$\pi \to \pi\pi$ transition generalized parton distributions and non-diagonal deeply virtual Compton scattering

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Generalized parton distributions (GPDs)

Electromagnetic form factor

$F(\Delta^2)$

1st Mellin moment of GPD

$\int dx\, H(x,\xi,\Delta^2)$

Ordinary parton distribution function (PDF)

Forward limit

$\Delta \to 0$

$q(x)$

Inclusive reaction such as the deep inelastic scattering

1-dimensional momentum distribution of parton

GPD (off-forward)

$H(x,\xi,\Delta^2)$

2nd Mellin moment of GPD

$\int dx\, x[H(x,\xi,\Delta^2) + E(x,\xi,\Delta^2)] = A(\Delta^2) + B(\Delta^2)$

Fourier transf. to impact parameter space

Angular momentum sum rule

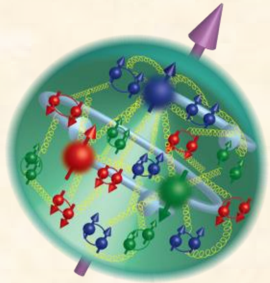
$J_{q,g} = \frac{1}{2}[A(0) + B(0)]$

Transverse spatial imaging

$\Delta \leftrightarrow b_T$

introduced by X. Ji, D. Mueller, and A. Radyushkin in the 90s

Δ dependence of GPD unlike PDF



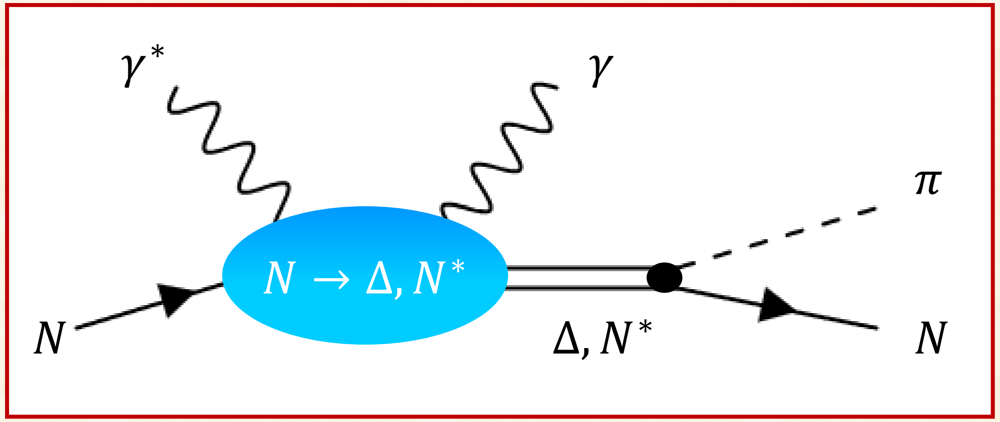
X. Ji, Phys. Rev. Lett. 78, 610 (1997)
X. Ji, Phys. Rev. D 55, 7114 (1997)
A. Radyushkin, Phys. Rev. D 56, 5524 (1997)
D. Mueller et al., Fortschr. Phys. 42, 101 (1994)

J. Ralston and B. Pire, Phys .Rev. D 66, 111501 (2002)

A, B: EMT form factors

x : momentum fraction of the active parton
ξ : skewedness (longitudinal component of momentum transfer)
Δ : momentum transfer between initial and final state hadrons

Non-diagonal deeply virtual Compton scattering



Transition GPDs in non-diagonal exclusive reactions

K. Goeke, M. Polyakov, and M. Vanderhaeghen,
Prog. Part. Nucl. Phys. 47, 401 (2001)

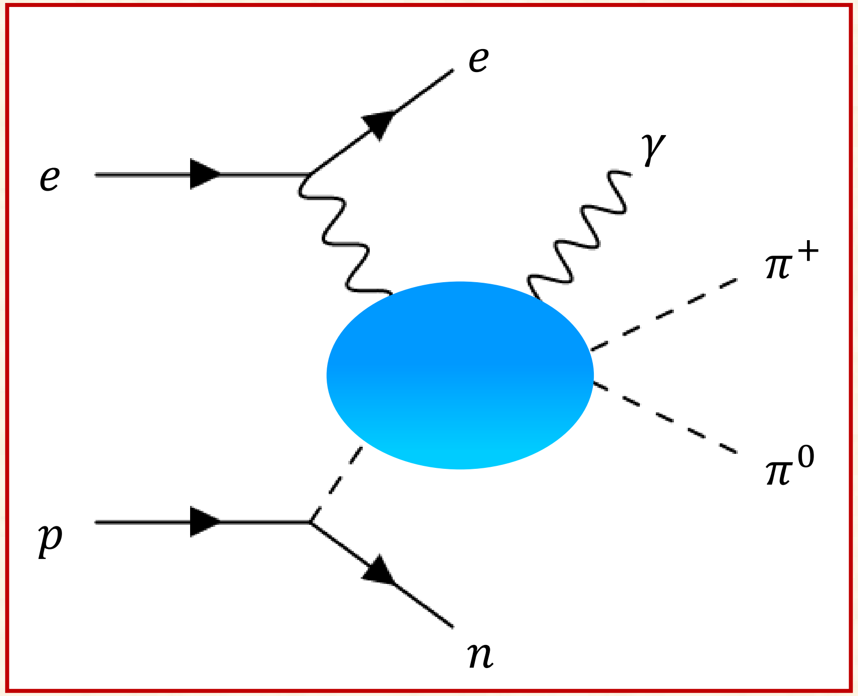
- ➢ Provide information about the dynamics of the hadron excitations in terms of quark and gluon degrees of freedom.

