# Gamow-Teller transition described by the Monte-Calro shell model

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Most of the elements heavier than iron are produced through the rapid neutron capture process(r-process). The r-process path passes through the neutron-rich side of $^{132}$Sn. We compute the low-energy nuclear properties contributing to the $\beta$-decay half lives by using the Monte-Carlo shell model (MCSM). In this paper, we focus on the southwest region of $^{132}$Sn. We employ the same model space with the large-scale shell model calculation in Ref. [1], which consists of the five neutron orbitals betweeen $N = 50$ and 82 and the nine proton orbitals between $Z = 28$ and 82. The effective interaction is taken from the SNV interaction and the remaining part is given by the phenomenological VMU interaction. Moreover, the monopole interaction strengths are decreased by 10% to reproduce the low-energy spectra as shown in Fig. 1.

The upper pannel of Fig. 2 shows the Gamow-Teller transition strength of $^{130}$Cd. In the MCSM calculation, we adopt several tens of basis vectors optimized for low-energy eigenstates. This prescription is not enough to reproduce the Gamow-Teller transition. We utilize an unitary transformation for the many-body bases to describe the states strongly connected to the ground state of the parent nucleus through the Gamow-Teller transition operator.

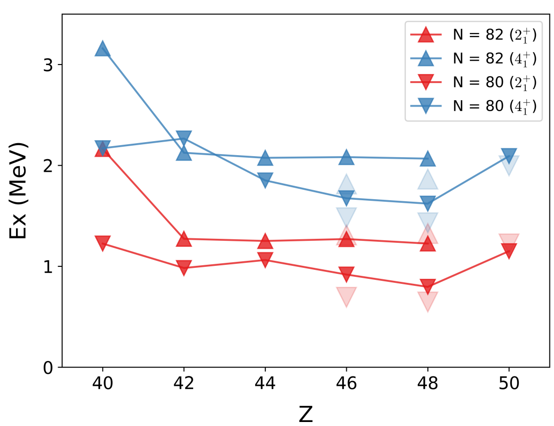


Figure 1. The energy spectra of $N = 80$ and 82 nuclei. The results of MCSM calculations are shown with filled symbols.

The Gamow-Teller transition operator is defined by

$$G_{\pm,\mu} = \sum_{a=1}^{A} \tau_\pm(a)\sigma_\mu(a), \tag{1}$$

where $\sigma_\mu(a)$ and $\tau_\pm(a)$ denote the spin and isospin operators, and $G^\dagger_{+,\mu} = G_{-,\mu}$. Using the commutation relations

$$\sum_{i,j=1}^{3} [G_{+,i}, G_{-,j}] = \sum_{a=1}^{A} 2i\varepsilon_{ijk}\sigma_k(a)\tau_+(a)\tau_-(a) + \sum_{a=1}^{A} \sigma_i(a)\sigma_j(a)\tau_z(a), \tag{2}$$

the sum rule is given as

$$\begin{aligned}\sum_{\mu,\nu=0,\pm1} &\langle\psi_{J,M}|[G_{+,\mu}, G_{-,\nu}]|\psi_{J,M}\rangle \\ &= \sum_{i=1}^{3} \langle\psi_{J,M}|G_{+,i}|\psi_n\rangle\langle\psi_n|G_{-,i}|\psi_{J,M}\rangle \\ &\quad - \sum_{i=1}^{3} \langle\psi_{J,M}|G_{-,i}|\psi_n\rangle\langle\psi_n|G_{+,i}|\psi_{J,M}\rangle \\ &= 3(N-Z),\end{aligned} \tag{3}$$

where $|\psi_{J,M}\rangle$ is the ground state and $|\psi_n\rangle$ is the intermediate states of the daughter nucleus. Our goal is to construct a set of basis vectors $\{|\psi_n\rangle\}$ that almost exhausts the sum rule.

In the MCSM calculation, the ground state is expressed with deformed Slater determinants, one of which is given by

$$|\phi\rangle = \prod_{k=1}^{N_f} a_k^\dagger |-\rangle, \tag{4}$$

$$a_k^\dagger = \sum_{\alpha=1}^{N_{\rm sp}} D_{\alpha k} c_\alpha^\dagger, \tag{5}$$

where $|-\rangle$ denotes the inert core occupied by a definite number of nucleons. The number of nucleons within the model space is $N_f$ and the number of the single-particle states is $N_{\rm sp}$.

The deformed Slater determinant is decomposed into the proton and neutron parts as

$$|\phi\rangle = |\phi^{(\pi)}\rangle \otimes |\phi^{(\nu)}\rangle, \tag{6}$$

where

$$|\phi^{(\tau)}\rangle = \prod_{k=1}^{N_\tau} a_{\tau,k}^\dagger |-\rangle, \tag{7}$$

$$a_{\tau,k}^\dagger = \sum_{\alpha=1}^{N_{\rm sp}^{(\tau)}} D_{\alpha k}^{(\tau)} c_{\tau,\alpha}^\dagger, \tag{8}$$

for $\tau = \pi, \nu$. Using the QR decomposition algorithm, we can construct the $N_{\rm sp}^{(\tau)} \times N_{\rm sp}^{(\tau)}$ unitary matrix $\overline{D}^{(\tau)}$ extended from $D^{(\tau)}$. The creation operator can then be extended for $k = 1, 2, \cdots, N_{\rm sp}^{(\tau)}$ as

$$a^\dagger_{\tau,k} = \sum_{\alpha=1}^{N_{\rm sp}^{(\tau)}} \overline{D}^{(\tau)}_{\alpha k} c^\dagger_{\tau,\alpha}. \tag{9}$$

