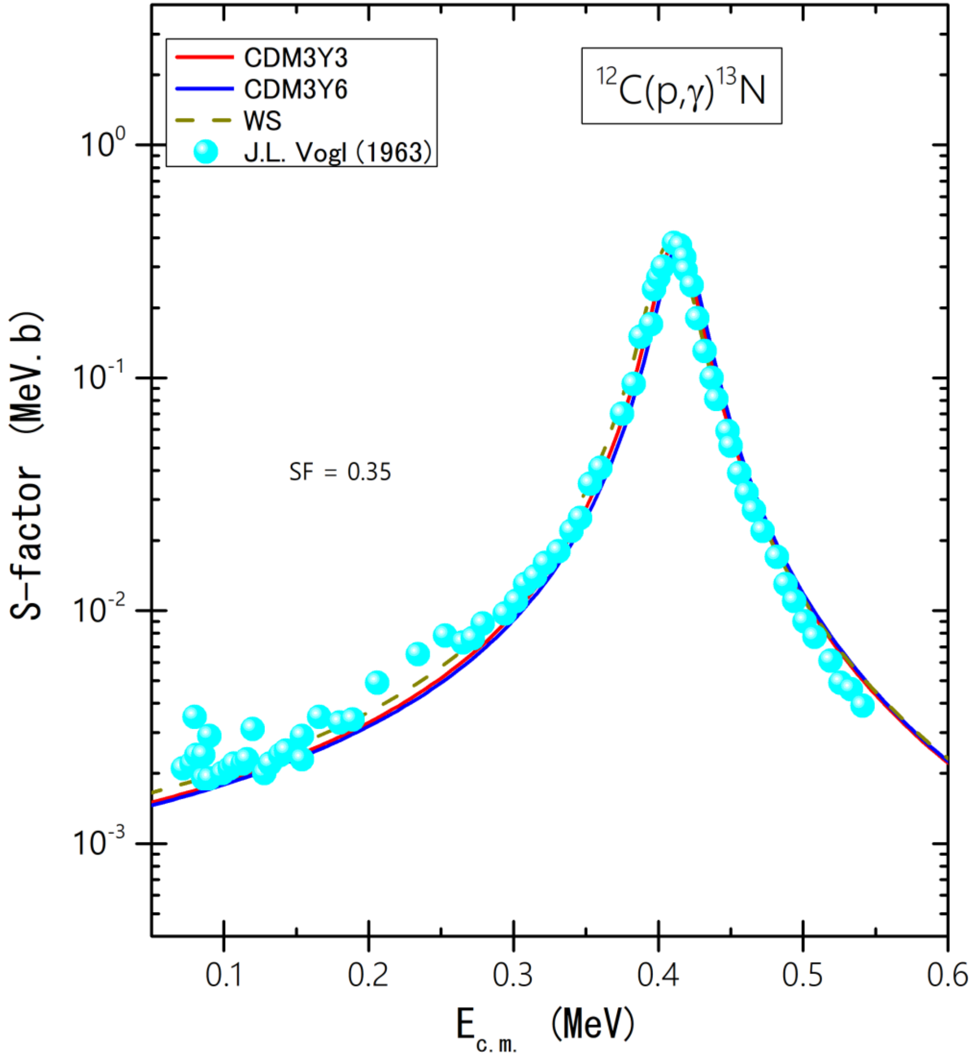
Q = 1.94 MeV

E_γ

G.S. (1/2⁻)