- ➢ Depend on more arguments such as the invariant mass and the angular structure, etc.

✓ In this work, we study the non-diagonal DVCS of $\gamma^*\pi \to \gamma\pi\pi$ to avoid complications due to target spin.

The Sullivan-type of process, $eN \to e\gamma N'\pi\pi$

- DVCS reaction of $e\pi \to e\gamma\pi\pi$ can be accessed through the pion emission from the Sullivan process.



- Near the threshold of pion production, the momentum transfer between nucleons is small.

Factorization into two subprocesses

$$\mathcal{M}_{ep\to en\gamma\pi\pi} = \mathcal{M}_{p\to n\pi}\mathcal{M}_{e\pi\to e\gamma\pi\pi}$$

✓ *The meson cloud can be approximated by the one-pion-exchange.*

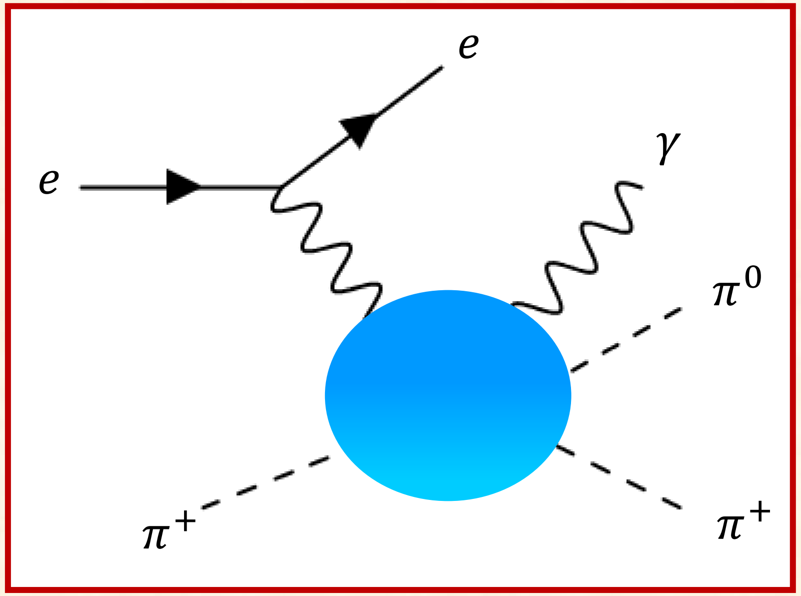
✓ *The intermediate pion is slightly off-shell.*

D. Amrath, M. Diehl, and J. P. Lansberg, Eur. Phys. J. C 58, 179 (2008)
J. Morgado et al., arXiv.2203.169472

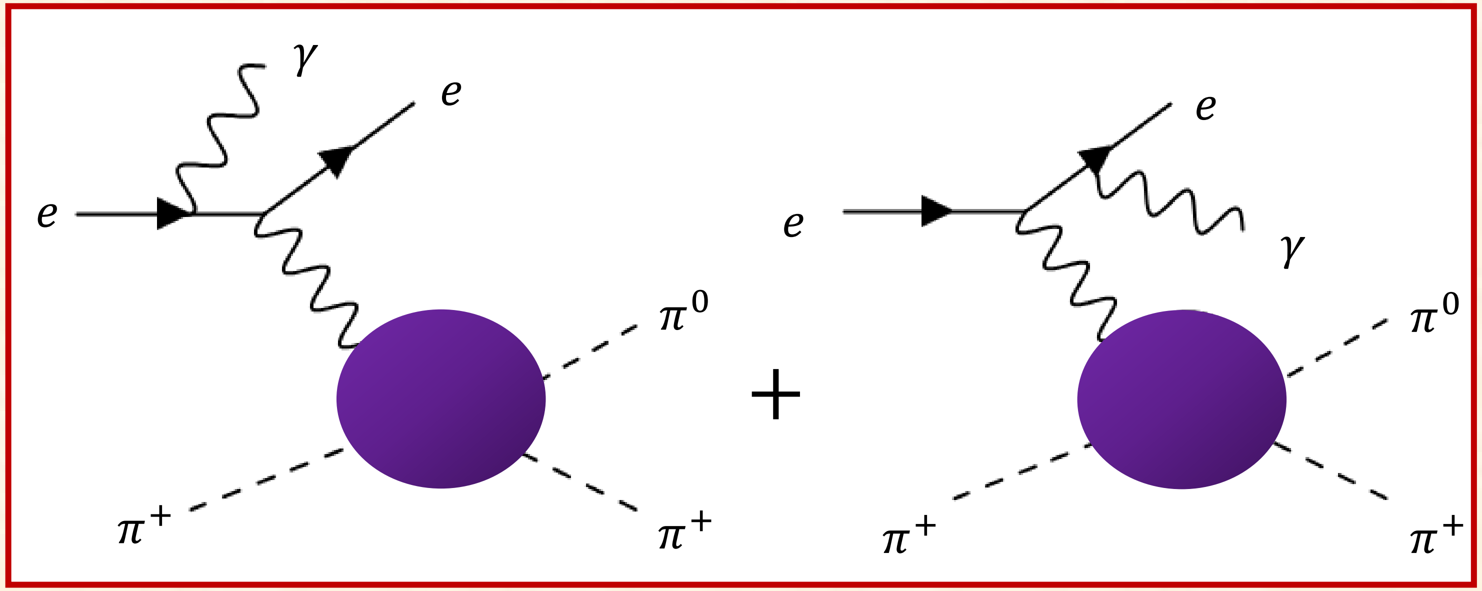
$$\left|\mathcal{M}_{e\pi\to e\gamma\pi\pi}\right|^2 = |BH|^2 + |DVCS|^2 + Re[BH^*\ DVCS]$$