The Gamow-Teller operator can be expressed as

$$\begin{aligned} G_{-,\mu} &= \sum_{\alpha\beta} \langle \alpha | \tau_- \sigma_\mu | \beta \rangle c^\dagger_\alpha c_\beta \\ &= \sum_{i=1}^{N_{\rm sp}^{(\pi)}} \sum_{j=1}^{N_{\rm sp}^{(\nu)}} \langle \pi, i | \sigma_\mu | \nu, j \rangle a^\dagger_{\pi,i} a_{\nu,j}, \end{aligned} \tag{10}$$

where the one-body matrix elements for the new basis vectors are defined by

$$\langle \pi, i | \sigma_\mu | \nu, j \rangle = \sum_{\alpha\beta} \langle \pi, \alpha | \sigma_\mu | \nu, \beta \rangle \overline{D}^{(\pi)*}_{\alpha i} \overline{D}^{(\nu)}_{\beta j}. \tag{11}$$

Considering a basis vector $|\phi\rangle$,

$$G_{-,\mu} |\phi\rangle = \sum_{j=1}^{N_\nu} |\phi_{\mu j}\rangle, \tag{12}$$

where

$$|\phi_{\mu j}\rangle = \prod_{k=1}^{N_\pi} a^\dagger_{\pi,k} b^\dagger_{\pi,\mu j} \prod_{k(\neq j)}^{N_\nu} a^\dagger_{\nu,k} |-\rangle, \tag{13}$$

$$b^\dagger_{\pi,\mu j} = \sum_{\alpha=1}^{N_{\rm sp}^{(\pi)}} B^{(\pi)}_{\mu,\alpha j} c^\dagger_{\pi,\alpha}, \tag{14}$$

$$B^{(\pi)}_{\mu,\alpha j} = (-1)^{j-1} \sum_{i=N_\pi+1}^{N_{\rm sp}^{(\pi)}} \overline{D}^{(\pi)}_{\alpha i} \langle \pi, i | \sigma_\mu | \nu, j \rangle. \tag{15}$$

To completely satisfy the sum rule, we need the $3N_\nu$ basis vectors for each $|\phi\rangle$. The total dimension $3N_\nu \times N_{\rm g.s.}$ becomes large as the number of basis vectors used to describe the ground state denoted by $N_{\rm g.s.}$. We should economize the computational costs by selecting a small number of basis vectors that almost satisfy the sum rule.

When the ground state is expressed with only one basis vector, the sum rule is given by

$$\langle \phi | G_{+,\mu} G_{-,\mu} | \phi \rangle = \sum_{i,j=1}^{N_\nu} \langle \phi_{\mu i} | \phi_{\mu j} \rangle. \tag{16}$$

This is conserved with the unitary transformation of the neutron single-particle levels

$$\begin{aligned} \sum_{j=1}^{N_\nu} |\phi_{\mu j}\rangle &= \det(U) \sum_{j=1}^{N_\nu} |\phi_{\mu j}\rangle \\ &= \sum_{j=1}^{N_\nu} |\phi'_{\mu j}\rangle. \end{aligned} \tag{17}$$

We utilize the unitary matrix $U$ to diagonalize the matrix (16), and adopt 50 bases from the largest eigenvalue. The lower pannel of Fig. 2 shows the Gamow-Teller transition strengths obtained by this prescription, where the sum rule is almost satisfied.

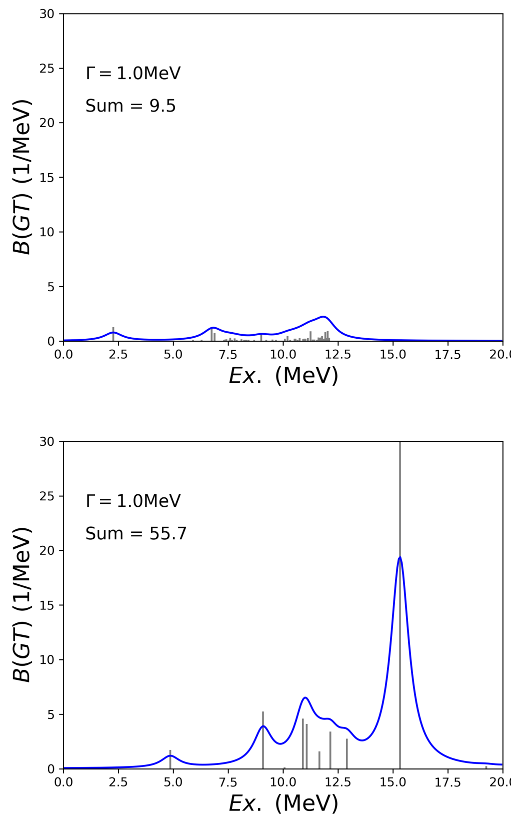


Figure 2. The Gamow-Teller transition strength of $^{130}$Cd. The blue lines are the folded strength functions by a Lorentzian function with 1 MeV width.

## References


[1] N. Shimizu, T. Togashi, and Y. Utsuno, PTEP **2021** (2021) 033D01.