S-factor

$$S(E) = E\exp(2\pi\eta)\,\sigma(E)$$

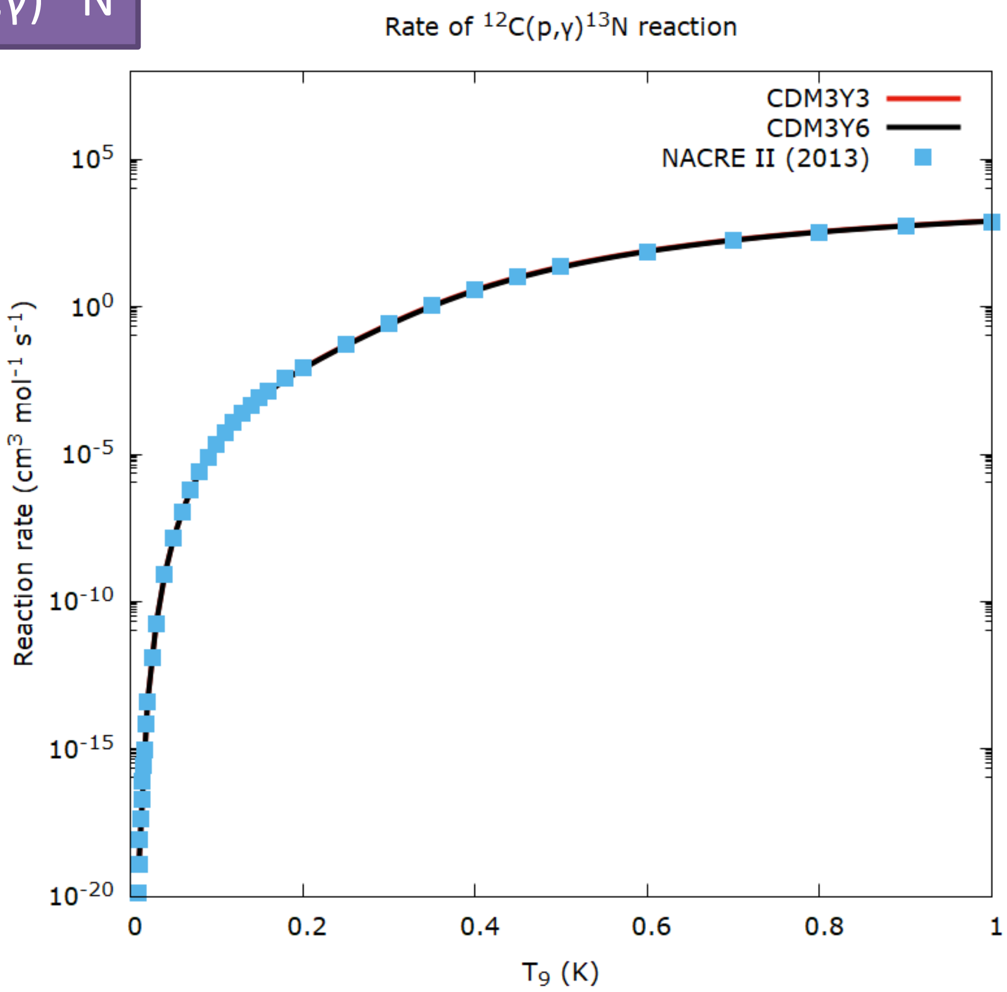


MEAN-FIELD DESCRIPTION OF THE ^{12}C(p,γ)^{13}N AND ^{13}C(p,γ)^{14}N REACTIONS

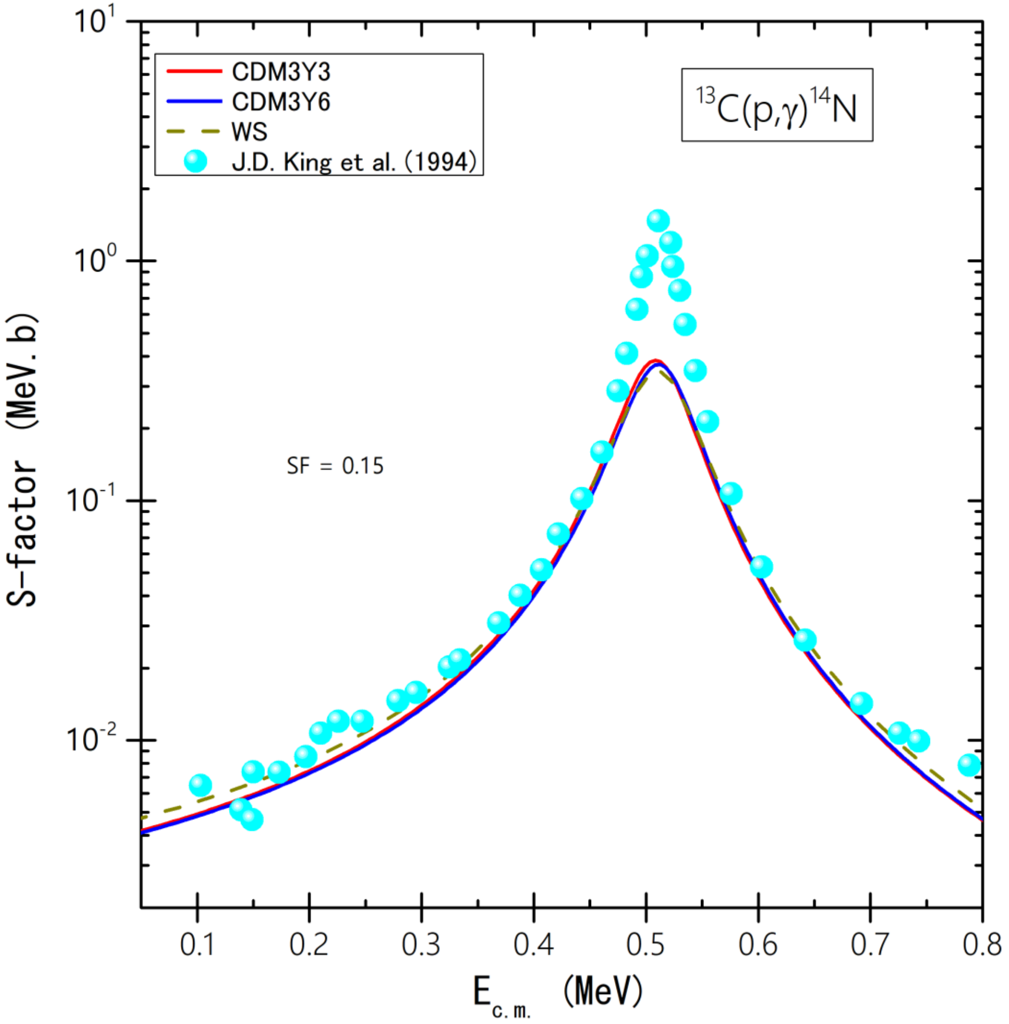
Reaction rate

$$\langle \sigma v \rangle = \left(\frac{8}{\pi\mu}\right)^{\frac{1}{2}} \frac{1}{(k_B T)^{\frac{3}{2}}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE$$

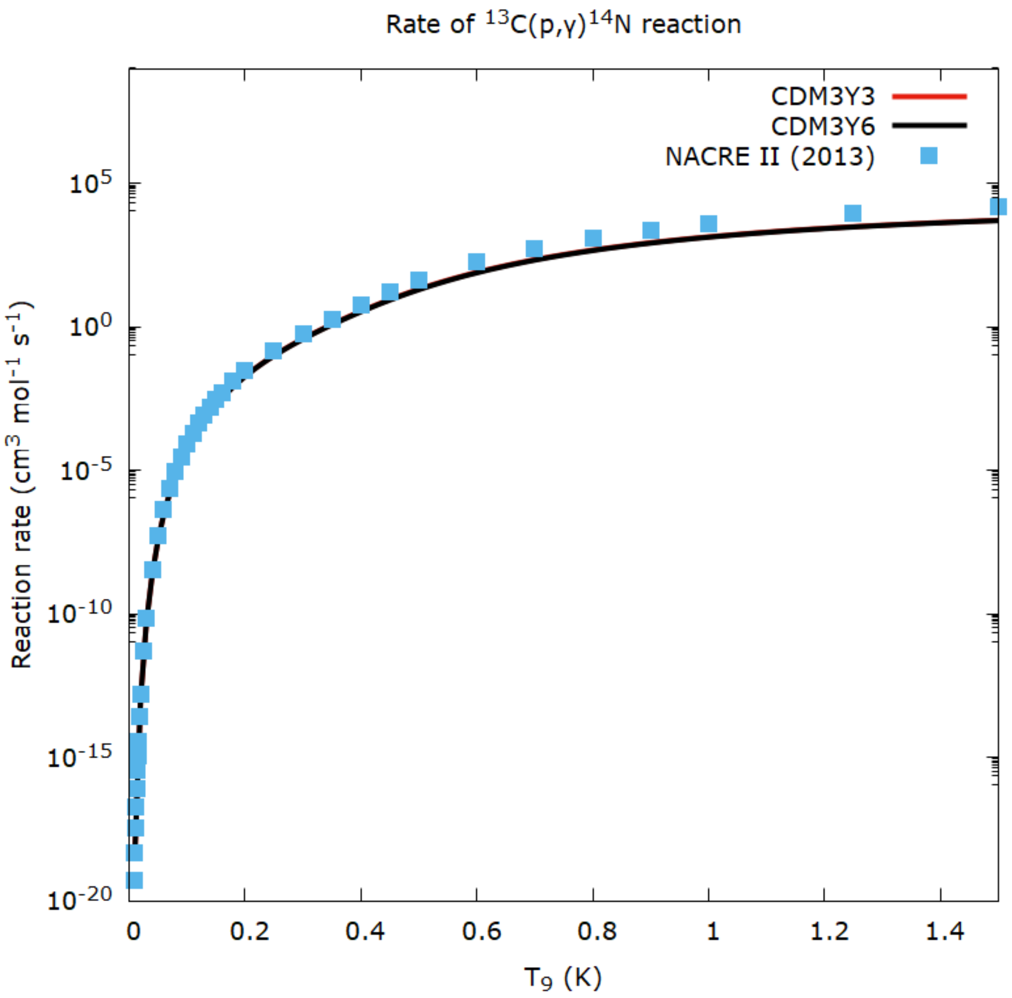
^{12}C(p,γ)^{13}N



MEAN-FIELD DESCRIPTION OF THE $^{12}C(p,\gamma)^{13}N$ AND $^{13}C(p,\gamma)^{14}N$ REACTIONS



$^{13}C(p,\gamma)^{14}N$



J. T. Huang et al. Atom.Data Nucl.Data Tabl. 96 (2010)

SUMMARY

This SFM approach is further used to calculate the nuclear mean-field potential for the study of the astrophysical S factor of the $^{12}C(p,\gamma)^{13}N$ and $^{13}C(p,\gamma)^{14}N$ reactions.

Reaction rates of the radiative capture reactions which are an importantly astrophysical quantity are produced to describe effectively the experimental data.

THANK YOU FOR YOUR ATTENTION!