- ✓ The interference term gives enhancement of the VCS signal
- ✓ Linearly proportional to the Compton FFs

DVCS (contains GPDs)



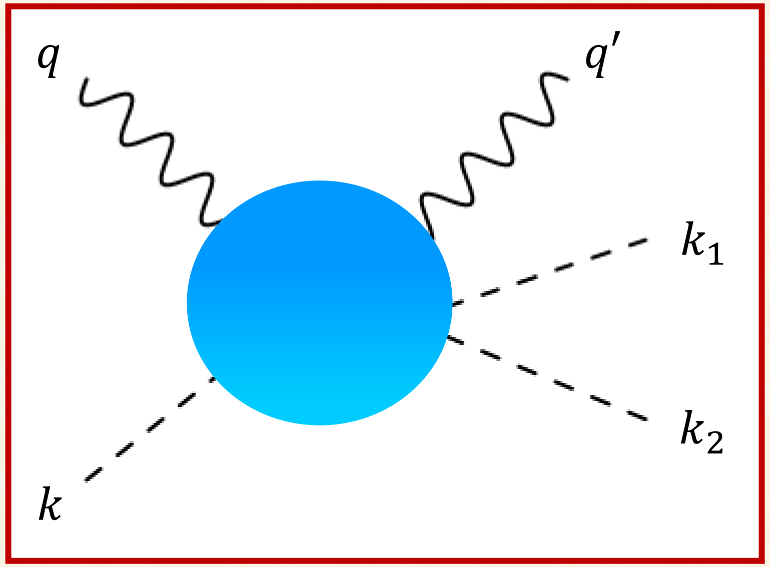
Bethe-Heitler (contains EM form factors)



Kinematics

5 kinematical invariants for $2 \to 3$ reaction

$$
\begin{aligned}
s &= (k+q)^2 = (q' + k_1 + k_2)^2, \\
t &= (q-q')^2 = (k_1 + k_2 - k)^2 \equiv \Delta^2, \\
W_{\pi\pi}^2 &= (k_1+k_2)^2, \\
t' &= (k-k_1)^2, \\
u' &= (k-k_2)^2.
\end{aligned}
$$



$$W_{\pi\pi}^2 + t' + u' = 3m_\pi^2 + \Delta^2$$

Quantify the fraction of longitudinal momentum of the final state pions

$$
\begin{aligned}
\alpha &= \frac{k_2 \cdot n}{(k_1+k_2)\cdot n} = \frac{k_2 \cdot n}{1-\xi} \\
1-\alpha &= \frac{k_1 \cdot n}{(k_1+k_2)\cdot n} = \frac{k_1 \cdot n}{1-\xi}
\end{aligned}
$$

- ✓ We treat $\pi^a(k)$ as a quasi-real state in this work
- ✓ The $\pi \to \pi\pi$ transition GPDs depend on this parameter and the invariant mass of 2 pion system.

$\pi \to \pi\pi$ transition GPDs

- We introduce the parameterizations of the $\pi \to \pi\pi$ transition GPDs up to the leading twist accuracy.

- 3+1 (un)polarized transition GPDs from the isovector and isoscalar lightcone operators.

  - ✓ Unpolarized and polarized isoscalar GPDs

$$
\begin{aligned}
& \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda n\cdot\bar{P}} \langle \pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle \\
= \; & \frac{1}{2\bar{P}\cdot n} i\epsilon(n,\bar{P},\Delta,k_1)\frac{1}{f_\pi^3} i\epsilon^{abc} H^{(S)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2), \\
& \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda n\cdot\bar{P}} \langle \pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\gamma_5\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle \\
= \; & \frac{1}{2\bar{P}\cdot n}(\bar{P}\cdot n)\frac{1}{f_\pi} i\epsilon^{abc} \tilde{H}^{(S)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2),
\end{aligned}
$$

ϵ: anti-symmetric tensor

$\epsilon(a,b,c,d) = \epsilon^{\mu\nu\alpha\beta} a_\mu b_\nu c_\alpha d_\beta$

$f_\pi = 93$ MeV: pion decay const.

$\pi \to \pi\pi$ transition GPDs

- The GPDs are defined along the longitudinal component of the lightcone operator at the leading twist.
- 6 arguments: the variables α and t' contain the decay angles of $\pi\pi$ system.

✓ Unpolarized and polarized isovector GPDs

$$
\begin{aligned}
& \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda n\cdot\bar{P}} \langle \pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\tau^d\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle \\
= & \frac{1}{2\bar{P}\cdot n} i\epsilon(n,\bar{P},\Delta,k_1)\frac{1}{f_\pi^3}\Big[\delta_{ab}\delta_{cd}H_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)+\delta_{ac}\delta_{bd}H_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) \\
& +\delta_{ad}\delta_{bc}H_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)\Big], \\
& \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda n\cdot\bar{P}} \langle \pi^b(k_1)\pi^c(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\gamma_5\tau^d\psi\left(\frac{\lambda n}{2}\right)|\pi^a(k)\rangle \\
= & \frac{1}{2\bar{P}\cdot n}(\bar{P}\cdot n)\frac{i}{f_\pi}\Big[\delta_{ab}\delta_{cd}\tilde{H}_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)+\delta_{ac}\delta_{bd}\tilde{H}_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) \\
& +\delta_{ad}\delta_{bc}\tilde{H}_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2)\Big],
\end{aligned}
$$

- We investigated the symmetric properties of these transition GPDs by interchanging two pions in the final state and using the invariance under the charge conjugate operation.

1. $k_1 \leftrightarrow k_2$ implies $t' \leftrightarrow u'$ and $\alpha \leftrightarrow 1-\alpha$
2. Exchange of k_1 and k_2 must be equivalent to that of isospin indices b and c

✓ Symmetric properties of the polarized transition GPDs under $x \to -x$ and $\alpha \to 1-\alpha$

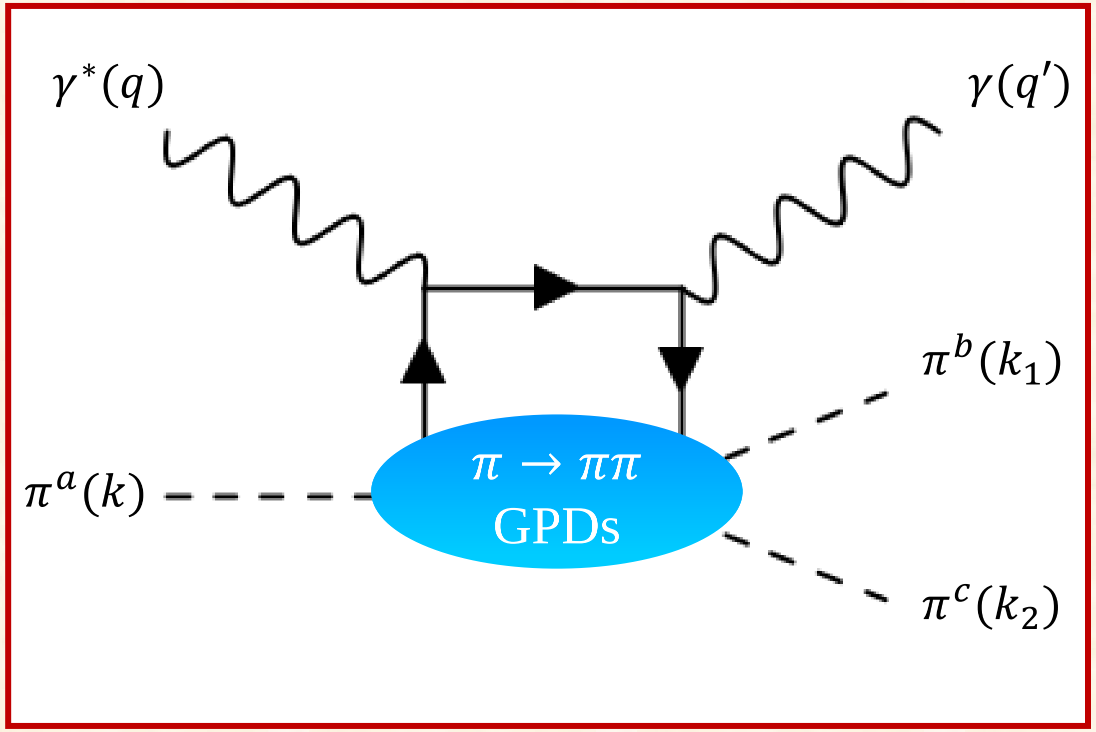
$$
\begin{aligned}
\tilde{H}^{(S)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= -\tilde{H}^{(S)}(-x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) = -\tilde{H}^{(S)}(x,\xi,1-\alpha,u',\Delta^2,W_{\pi\pi}^2),\\
\tilde{H}_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= \tilde{H}_1^{(V)}(-x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) = \tilde{H}_2^{(V)}(x,\xi,1-\alpha,u',\Delta^2,W_{\pi\pi}^2),\\
\tilde{H}_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= \tilde{H}_2^{(V)}(-x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) = \tilde{H}_1^{(V)}(x,\xi,1-\alpha,u',\Delta^2,W_{\pi\pi}^2),\\
\tilde{H}_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= \tilde{H}_3^{(V)}(-x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) = \tilde{H}_3^{(V)}(x,\xi,1-\alpha,u',\Delta^2,W_{\pi\pi}^2).
\end{aligned}
$$

✓ Similar for the unpolarized ones

DVCS amplitude

The hadronic tensor is required to calculate the DVCS amplitude

$$H^{\mu\nu}(\gamma^*\pi^a \to \gamma\pi^b\pi^c) \equiv -i\int d^4x e^{iq'\cdot x}\langle\pi^b(k_1)\pi^c(k_2)|\mathcal{T}\{J^\mu(x)J^\nu(0)\}|\pi^a(k)\rangle$$



- ✓ Factorized into the hard part and the soft part at high value of photon virtuality

- The GPDs are convoluted with the hard kernel in the hadronic tensor.

- The convolution of C^+ (C^-) and $H^{(S)}$ ($\widetilde{H}^{(S)}$) vanish due to the symmetry properties of the hard kernel under $x \to -x$.

➡ *The isoscalar $\pi \to \pi\pi$ GPDs need to be investigated from other hard exclusive reactions.*

$$H^{\mu\nu}(\gamma^*\pi^\pm \to \gamma\pi^\pm\pi^0) = -\frac{g_\perp^{\mu\nu}}{2}\frac{1}{f_\pi^3} i\epsilon(n,\bar{P},\Delta,k_1)\int_{-1}^{1} dx\, C^+(x,\xi)\left(\frac{5}{18}H^{(S)} + \frac{1}{6}H_1^{(V)}\right) + \frac{i}{2}\epsilon_\perp^{\mu\nu}\frac{1}{f_\pi}\int_{-1}^{1} dx\, C^-(x,\xi)\left(\frac{5}{18}\tilde{H}^{(S)} + \frac{i}{6}\tilde{H}_1^{(V)}\right),$$

$$C^\pm(x,\xi) \equiv \frac{1}{x-\xi+i\epsilon} \pm \frac{1}{x+\xi-i\epsilon}$$: Hard kernel

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \tilde{p}^\mu n^\nu - \tilde{p}^\nu n^\mu,$$
$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\tilde{p}_\rho n_\sigma.$$

Soft-pion theorem

- Near the two-pion threshold, $W_{\pi\pi} = 2m_\pi$, the emitted pion is *soft*.

- The soft-pion theorem provides the normalization conditions of $\pi \to \pi\pi$ transition GPDs at threshold in terms of the pion GPD.

P. Pobylitsa, M. Polyakov, and M. Strikman, Phys. Rev. Lett. 87, 022001 (2001)

- PCAC relation lets us to write the pion field in terms of the axial current and by the LSZ reduction *soft pion reduces to the chiral rotation of the operator.*

→ *Soft-pion theorem*

No poles in this case

$$\langle \pi^b(k_1)\pi^c(k_2)|\mathcal{O}(z)|\pi^a(k)\rangle \Big|_{k_2\to 0} = -\frac{i}{f_\pi}\langle \pi^b(k_1)|[Q_5^c, \mathcal{O}(z)]|\pi^a(k)\rangle + k_2^\mu R_\mu^c(k_2)\Big|_{k_2\to 0}$$

✓ The chiral rotation of the isoscalar (isovector) lightcone operator

$$[Q_5^a, \bar\psi(0)\gamma^\mu(1,\gamma_5)\tau^b\psi(z)] = i\epsilon^{abc}\bar\psi(0)\gamma^\mu(\gamma_5, 1)\tau^c\psi(z)$$

$$[Q_5^a, \bar\psi(0)\gamma^\mu(1,\gamma_5)\psi(z)] = 0$$

Q_5^a: axial charge

$R^a(k_2)$: pole contribution

Soft-pion theorem

Pion GPD in the leading twist

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{iy\lambda n\cdot\bar{P}_\pi} \langle \pi^b(p'_\pi)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\not{n}\tau^c\psi\left(\frac{\lambda n}{2}\right)|\pi^a(p_\pi)\rangle = 2(\bar{P}_\pi\cdot n)i\epsilon^{abc}H_\pi^{(V)}(y,\zeta,t_\pi)$$

- ✓ No polarized pion GPD due to the parity invariance.
- ✓ $\pi \to \pi\pi$ transition GPDs is normalized by the usual pion GPD at the threshold.

$$\zeta_1 = \frac{2\xi-(1-\xi)\alpha}{2-(1-\xi)\alpha} \quad \text{and} \quad \zeta_2 = \frac{2\xi-(1-\xi)(1-\alpha)}{2-(1-\xi)(1-\alpha)}$$

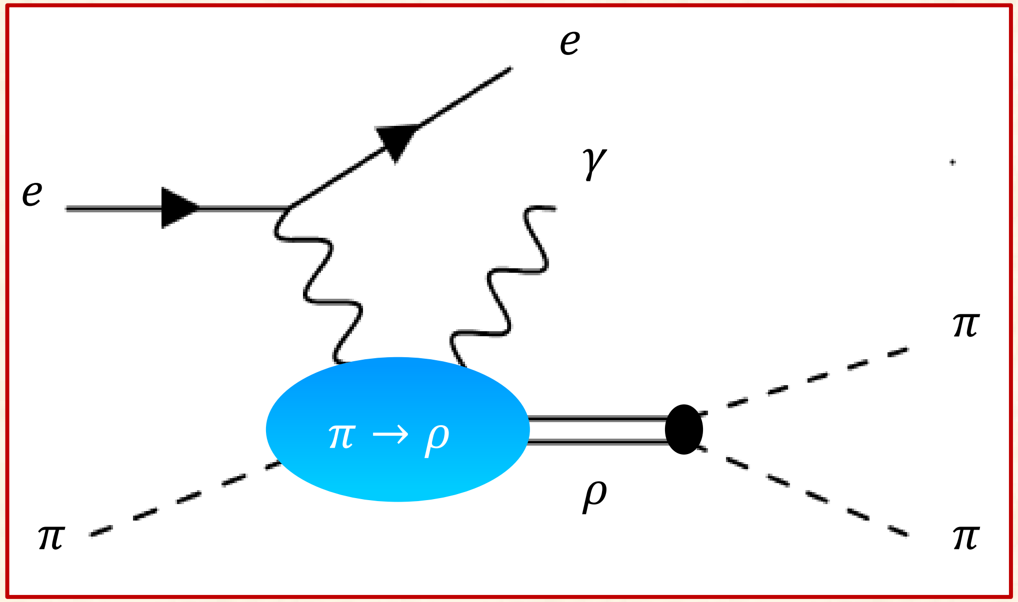
Ex) In the case that $\pi(k_2)$ is taken to be soft

$$\begin{aligned}
\tilde{H}_1^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= 0\\
\tilde{H}_2^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= 2n\cdot(k+k_1)H_\pi^{(V)}\left(\frac{2x}{n\cdot(k+k_1)},\zeta_1,t'\right)\theta\left(1-\left|\frac{2x}{n\cdot(k+k_1)}\right|\right)\\
\tilde{H}_3^{(V)}(x,\xi,\alpha,t',\Delta^2,W_{\pi\pi}^2) &= -2n\cdot(k+k_1)H_\pi^{(V)}\left(\frac{2x}{n\cdot(k+k_1)},\zeta_1,t'\right)\theta\left(1-\left|\frac{2x}{n\cdot(k+k_1)}\right|\right)
\end{aligned}$$

$\pi \to \rho$ transition

- As ρ meson is likely to decay into two pions the $\pi \to \rho$ transition in the intermediate resonance state can be considered.

- $\pi \to \rho$ transition GPDs (FFs) are accessed through the VCS (BH) amplitude.



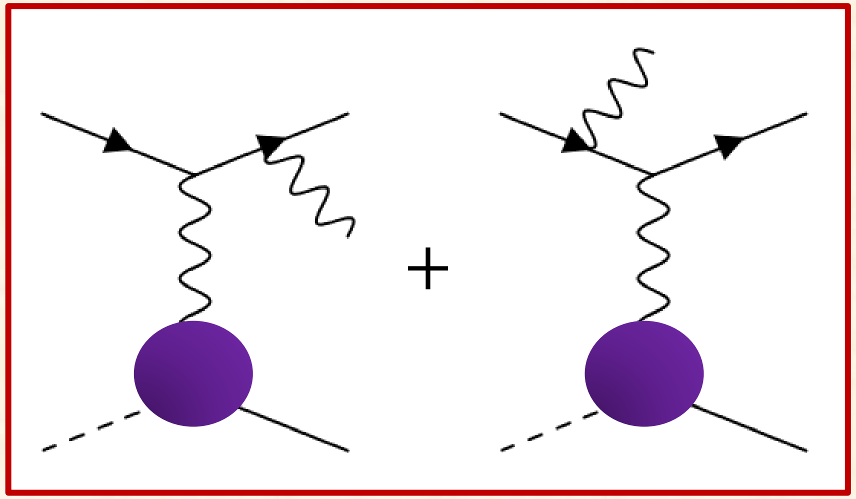
✓ We first put the decay part aside for simplicity and investigate the size of the BH cross section.

Bethe-Heitler cross section

A. Khodjamirian, Eur. Phys. J. C 6, 477 (1999)

I. Danilkin, C. Redmer, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 107, 20 (2019)

$\pi \rightarrow \rho$ BH contribution



$\pi \rightarrow \rho$ transition form factor

$$F_{\rho\pi}(\Delta^2)\Delta^4 = \frac{A^{\rho\pi}}{1 - B^{\rho\pi}/\Delta^2 + C^{\rho\pi}/\Delta^4}$$

C_V: $\gamma\rho\pi$ coupling
x_B: Bjorken variable
y: lepton energy loss faction
$\epsilon \equiv 2m_\pi x_B/Q$

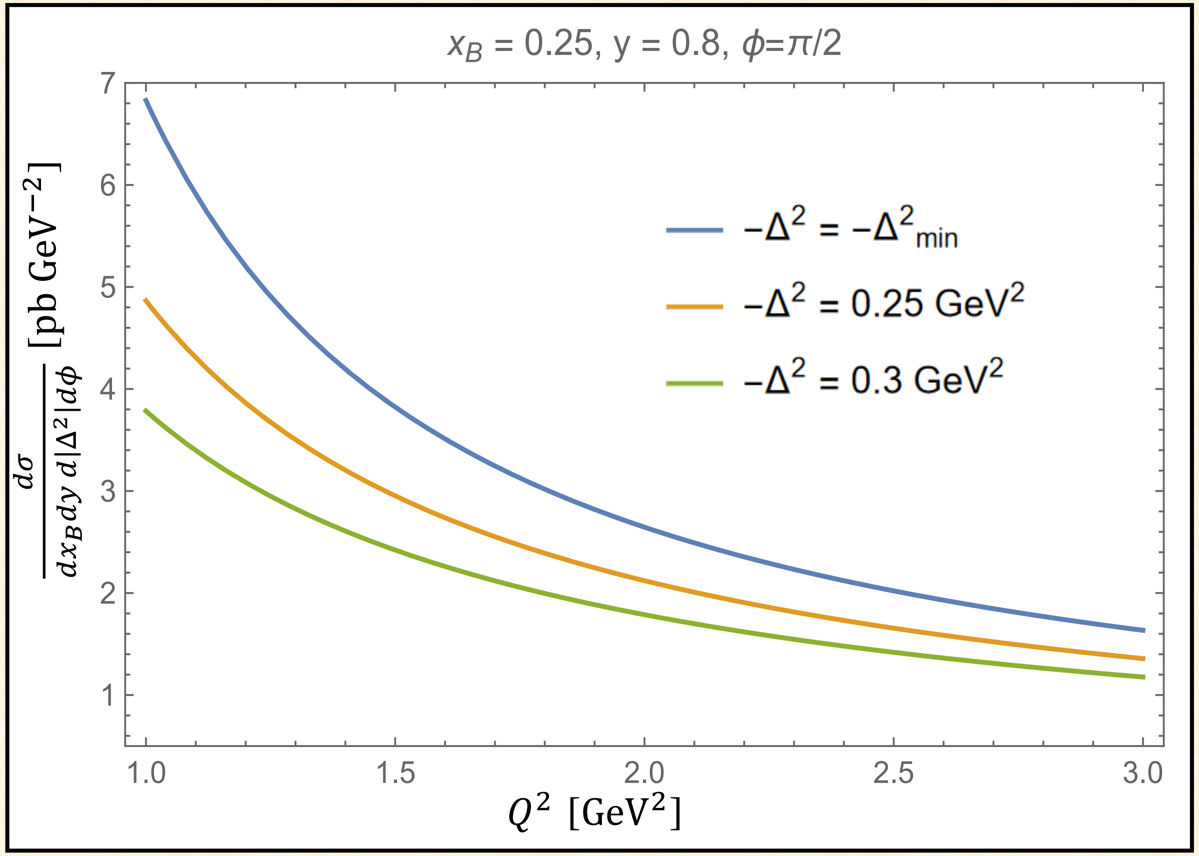
$A^{\rho\pi} = 0.92\ \mathrm{GeV}^4$
$B^{\rho\pi} = 3.96\ \mathrm{GeV}^2$
$C^{\rho\pi} = 2.48\ \mathrm{GeV}^2$

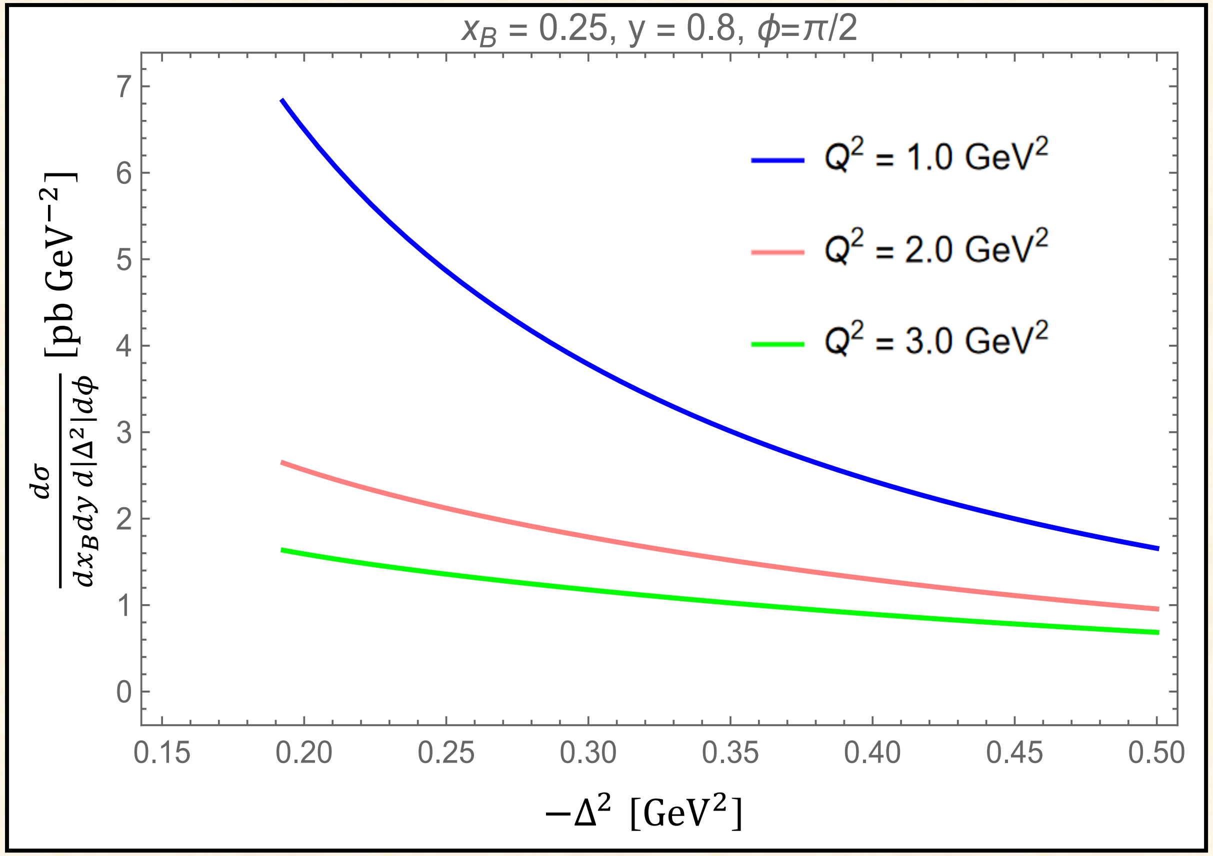
$$\mathcal{M}_{BH} = \frac{2e^3 C_V^2}{\Delta^2} F_{\rho\pi}(\Delta^2)\epsilon_{\mu\alpha\beta\gamma}p_2^\alpha p_1^\beta \epsilon^{*\gamma}(p_2)\epsilon_\nu^*(q_2)\bar{u}(k')\left[\gamma^\nu \frac{1}{\not{k}' + \not{q}_2}\gamma^\mu + \gamma^\mu \frac{1}{\not{k} - \not{q}_2}\gamma^\nu\right]u(k)$$

A. Belitsky et al., Phys. Rev. D 64, 116002 (2001)

$$|\mathcal{M}_{BH}|^2 = 2C_V^2 \frac{F_{\rho\pi}^2(\Delta^2)}{x_B^2 y^2 (1+\epsilon^2)^2 \Delta^2 \mathcal{P}_1 \mathcal{P}_2} \sum_{m=0}^{2} c_m^{BH} K^m \cos(m\phi)$$

✓ Following the approach by Belitsky et al., we express it by a series of $K^m \cos(m\phi)$





4-fold diff. cross section

$$\frac{d\sigma}{dx_B dy d|\Delta^2| d\phi} = \frac{\alpha_{em}^3 x_B y}{8\pi Q^2 \sqrt{1+\epsilon^2}} |\mathcal{M}|^2$$

- Increasing cross section as $-\Delta^2$ decreases as expected
- Values about few picobarn

Summary

- The transition GPDs arise from the non-diagonal hard exclusive reaction can be used as a tool to investigate the physics of the hadron excitation at the fundamental level.

- We study the $\pi \to \pi\pi$ GPDs in $e\pi \to e\gamma\pi\pi$ reaction which is not only the theoretical interest as a spinless hadron example but, also can be accessed through the Sullivan-type reaction in experiments in JLab and at future EIC.

- Symmetry properties of the transition GPDs are addressed and relation to the usual pion GPD near the threshold is studied by the soft-pion theorem.

- As $\pi \to \rho$ meson transition can occur with subsequent decay into two pions we calculated $\pi \to \rho$ BH cross section to estimate its size roughly. *BH amplitude can be seen as amplifier for the DVCS signal through the interference term.*

→ One needs to include the $\rho \to \pi\pi$ decay part.